Multiobjective Optimization of Three-Stage Spur Gear Reduction Units Using Interactive Physical Programming

Hong-Zhong Huang*
School of Mechatronics Engineering, University of Electronic Science and Technology of China, Chengdu, Sichuan 610054, P.R. China
Zhi-Gang Tian, Ming J. Zuo
Department of Mechanical Engineering, University of Alberta, Edmonton, Alberta, T6G 2G8, Canada

The preliminary design optimization of multi-stage spur gear reduction units has been a subject of considerable interest, since many high-performance power transmission applications (e.g., automotive and aerospace) require high-performance gear reduction units. There are multiple objectives in the optimal design of multi-stage spur gear reduction unit, such as minimizing the volume and maximizing the surface fatigue life. It is reasonable to formulate the design of spur gear reduction unit as a multi-objective optimization problem, and find an appropriate approach to solve it. In this paper an interactive physical programming approach is developed to place physical programming into an interactive framework in a natural way. Class functions, which are used to represent the designer's preferences on design objectives, are fixed during the interactive physical programming procedure. After a Pareto solution is generated, a preference offset is added into the class function of each objective based on whether the designer would like to improve this objective or sacrifice the objective so as to improve other objectives. The preference offsets are adjusted during the interactive physical programming procedure, and an optimal solution that satisfies the designer's preferences is supposed to be obtained by the end of the procedure. An optimization problem of three-stage spur gear reduction unit is given to illustrate the effectiveness of the proposed approach.

Key Words : Interactive Physical Programming, Physical Programming, Multiobjective Optimization, Three-Stage Spur Gear Reduction Unit, Preference

1. Introduction

The preliminary design optimization of multi-stage spur gear reduction units has been a subject of considerable interest, since many high-performance power transmission applications (e.g., automotive and aerospace) require high-performance gear reduction units (David et al., 2000). There are multiple objectives in the optimal design of multi-stage spur gear reduction unit, such as minimizing the volume and maximizing the surface fatigue life. It is reasonable to formulate the design of spur gear reduction unit as a multi-objective optimization problem, and find an appropriate approach to solve it.

Park, Kim, and Choi proposed a new decomposition method for parallel processing of multidisciplinary design optimization, such as collaborative optimization (CO) and individual discipline feasible (IDF) method (Park et al., 2002). Park, Lee, and Choi proposed a decomposition method which adaptively determines the number

* Corresponding Author,
E-mail : hzhuang@uestc.edu.cn
TEL : +86-28-83206916; **FAX** : +86-28-83206916
School of Mechatronics Engineering, University of Electronic Science and Technology of China, Chengdu, Sichuan 610054, P.R. China. (Manuscript **Received** August 31, 2004; **Revised** March 31, 2005)

and sequence of analyses in each sub-problem corresponding to the available number of processors in parallel (Lee et al., 2004). Physical programming has shown its effectiveness and ease of use in a variety range of problems (Chen et al., 2000 ; Messac, 1996 ; Messac et al., 2001). Physical programming captures the designer's preferences using class functions with physically meaningful parameters. Changing the class functions of design objectives, each Pareto points on the Pareto frontier can be reached. Physical programming is a non-interactive approach. However, the designer's knowledge on the multiobjective optimization problem is increasing during the design process. After obtaining a Pareto design, the designer may want to explore other Pareto designs around it, in order to improve some objectives, or sacrifice some objectives to improve the other objectives. This entails the interactive multiobjective optimization design.

Huang and Tian developed an interactive physical programming approach, and applied it to the design optimization of gear reduction unit (Tian et al., 2002) and reliability and redundancy allocation problem (Huang et al., 2004). This approach first placed physical programming into an interactive framework and attained more flexibilities. However, the way this approach used to adjust the optimization model during the interactive process is not so natural. After obtaining a Pareto solution, for instance, if the designer would like to improve an objective to be minimized, the preference ranges of this objective will move to the left by the same distance. This objective will be improved by adjusting its class function setting in this way, but the resulted preference ranges can not actually represent the designer's preferences on this objective anymore.

In this paper, we propose another interactive physical programming approach that adjusts the physical programming model in a natural way. The preference ranges for each objective are fixed during the interactive physical programming procedure. That is, the preference ranges will actually represent the designer's preferences on the objectives all through the optimization process. After a Pareto solution is generated, a preference offset is added into the class function of each objective based on whether the designer would like to improve this objective or sacrifice the objective so as to improve other objectives. These preference offsets, in fact, represent the designer's tradeoff on the design objectives. The preference offsets are adjusted during the interactive physical programming procedure, and an optimal solution that satisfies the designer's preferences is supposed to be obtained by the end of the procedure. An optimization problem of three-stage spur gear reduction unit is given to illustrate the effectiveness of the proposed approach.

2. Physical Programming Synopsis (Messac, 1996)

Physical programming is an effective and computational efficient approach for multiobjective optimization design which includes the following steps in the optimization process : (1) Choose design metrics, (2) Choose design parameters, (3) Develop mapping between design parameters and design metrics, (4) Develop aggregate objective function using class functions, and (5) Perform computational optimization.

Class functions are used within physical programming for the designer to express his (or her) preferences over each design metric. The design metrics are classified into four classes : smaller is better, larger is better, value is better, and range is better. There are two class functions, one soft and one hard, with respect to each class. The hard class functions are used to represent constraints, while the soft class functions become additive constituent components of the aggregate objective function (to be minimized) of the optimization model. For example, the qualitative meaning of the class function of class 1 soft metric (class 1-S) is depicted in Figure 1. The value of the objective function, g_i, is on the x-axis, and the corresponding class function value, $\bar{g}_i$, is on the y-axis.

There are six preference ranges for class 1-S design metrics : Highly desirable ($g_i \leq g_{i1}$), Desirable ($g_{i1} \leq g_i \leq g_{i2}$), Tolerable ($g_{i2} \leq g_i \leq g_{i3}$), Undesirable ($g_{i3} \leq g_i \leq g_{i4}$), Highly undesirable

($g_{i4} \le g_i \le g_{i5}$), and Unacceptable ($g_i \ge g_{i5}$). The parameters g_{i1} to g_{i5} are specified by the designer to quantify his (or her) preferences on design metric g_i.

The aggregate objective function is formed by combining the class functions of all the soft design metrics. The physical programming model takes the following form (Messac, 1996).

$$\lim_x J(x) = \ln\left\{\frac{1}{n_{sc}}\sum_{i=1}^{n_{sc}} \bar{g}_i[g_i(\mathbf{x})]\right\} \quad \text{(For soft classes)}$$

$$\begin{aligned} \text{s.t. } & g_i(\mathbf{x}) \le g_{i5} && \text{(For class 1-S design metrics)} \\ & g_i(\mathbf{x}) \ge g_{i5} && \text{(For class 2-S design metrics)} \\ & g_{i5L} \le g_i(\mathbf{x}) \le g_{i5R} && \text{(For class 3-S and 4-S design metrics)} \quad (1) \\ & g_i(\mathbf{x}) \le g_{iM} && \text{(For class 1-H design metrics)} \\ & g_i(\mathbf{x}) \ge g_{im} && \text{(For class 2-H design metrics)} \\ & g_{im} \le g_i(\mathbf{x}) \le g_{iM} && \text{(For class 3-H and 4-H design metrics)} \\ & x_{jm} \le x_j \le x_{jM} \end{aligned}$$

where n_{sc} is the number of soft criteria, g_{im}, g_{im}, x_{jm} and x_{jM} represent minimum and maximum values.

3. Interactive Physical Programming Procedure

The concept of preference offset is introduced, and the physical programming model that includes the preference offsets is presented in this section. Finally, the procedure of the proposed interactive physical programming approach is presented in details.

3.1 Preference offset

The preference offsets are used to describe the designer's tradeoff on design objectives in the physical programming model of the optimization problem. There is a preference offset with respect to each design objective. The physical programming model that includes preference offsets is formulated as

$$\lim_x J(x) = \ln\left\{\frac{1}{n_{sc}}\sum_{i=1}^{n_{sc}} \exp[\ln(\bar{g}_i) + d_i]\right\} \quad \text{(For soft classes)}$$

$$\begin{aligned} \text{s.t. } & g_i(x) \le g_{i5} && \text{(For class 1-S design metrics)} \\ & g_i(x) \ge g_{i5} && \text{(For class 2-S design metrics)} \\ & g_{i5L} \le g_i(x) \le g_{i5R} && \text{(For class 3-S and 4-S design metrics)} \quad (2) \\ & g_i(\mathbf{x}) \le g_{iM} && \text{(For class 1-H design metrics)} \\ & g_i(\mathbf{x}) \ge g_{im} && \text{(For class 2-H design metrics)} \\ & g_{im} \le g_i(\mathbf{x}) \le g_{iM} && \text{(For class 3-H and 4-H design metrics)} \\ & x_{jm} \le x_j \le x_{jM} \end{aligned}$$

where d_i is the preference offset with respect to objective i. If each d_i takes the value 0, then the model in equation (2) is reduced to that in equation (1).

In equation (2), d_i is added to $\ln(\bar{g}_i)$, the logarithm value of the class function of objective i, so that the designer's tradeoff on this objective will vary evenly in preference space with the variation of value d_i. The preference offsets are adjusted during the interactive physical programming procedure, depending on whether the objectives are supposed to be improved or sacrificed. If an objective i is supposed to be improved, the preference offset d_i should be increased, and vice versa.

3.2 The interactive physical programming procedure

The procedure of the proposed interactive physical programming is presented below, with detailed explanations.

Step 1 : Generate the initial Pareto design using physical programming formulated in equation (1), or using the model in equation (2) with the preference offsets equal to 0.

Step 2 : Visualize the generated Pareto designs. This step enables the designer to analyze the generated designs directly, and specify which objectives need to be improved and which can be sacrificed. The visualization of Pareto designs is implemented in the form of bar chart in the preference space.

Step 3 : Select the most satisfying one from the Pareto designs that already generated. If the designer is satisfied with this design, the procedure will be terminated and the design will be outputted. Otherwise, go to step 4.

Step 4 : Adjust the preference offsets for the design objectives. In this way, we will specify which objectives need to be improved and which can be sacrificed, and how much these objectives should be improved or sacrificed. If an objective i is supposed to be improved, the preference offset d_i should be increased, and vice versa. How much the preference offset should be depends on how much the objective is supposed to be improved or sacrificed, and how many objectives there are.

Step 5: Generate a new Pareto design by solving the optimization model in equation (2) with the preference offsets determined in step 4. Here the MATLAB constrained optimization package is used to solve this nonlinear programming problem. Then go to step 2.

4. Problem Formulation

The optimization model of three-stage spur gear reduction unit is formulated in this section, with minimum volume, minimum surface fatigue life and maximum load-carrying capacity as design objectives. The schematic illustration of the three-stage spur gear reduction unit is shown in Figure 2 (David et al., 2000). The design vector $\mathbf{x}$ is

$$\mathbf{x}=[H_1, H_2, H_3, m_1, m_2, m_3, N_{p1}, N_{p2}, N_{p3}, N_{g1}, N_{g2}, b_1, b_2, b_3, d_{s1}, d_{s2}] \quad (3)$$

where H_i is the core hardness, m_i is the module value, b_i is the face width of the i-th gear set. N_{pi} represents the tooth numbers of the i-th pinion, N_{gi} represents the tooth numbers of the i-th gear. d_{si} is the diameter of the i-th shaft.

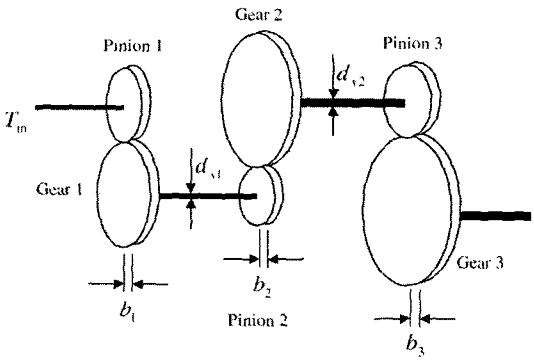

Fig. 2 The schematic illustration of the three-stage spur gear reduction unit

Let d_{p1}, d_{p2} and d_{p3} represent the pitch circle diameters of the pinions and d_{g1}, d_{g2} and d_{g3} the pitch circle diameters of the gears. L_s represents the length of the two shafts, assuming that they are equal. We can calculate d_{g3} with the following equation:

$$d_{g3}=\frac{d_{p1}d_{p2}d_{p3}}{ed_{g1}d_{g2}} \quad (4)$$

where e is the overall speed reduction ratio of the gearbox (the value of e=0.1 is used in this paper).

The optimization model of three-stage spur gear reduction unit takes the following form (David et al., 2000; Tian et al., 2002):

$$\min V=\frac{\pi}{4}[(d_{p1}^2+d_{g1}^2)b_1+(d_{p2}^2+d_{g2}^2)b_2+(d_{p3}^2+d_{g3}^2)b_3+(d_{s1}^2+d_{s2}^2)L_s] \quad (5)$$

$$\min SF=\min\{SF1, SF2, SF3\} \quad (6)$$

$$\max T_{in}=\{T_{in}^1, T_{in}^2, T_{in}^3\} \quad (7)$$

s.t.

$$\frac{2T_{in}}{m_1b_1J(N_{p1})d_{p1}}K_vK_oK_m-S_n'(H_1)C_s(H_1)k_rk_{ms}\le 0 \quad (8)$$

$$\frac{2T_{in}d_{g1}}{m_2b_2J(N_{p2})d_{p1}d_{p2}}K_vK_oK_M-S_n'(H_2)C_s(H_2)k_rk_{ms}\le 0 \quad (9)$$

$$\frac{2T_{in}d_{g1}d_{g2}}{m_3b_3J(N_{p3})d_{p1}d_{p2}d_{p3}}K_vK_oK_M-S_n'(H_3)C_s(H_3)k_rk_{ms}\le 0 \quad (10)$$

$$\frac{16T_{in}d_{g1}}{\pi d_{s1}^3d_{p1}}-\tau_{\max}\le 0 \quad (11)$$

$$\frac{16T_{in}d_{g1}d_{g2}}{\pi d_{s2}^3d_{p2}d_{p2}}-\tau_{\max}\le 0 \quad (12)$$

$$9m_1-b_1\le 0 \quad (13)$$

$$9m_2-b_2\le 0 \quad (14)$$

$$9m_3-b_3\le 0 \quad (15)$$

$$b_1-14m_1\le 0 \quad (16)$$

$$b_2-14m_2\le 0 \quad (17)$$

$$b_2-14m_3\le 0 \quad (18)$$

$$d_{g1}+2m_1-2\sqrt{\left(\frac{d_{g1}\cos\phi}{2}\right)^2+\left(\frac{d_{p1}+d_{g1}}{2}\right)^2\sin^2\phi}\le 0 \quad (19)$$

$$d_{g2}+2m_2-2\sqrt{\left(\frac{d_{g2}\cos\phi}{2}\right)^2+\left(\frac{d_{p2}+d_{g2}}{2}\right)^2\sin^2\phi}\le 0 \quad (20)$$

$$d_{g3}+2m_3-2\sqrt{\left(\frac{d_{g3}\cos\phi}{2}\right)^2+\left(\frac{d_{p3}+d_{g3}}{2}\right)^2\sin^2\phi}\le 0 \quad (21)$$

$$17-N_{p1}\le 0 \quad (22)$$

$$17-N_{p2}\le 0 \quad (23)$$

$$17-N_{p3}\le 0 \quad (24)$$

Equation (5) represents the volume objective. SF in equation (6) represents the surface fatigue life index, where

$$SF1=\frac{4C_p^2K_VK_OK_mT_{in}}{\cos\phi\sin\phi S_{fe}^2C_R^2}\cdot\frac{d_{p1}+d_{g1}}{b_1d_{p1}^2d_{g1}} \quad (25)$$

$$SF2=\frac{4C_p^2K_VK_OK_mT_{in}}{\cos\phi\sin\phi S_{fe}^2C_R^2}\cdot\frac{d_{p2}+d_{g2}}{b_2d_{p2}^2d_{g2}}\cdot\frac{d_{g1}}{d_{p1}} \quad (26)$$

$$SF3=\frac{4C_p^2K_VK_OK_mT_{in}}{\cos\phi\sin\phi S_{fe}^2C_R^2}\cdot\frac{d_{p3}+d_{g3}}{b_3d_{p3}^2d_{g3}}\cdot\frac{d_{g1}d_{g2}}{d_{p1}d_{p2}} \quad (27)$$

where $SF1$, $SF2$, and $SF3$ represents the squared value C_{Li}^2 of the surface fatigue life factor C_{Li} of the first, second, and third gear set, respectively. A smaller value of C_{Li} indicates longer surface fatigue life (Juvinall et al., 1991).

T_{in} in equation (7) represents the load-carrying capacity index, where

$$T_{in}^1=\frac{S_n'(H_1)\,C_s(H_1)\,k_rk_{ms}m_1b_1J\,(N_{p1})\,d_{p1}}{2K_VK_OK_m} \quad (28)$$

$$T_{in}^1=\frac{S_n'(H_2)\,C_s(H_2)\,k_rk_{ms}m_2b_2J\,(N_{p2})\,d_{p1}d_{p2}}{2K_VK_OK_md_{g1}} \quad (29)$$

$$T_{in}^1=\frac{S_n'(H_3)\,C_s(H_3)\,k_rk_{ms}m_3b_3J\,(N_{p3})\,d_{p1}d_{p2}d_{p3}}{2K_VK_OK_md_{g1}d_{g2}} \quad (30)$$

where J is geometry factor. The standard R.R. Moore endurance limit S_n' and the surface factor C_s are the functions of the core hardness.

Equations (8)-(10) represent the tooth bending fatigue failure constraints, equations (11)-(12) represent the shaft torsional stress constraints, equations (13)-(18) reflect the face width constraints, equations (19)-(21) represent the interference constraints, equations (22)-(24) represent the tooth number constraints. Other symbols used here and their values are summarized in Table 1 (David et al., 2000).

Table 1 Parameter symbols and their values

Description	Symbol	Value	Units
Pressure angle	ϕ	20	degree
Shaft length	L_s	0.1	m
Elastic coefficient	C_p	190,910	$\sqrt{\text{Pa}}$
Velocity factor	K_V	2.0	none
Overload factor	K_O	1.0	none
Mounting factor	K_m	1.6	none
Surface fatigue strength	S_{fs}	1309.1	MPa
Surface reliability factor (99.0%)	C_R	1.0	none
Bending reliability factor (99.0%)	k_r	0.814	none
Mean stress factor	k_{ms}	1.4	none
Torsional stress limit	$\tau_{\max}$	172.25	MPa

5. Results and Discussions

Interactive physical programming is used to solve the optimization model of three-stage spur gear reduction unit formulated in section 4. The objectives V and SF belong to Class-1S design metrics, while objective T_{in} belongs to Class-2S design metric. The class functions settings for the three objectives are listed in Table 2. These class functions settings will not change all through the interactive physical programming procedure.

The MATLAB constrained optimization package is used to solve the optimization model formulated in equation (2). The initial Pareto design generated using physical programming is visualized in preference space, as shown in Figure 3. The optimal volume value is 0.0015 m^3, the optimal load-carrying capacity is 31.4082 N·m, and the optimal surface fatigue life index is 0.3868. Assume that the class function values of

Table 2 The class functions settings

Objective	Class	g_{i1}	g_{i2}	g_{i3}	g_{i4}	g_{i5}
V (m^3)	1S	0.0010	0.0015	0.0020	0.0050	0.0100
T_{in} (N·m)	2S	40	25	15	10	5
SF (None)	1S	0.3	0.5	0.8	1.0	1.5

the three objectives are supposed to be at the same level. Therefore, on the basis of the initial Pareto design, V needs to be improved.

There are three objectives in this problem. We need to fix the preference offset with respect to one objective to 0 in the interactive physical programming procedure, and adjust the preference offsets with respect to other objectives. If two objectives are improved, it is equivalent to sacrifice the left objective, since we can not improve all the objectives simultaneously. Here we fix the preference offset of SF to zero.

After running interactive physical programming for several iterations, we get the optimal solution with desirable characteristic we are seeking. The final optimal solution is visualized in Figure 4. The preference offset vector with respect to this optimal solution is [0.49, 0.21, 0]. The optimal values for the three objectives are

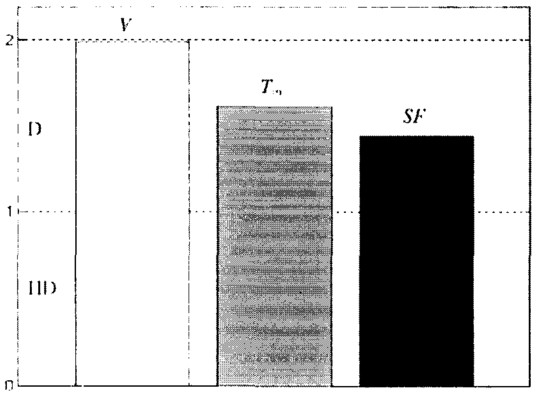

Fig. 3 The initial optimal design

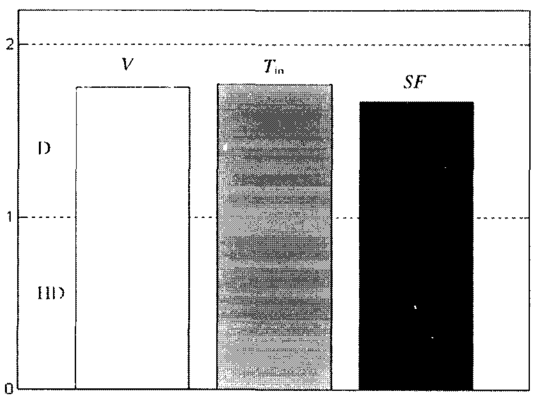

Fig. 4 The final optimal design

0.0014 m^3, 28.9987 and 0.4256, respectively. The volume objective is improved, and the other two objectives are sacrificed. The maximum class function value of the three objectives decreases form 0.5357 in the initial optimal solution to 0.3750 in the final optimal solution.

6. Conclusions

In this paper, we proposed an interactive physical programming approach that adjusts the physical programming model in a natural way. The preference ranges for each objective are fixed during the interactive physical programming procedure. The concept of preference offset is introduced, and used for adjusting the optimization model during the interactive physical programming procedure based on the designer's preferences on improving or sacrificing some objectives. The preference offsets actually represent the designer's tradeoff on the design objectives. The preference offsets are adjusted during the interactive physical programming procedure, and an optimal solution that satisfies the designer's preferences is supposed to be obtained by the end of the procedure. The optimization problem of three-stage spur gear reduction unit illustrates the effectiveness of the proposed approach.

Acknowledgments

This research was partially supported by the National Natural Science Foundation of China under the contract number 50175010, the Excellent Young Teachers Program of the Ministry of Education of China under the contract number 1766, the National Excellent Doctoral Dissertation Special Foundation of China under the contract number 200232, and the Natural Sciences and Engineering Research Council of Canada.

References

Chen, W., Sahai, A., Messac, A. and Sundararaj, G. J., 2000, "Exploration of the Effectiveness of Physical Programming in Robust Design," *Journal of Mechanical Design, Trans-*

actions of the ASME, Vol. 122, No. 2, pp. 155~163.

David, F. T., Shubhagm, G. and Amit, S., 2000, "Tradeoff Analysis in Minimum Volume Design of Multi-Stage Spur Gear Reduction Units," *Mechanism and Machine Theory*, Vol. 35, No. 5, pp. 609~627.

Huang, H. Z., Tian, Z. and Gu, Y., 2004, "Reliability and Redundancy Apportionment Optimization Using Interactive Physical Programming," *International Journal of Reliability, Quality And Safety Engineering*, Vol. 11. No. 3, pp. 213~222.

Juvinall, R. C. and Marshek, K. M., 1991, Fundamentals of Machine Component Design, 2nd ed., Wiley, New York, pp. 550~596.

Messac, A., 1996, "Physical Programming: Effective Optimization for Computational Design," *AIAA Journal*, Vol. 34, No. 1, pp. 149~158.

Messac, A., Sukam, C. P. and Melachrinoudis, E., 2001, "Mathematical and Pragmatic Perspectives of Physical Programming," *AIAA Journal*, Vol. 39, No. 5, pp. 885~893.

Park, H. -W., Kim, M. -S., Choi, D. -H., 2002, "A New Decomposition Method for Parallel Processing Multi-Level Optimization," *KSME International Journal*, Vol. 16, No. 5, pp. 609~618.

Park, H. -W., Lee, S. J., Lee, H. -S., Choi, D. -H., 2004, "Adaptive Parallel Decomposition for Multidisciplinary Design," *KSME International Journal*, Vol. 18, No. 5, pp. 814~819.

Tian, Z. and Huang, H. Z., 2002, "Interactive Physical Programming and Its Application to Mechanical Design," *Proceedings of the 5th International Conference on Frontiers of Design and Manufacturing*, Dalian, pp. 502~506.