A Study on the Configuration Control of a Mobile Manipulator Based on the Optimal Cost Function

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Abstract—One of the most important feature of the Mobile Manipulator is redundant freedom. Using the redundant freedom, Mobile Manipulator can move various mode, perform dexterous motion. In this paper, to improve robot job ability, as two robots perform a job in co-operation control, we studied optimal position and posture of Mobile Manipulator with minimum movement of each robot joint. Kinematics of mobile robot and task robot is solved. Using mobility of Mobile robot, weight vector of robots is determined. Using Gradient methode, global motion trajectory is minimized. so the job which Mobile Manipulator perform is optimized. The proposed algorithm is verified with PURL-II which is Mobile Manipulator combined Mobile robot and task robot. and discussed the result.

Index Terms— Mobile Manipulator, Gradient method PURL-II, Co-operation,

I. INTRODUCTION

A mobile robot only extends the work area but cannot work, and a vertically articulated robot is dependent on the work environment and conditions since it has a fixed base structure and its work areas are limited. Therefore, the interest in multi-robots, particularly in the work performance of mutually cooperating multiple robots, is growing [3]. Recently, it has been found that the work performance of the surplus robot, which has a greater degree of freedom than that required in the given workspace, can be optimized by optimizing its configuration with a surplus degree of freedom [1][2]. While there are a number of studies, however, in the area of the automatic operation of mobile robots and the automatic operation control of fixed manipulators, there are hardly any reports about studies that have been conducted in the area of cooperative control by robots in which mobility and operation ability have been combined [3]. In this study, the vertically articulated robot serially combined with the mobile robot is defined as a mobile manipulator, and the vertically articulated robot as a work robot. Furthermore, based on the kinematics of a mobile manipulator, the mobility of a mobile robot is defined herein. The optimal locations and configurations of a mobile manipulator were processed so all robots could work with a minimal joint displacement, and using the proposed mobility, the weight of a mobile manipulator was adjusted. The total movements of the mobile and work robots mutually cooperating to perform work were determined using the gradient method, the optimal standard of the work was examined, and the algorithm proposed by a simulation was discussed.

II. MOBILE MANIPULATOR

A. Composition of a Mobile Manipulator and the Kinematics Analysis

The composition diagram of the mobile manipulator used in the experiments conducted in this study is shown in Figure 1. By combining two robots designed for independent purposes, each of the robots would be made to perform work spontaneously. Trace planning can lead to a more efficient work performance through the production of a surplus degree of freedom by combining the robots. Therefore, a kinematics analysis of the entire system is required.

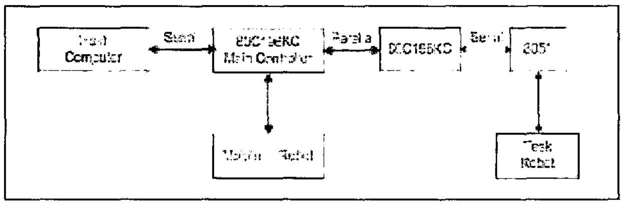

Fig. 1 Composition of a mobile manipulator

Figure 2 shows the formation diagram of a line coordinate system of each joint on the orthogonal coordinate space of the mobile/work robot system produced [4].

In Figure 2, the mobile robot is the robot with 3 degrees of freedom and 3 axial individual operations, which is able to move in three directions: X-Y-Z. It set $P_1^0 = [P_x, P_y, P_z]^T$, which is the end-plate location of the mobile robot (the base of the vertically articulated robot) formed as the frame by the joint variables of each axis of the mobile robot.

P_1^0 is the location vector to frame $\{1\}$ based on frame $\{0\}$. In the kinematics of the mobile manipulator, first, since the mobile robot has three axes, the joint variable $\dot{\theta}_m$ is the same as the line and speed of the mobile

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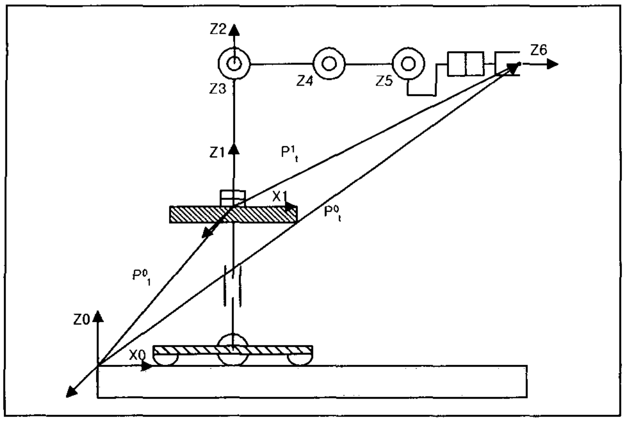


Fig. 2 Coordinate system of a mobile/work robot

Robot $\overline{P}_m^0 = [V_m^0 \quad \omega_m^0]^T$ on the orthogonal coordinate space for the fixed world frame, and Equation(1) below was used.

$$\dot{P}_m^0 = \begin{bmatrix} V_m^o \\ \omega_m^o \end{bmatrix} = \begin{bmatrix} J_{m,v}^0 \\ J_{m,\omega}^0 \end{bmatrix} \dot{\theta}_m = J_m^0 \dot{\theta}_m \tag{1}$$

The Jacobian, the vector $\dot{\theta}_t$, of the joint variables of the work robot, is explained based on frame $\{1\}$ in Figure 2, using Equation (2), as follows:

$$\dot{P}_t^1 = \begin{bmatrix} V_t^1 \\ \omega_t^1 \end{bmatrix} = \begin{bmatrix} J_{t,v}^1 \\ J_{t,\omega}^1 \end{bmatrix} \dot{\theta}_t = J_t^1 \dot{\theta}_t \tag{2}$$

If the Jacobian of the mobile manipulator is set as J_t^0 when the Jacobians for each robot are given as $J_m^0, J_t^1, \dot{P}_t^0 = [V_t^0 \ \ \omega_t^0]^T$, the line and angular velocity of the end-effector for the world frame would be expressed in Equation (3), as follows:

$$\begin{aligned} \dot{P}_t^0 &= \begin{bmatrix} V_t^o \\ \omega_t^o \end{bmatrix} = \begin{bmatrix} V_m^0 \\ \omega_m^0 \end{bmatrix} + \begin{bmatrix} \omega_m^0 + R_1^0 V_t^1 \\ R_1^0 \omega_t^1 \end{bmatrix} \\ &= J_m^0 \dot{\theta}_m + J_t^0 \dot{\theta}_t = [J_m^0 \ \ J_t^0] \begin{bmatrix} \dot{\theta}_m \\ \dot{\theta}_t \end{bmatrix} \end{aligned} \tag{3}$$

R_1^0 is the rotational transformation into the basic frame of the work robot from the world frame. By using Equations(1)-(3), it was found that the movements of both the mobile and work robots affect the end-effector's movement [6].

III. SYSTEM OPERATION ALGORITHM

A. Work Plan for Minimal Movement

To move the operation point of the work robot from a random location to a certain location, the location of the base frame of the mobile manipulator's work robot was changed according to the movement of its mobile robot. Therefore, the work plan involving inverse kinematics will have many solutions, depending on the configurations of the robot.

This study aims to minimize the total movement of the robot to promote optimal work performance and efficiency. To enable the robot to carry out more efficient work, its restraints should be defined, and the optimal solution should be found. The vector showing the status of the mobile manipulator is expressed in Equation(4) below.

$$p = \begin{bmatrix} x \\ q \end{bmatrix}, \quad x = [x, \ y, \ z, \ \theta]^T, \qquad q = [q_1, \ q_2, \ \ldots, \ q_n]^T \tag{4}$$

Here, p is the vector presenting the condition of the robot, indicates the location and direction of the robot on the orthogonal coordinate space q, and represents the articulation variables for each n number of the links of the work robot. Therefore, the movements of the mobile and work robots can be expressed as in Equations(5) and (6) below.

$$\begin{aligned} L_m &= \int_{t_0}^{t_f} \dot{x}_m^2 + \dot{y}_m^2 + \dot{z}_m^2 + \dot{\theta}_m^2 \, dt \\ &= \int_{t_0}^{t_f} \dot{x}^T \cdot \dot{x} dt \end{aligned} \tag{5}$$

$$L_t = \int_{t_0}^{t_f} \sum_{k=1}^{n} \dot{q}_k^2 dt = \int_{t_0}^{t_f} \dot{q}^T \cdot \dot{q} \, dt \tag{6}$$

n is link number. The entire movement of the mobile manipulator is expressed in Equation(7), and the work is planned to minimize the price function L.

$$\begin{aligned} L &= \int_{t_0}^{t_1} \dot{x}^T \cdot \dot{x} + \dot{q}^T \cdot \dot{q} \, dt \\ &= \int_{t_i}^{t_f} \dot{p}^T \cdot \dot{p} dt \end{aligned} \tag{7}$$

If the price function L of Equation(7) would be developed, it will be expressed as in Equation(8) below.

$$\begin{aligned} L &= \int_{t_1}^{t_f} \dot{p}^T \cdot \dot{p} \, dt = \Delta p^T \Delta p \\ &= (p_f - p_i)^T (p_f - p_i) + (x_f - x_i)^T (x_f - x_i) \end{aligned} \tag{8}$$

Here, $p_i = [x_i, q_i]^T$ is the initial configuration of the mobile manipulator, and $p_i = [x_f, q_f]^T$ indicates the final configuration after the work performance. Since the operation point of the work robot should be located in the desirable spot x_d, in the final configuration, the conditions in Equation(9) should be met. Here, $R(\theta_f)$ shows the rotational transformation on the XY plane, and $f(q_f)$ indicates the kinematics equation of the work robot.

$$x_d = R(\theta_f)f(q_f) + x_f \tag{9}$$

The mobile robot's final location, x_f can be expressed as the equation of the target coordinate x_d and the variables θ_f and q_f, and the price function showing the movement of the robot can be expressed as the space functions θ_f, q_f and $n \times 1$ as in Equation (10).

$$\begin{aligned} L = &\{x_d - R(\theta_f)f(q_f) - x_i\}^T \\ &*\{x_d - R(\theta_f)f(q_f) - x_i\} \\ &+\{q_f - q_i\}^T\{q_f - q_i\} \end{aligned} \tag{10}$$

At this point, θ_f and q_f, which minimize the price function L, should be able to meet the conditions in Equation(11).

$$\nabla L = \begin{bmatrix} \dfrac{\partial L}{\partial \theta_f} \\ \dfrac{\partial L}{\partial q_f} \end{bmatrix} = 0 \tag{11}$$

Since the price function is a non-linear function, the optimal solution satisfying the conditions in Equation 11 is difficult to arrive at through analysis. The numerical value was determined in this study using the gradient method, as in Equation 12.

$$\begin{bmatrix} \theta_{f(k+1)} \\ q_{f(k+1)} \end{bmatrix} = \begin{bmatrix} \theta_{f(k)} \\ q_{f(k)} \end{bmatrix} - \eta \nabla L \Big|_{\theta_{f(k)}, q_{f(K)}} \tag{12}$$

$If \|\nabla L\| < \varepsilon \approx 0, \quad \theta_{f(k)}, q_{f(K)}$: *optimum solution*

The final configuration of the robot, p_f based on the optimal solutions θ_f and q_f are expressed in Equation (13).

$$p_f = \begin{bmatrix} x'_f \\ q'_f \end{bmatrix} = \begin{bmatrix} x_d - R(\theta'_f)f(q'_f) \\ q'_f \end{bmatrix} \tag{13}$$

B. Allowance of Weight by Mobility

Through mobility, the mobile robot's kinesis in random directions could be found, and the appropriateness of the work could be seen in the current configuration of the mobile robot. Currently, if the configuration of the mobile robot is found to be appropriate, so mobility would have a large value, the movement of the mobile robot should be made small by giving a low weight value in the items showing the movement of the mobile robot in the price function in Equation(14), and if the configuration is not appropriate for work, a high weight should be given to limit the movement of the mobile robot. Equation(14) expresses the price function weights that were given.

$$\begin{aligned} L = &\{x_d - R(\theta_f)f(q_f) - x_i\}^T \\ &*W_M\{x_d - R(\theta_f)f(q_f) - x_i\} \\ &+\{q_f - q_i\}^T W_T\{q_f - q_i\} \end{aligned} \tag{14}$$

Here, W_M and W_T are the weighted matrices, which are affected by the movements of the mobile and work robots. In the price function, the movement of the mobile robot on the orthogonal cooperate space was shown, so that the weighted matrix W_M should be allowed after the factors for each axis of the orthogonal cooperates are resolved.

As shown in Figure 3, the mobility can be resolved against the X and Y axes of the orthogonal coordinate space. To satisfy the relation between mobility and weight that was mentioned earlier, a certain weight should be given to the mobile robot, as in Equation(15) below.

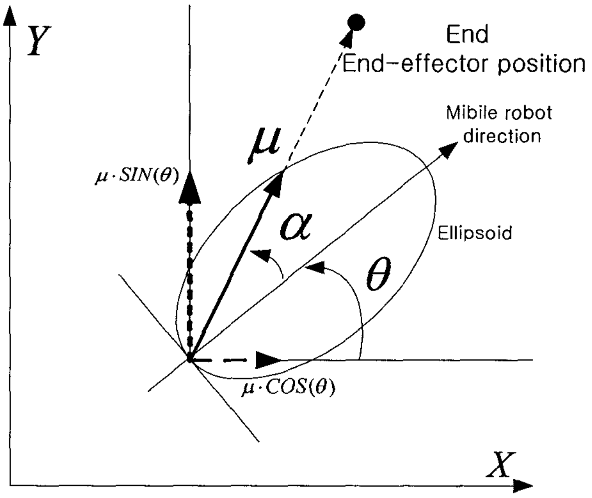


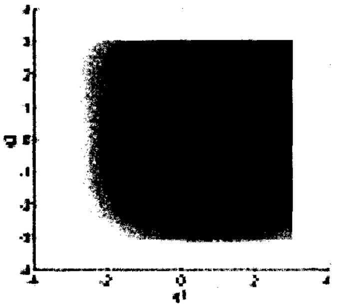
Fig.3 Resolution of mobility

$$W_M = \begin{bmatrix} w_x & 0 & 0 & 0 \\ 0 & w_y & 0 & 0 \\ 0 & 0 & w_z & 0 \\ 0 & 0 & 0 & w_\theta \end{bmatrix} \tag{16}$$

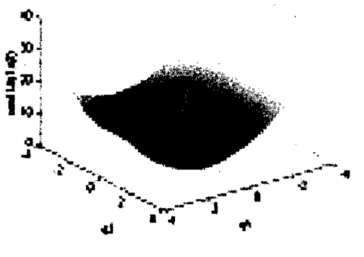
$$= \begin{bmatrix} \dfrac{1}{\mu\cos(\theta)+e} & & & 0 \\ & \dfrac{1}{\mu\cos(\theta)+e} & & \\ & & k_1(z_d - f_z(q))^2 + k_2 & \\ 0 & & & 1 \end{bmatrix}$$

For the weight in the Z direction, since the range of the mobile robot's movement is limited, the function was set as a quadratic function so there would be a higher weight value. Moreover, the weight of the work robot w_T was fixed as an identical matrix so that the relative importance can be changed based on the weight of the mobile robot. Therefore, a more efficient cooperative work performance of the mobile robot could be promoted by considering mobility in the mobile robot's work direction.

IV. SIMULATION

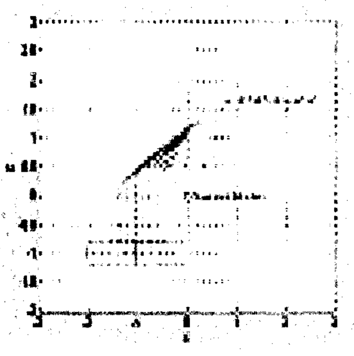


(a) Cost function plane in the [q1, q2] areas (the dark areas are the parts with low cost)

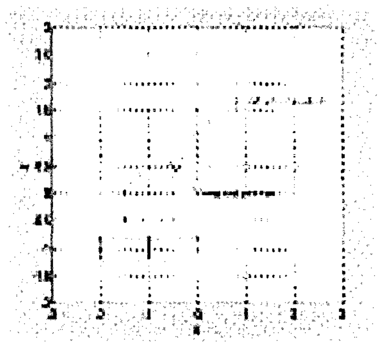


(b) Optimization through the gradient method

Initial configuration: mobile robot$(-0.5, 0.3)$,
work robot $(60^\circ, 30^\circ)$
Final configuration: mobile robot $(-1.5373, 0.0921)$,
work robot $(36.2151^\circ, 79.2887^\circ)$

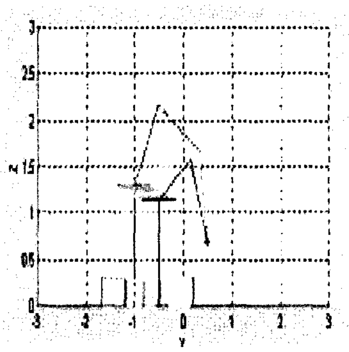


(c) Robot's operation point shift work in the optimal configuration plan (0.866, 1.666) → (0, 0)

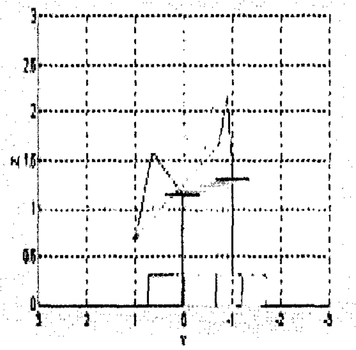


(d) Robot's initial and final configurations

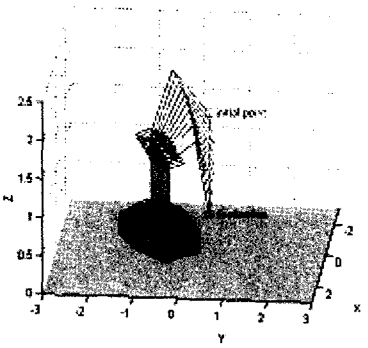
Fig. 4 Optimal configuration plan minimizing the entire displacement of the robot under a surplus condition.



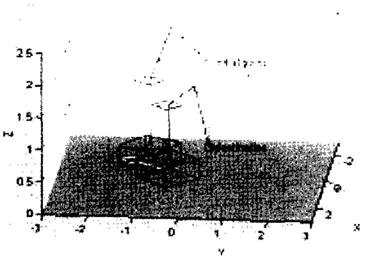
(a) Movement of the robot as viewed in the X axis



(b) Movement of the robot as viewed in the Y axis



(c) Movement work of the robot's operation point



(d) Robot's initial and final configurations

Fig. 5 Optimal configuration plan to move the operation point of the robot to (1, 0.5, 0.7).

Initial configuration: mobile robot (-1, -1, 1.3, 60°)
work robot (18°, 60°, 90°)
Robot's operation point: (-0.716, 0.3362, 1.666)
Final configuration:
mobile robot (0.0368, -0.4965, 1.1435, 44.051°)
work robot (1.986°, 25.5701°, 86.6327°)
Robot's operation point (1, 0.5, 0.7)

IV. CONCLUSIONS

A mobile robot only expands the workspace, and the workspace of a work robot is limited since it has a fixed base structure.

Therefore, to overcome these limits, a manipulator system combining the mobile and work robots was formulated, and the surplus joints of the surplus robot were efficiently used. In case the workspace would be changed due to the operation of the mobile robot, the work robot can be controlled to make the two robots work together by preparing it for the next work, with an appropriate configuration during the movement time.

By studying the optimal location and configuration to minimize the amount of the joint displacement through the cooperative control of the two robots in the main work, the robots' work efficiency was improved, and the optimal condition was met.

In this study, the mobile and work robots were analyzed in terms of their kinetics, and the weight of the mobile robot was adjusted using the mobility of the mobile robot, based on the analysis. This study also provided the foundation for the robot's path planning, with a good configuration in the configuration space based on the optimal condition. In addition, a way of minimizing the total movement of the robot was arrived at using the gradient method. A simulation of the cooperative control of the mobile and work robots was accomplished based on the theoretical background.

In the future, gain training, which is suitable for real time, using Auto Tuning Adaptive PID[7] and based on the sensor, is required to make it possible to precisely control the location by correcting the location error through velocity control.

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