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# A STUDY OF THE BILATERAL FORM OF THE MOCK THETA FUNCTIONS OF ORDER EIGHT

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Abstract. We give a generalization of bilateral mock theta functions of order eight and show that they are $F_q$-functions. We also give an integral representation for these functions. We give a relation between mock theta functions of the first set and bilateral mock theta functions of the second set.

## 1. Introduction

Bilateral forms of mock theta functions of order three, five, and six were defined by Watson [5, 6], A. Gupta [3], Srivastava [4]. Recently Gordon and McIntosh [2], using half-shift transformation on theta series, constructed two sets of eight functions-four functions in each set - and called them mock theta functions of order eight. In this paper we define bilateral generalized functions which, on specialization, reduce to bilateral mock theta functions of order eight. In bilateral form the summation is from $-\infty$ to $+\infty$.

Truesdell [5] calls the functions which satisfy the difference equation

$$\frac{\partial}{\partial z}F(z,\alpha) = F(z,\alpha+1)$$

as $F$-functions. He has tried to unify the study of these $F$-functions. The functions which satisfy the $q$-analogue of the difference equation

$$D_{q,z}F(z,\alpha) = F(z,\alpha+1),$$

where

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$$zD_{q,z}F(z,\alpha) = F(z,\alpha) - F(zq,\alpha),$$

are called $F_q$-functions.

In Section 5, we show that these bilateral generalized functions satisfy $q$-difference equation and are $F_q$-functions and consequently the bilateral mock theta functions are $F_q$-functions.

In Section 6, we have given an integral representation for these generalized functions and consequently for bilateral mock theta functions.

Later we have connected the bilateral mock theta functions with mock theta functions. We have also given alternate definition for the bilateral mock theta functions by using a transformation of Bailey. Using this definition we have connected the bilateral mock theta functions of the first set with bilateral mock theta functions of the second set.

Not much is known about mock theta functions. They are mysterious functions. Nobody, even Ramanujan, was able to prove that they are mock theta functions, by the definition of the mock theta function given by Ramanujan. By considering their bilateral form, we presume, that they will be helpful in knowing more about these functions. Moreover their belonging to the $F_q$-family will be a great help.

**Notations and symbols.** We shall use the following usual basic hypergeometric notations :

For $|q^k| < 1$,

$$\begin{aligned}
(a;q^k)_n &= (1-a)(1-aq^k)\ldots(1-aq^{k(n-1)}), n \geq 1 \\
(a)_n &= (a;q)_n, \\
(a)_0 &= 1, \\
(a)_{-n} &= (a;q)_{-n} = \frac{1}{(aq^{-n};q)_n} = \frac{(-q/a)^n}{(q/a;q)_n} q^{n(n-1)/2}, \\
(a_1,a_2,\ldots,a_k;q)_n &= (a_1;q)_n(a_2;q)_n\ldots(a_k;q)_n.
\end{aligned}$$

$$ {}_r\psi_r\left[\begin{matrix} a_1,\ldots,a_r \\ b_1,\ldots,b_r \end{matrix};q;z\right] = \sum_{n=-\infty}^{\infty} \frac{(a_1,a_2,\ldots,a_r;q)_n}{(b_1,b_2,\ldots,b_r;q)_n} z^n, $$

where $|b_1 b_2 \ldots b_r / a_1 a_2 \ldots a_r| < |z| < 1$.

## 2. Definition of the mock theta function of order eight

Gordon and McIntosh [2] have given the following eight mock theta function of order eight as :

$$ S_0(q) := \sum_{n=0}^{\infty} \frac{q^{n^2}(-q;q^2)_n}{(-q^2;q^2)_n}, $$

$$ T_0(q) := \sum_{n=0}^{\infty} \frac{q^{(n+1)(n+2)}(-q^2;q^2)_n}{(-q;q^2)_{n+1}}, $$

$$ U_0(q) := \sum_{n=0}^{\infty} \frac{q^{n^2}(-q;q^2)_n}{(-q^4;q^4)_n}, $$

$$ V_0(q) := -1 + 2\sum_{n=0}^{\infty} \frac{q^{n^2}(-q;q^2)_n}{(q;q^2)_n} = -1 + 2\sum_{n=0}^{\infty} \frac{q^{2n^2}(-q^2;q^4)_n}{(q;q^2)_{2n+1}}, $$

and

$$ S_1(q) := \sum_{n=0}^{\infty} \frac{q^{n(n+2)}(-q;q^2)_n}{(-q^2;q^2)_n}, $$

$$ T_1(q) := \sum_{n=0}^{\infty} \frac{q^{n(n+1)}(-q^2;q^2)_n}{(-q;q^2)_{n+1}}, $$

$$ U_1(q) := \sum_{n=\infty}^{\infty} \frac{q^{(n+1)^2}(-q;q^2)_n}{(-q^2;q^4)_{n+1}}, $$

$$ V_1(q) := \sum_{n=0}^{\infty} \frac{q^{(n+1)^2}(-q;q^2)_n}{(q;q^2)_{n+1}} = \sum_{n=0}^{\infty} \frac{q^{2n^2+2n+1}(-q^4;q^4)_n}{(q;q^2)_{2n+2}}. $$

## 3. Definition of bilateral mock theta function of order eight

We shall denote by $S_{0c}(q)$, the bilateral form of $S_0(q)$ with similar notation for other functions.

$$S_{0c}(q) := \sum_{n=-\infty}^{\infty} \frac{q^{n^2}(-q;q^2)_n}{(-q^2;q^2)_n},$$

$$T_{0c}(q) := \sum_{n=-\infty}^{\infty} \frac{q^{(n+1)(n+2)}(-q^2;q^2)_n}{(-q;q^2)_{n+1}},$$

$$U_{0c}(q) := \sum_{n=-\infty}^{\infty} \frac{q^{n^2}(-q;q^2)_n}{(-q^4;q^4)_n},$$

$$V_{0c}(q) := -1 + 2\sum_{n=-\infty}^{\infty} \frac{q^{n^2}(-q;q^2)_n}{(q;q^2)_n},$$

and

$$S_{1c}(q) := \sum_{n=-\infty}^{\infty} \frac{q^{n(n+2)}(-q;q^2)_n}{(-q^2;q^2)_n},$$

$$T_{1c}(q) := \sum_{n=-\infty}^{\infty} \frac{q^{n(n+1)}(-q^2;q^2)_n}{(-q;q^2)_{n+1}},$$

$$U_{1c}(q) := \sum_{n=-\infty}^{\infty} \frac{q^{(n+1)^2}(-q;q^2)_n}{(-q^2;q^4)_{n+1}},$$

$$V_{1c}(q) := \sum_{n=-\infty}^{\infty} \frac{q^{(n+1)^2}(-q;q^2)_n}{(q;q^2)_{n+1}}.$$

## 4. Bilateral generalized functions

We define the eight bilateral generalized functions as follows:

(1) $$S_{0c}(z,\alpha) := \frac{1}{(z)_\infty} \sum_{n=-\infty}^{\infty} \frac{(z)_n(-q;q^2)_n q^{n^2+n\alpha-n}}{(-q^2;q^2)_n},$$

$$T_{0c}(z,\alpha) := \frac{q^2}{(z)_\infty} \sum_{n=-\infty}^{\infty} \frac{(z)_n(-q^2;q^2)_n q^{n^2+n\alpha+2n}}{(-q;q^2)_{n+1}}, \tag{2}$$

$$U_{0c}(z,\alpha) := \frac{1}{(z)_\infty} \sum_{n=-\infty}^{\infty} \frac{(z)_n(-q;q^2)_n q^{n^2+n\alpha-n}}{(-q^4;q^4)_n}, \tag{3}$$

$$V_{0c}(z,\alpha) := -1 + \frac{2}{(z)_\infty} \sum_{n=-\infty}^{\infty} \frac{(z)_n(-q;q^2)_n q^{n^2+n\alpha-n}}{(q;q^2)_n}, \tag{4}$$

and

$$S_{1c}(z,\alpha) := \frac{1}{(z)_\infty} \sum_{n=-\infty}^{\infty} \frac{(z)_n(-q;q^2)_n q^{n^2+n\alpha+n}}{(-q^2;q^2)_n}, \tag{5}$$

$$T_{1c}(z,\alpha) := \frac{1}{(z)_\infty} \sum_{n=-\infty}^{\infty} \frac{(z)_n(-q^2;q^2)_n q^{n^2+n\alpha}}{(-q;q^2)_{n+1}}, \tag{6}$$

$$U_{1c}(z,\alpha) := \frac{q}{(z)_\infty} \sum_{n=-\infty}^{\infty} \frac{(z)_n(-q;q^2)_n q^{n^2+n\alpha+n}}{(-q^2;q^4)_{n+1}}, \tag{7}$$

$$V_{1c}(z,\alpha) := \frac{q}{(z)_\infty} \sum_{n=-\infty}^{\infty} \frac{(z)_n(-q;q^2)_n q^{n^2+n\alpha+n}}{(q;q^2)_{n+1}}. \tag{8}$$

For $\alpha = 1$, $z = 0$, these bilateral generalized functions reduce to bilateral mock theta functions of order eight.

## 5. $F_q$-functions

We shall show that these generalized functions are $F_q-$ functions.

THEOREM 5.1. *$S_{0c}(z,\alpha)$, $T_{0c}(z,\alpha)$, $U_{0c}(z,\alpha)$, $V_{0c}(z,\alpha)$, and $S_{1c}(z,\alpha)$, $T_{1c}(z,\alpha)$, $U_{1c}(z,\alpha)$, $V_{1c}(z,\alpha)$, are $F_q$-functions.*

*Proof.*

$$\begin{aligned} zD_{q,z}S_{0c}(z,\alpha) &= S_{0c}(z,\alpha) - S_{0c}(zq,\alpha) \\ &= \frac{1}{(z)_\infty}\sum_{n=-\infty}^{\infty}\frac{(z)_n(-q;q^2)_n q^{n^2+n\alpha-n}}{(-q^2;q^2)_n} \\ &- \frac{1}{(zq)_\infty}\sum_{n=-\infty}^{\infty}\frac{(zq)_n(-q;q^2)_n q^{n^2+n\alpha-n}}{(-q^2;q^2)_n} \\ &= \frac{1}{(z)_\infty}\sum_{n=-\infty}^{\infty}\frac{(z)_n(-q;q^2)_n q^{n^2+n\alpha-n}}{(-q^2;q^2)_n}[1-(1-zq^n)] \\ &= \frac{z}{(z)_\infty}\sum_{n=-\infty}^{\infty}\frac{(z)_n(-q;q^2)_n q^{n^2+n(\alpha+1)-n}}{(-q^2;q^2)_n} \\ &= zS_{0c}(z,\alpha+1). \end{aligned}$$

Hence $S_{0c}(z,\alpha)$ is a $F_q$-function.

The proofs for other functions are similar. □

## 6. Integral representation for the generalized bilateral functions

Jackson [1, (1.11.4) p.19] defined $q$-integral on $(0,\infty)$ by

$$\int_0^\infty f(t)d_qt = (1-q)\sum_{n=-\infty}^{\infty} f(q^n)q^n. \tag{9}$$

Taking $f(t) = t^{x-1}(tq;q)_\infty$, we have

$$\int_0^\infty t^{x-1}(tq;q)_\infty d_qt = (1-q)\sum_{n=-\infty}^{\infty}(q^{n+1};q)_\infty q^{nx} = \frac{(1-q)(q;q)_\infty}{(q^x;q)_\infty}.$$

So

$$\frac{1}{(q^x;q)_\infty} = \frac{(1-q)^{-1}}{(q;q)_\infty}\int_0^\infty t^{x-1}(tq;q)_\infty d_qt. \tag{10}$$

Using (10) we shall give integral representation for these generalized bilateral functions.

Theorem 6.1.

$$S_{0c}(q^z,\alpha) = \frac{(1-q)^{-1}}{(q;q)_\infty} = \int_0^\infty t^{z-1}(tq;q)_\infty S_{0c}(0,at)d_qt.$$

$$T_{0c}(q^z,\alpha) = \frac{(1-q)^{-1}}{(q;q)_\infty} = \int_0^\infty t^{z-1}(tq;q)_\infty T_{0c}(0,at)d_qt.$$

$$U_{0c}(q^z,\alpha) = \frac{(1-q)^{-1}}{(q;q)_\infty} = \int_0^\infty t^{z-1}(tq;q)_\infty U_{0c}(0,at)d_qt.$$

$$V_{0c}(q^z,\alpha) = \frac{(1-q)^{-1}}{(q;q)_\infty} = \int_0^\infty t^{z-1}(tq;q)_\infty V_{0c}(0,at)d_qt.$$

$$S_{1c}(q^z,\alpha) = \frac{(1-q)^{-1}}{(q;q)_\infty} = \int_0^\infty t^{z-1}(tq;q)_\infty S_{1c}(0,at)d_qt.$$

$$T_{1c}(q^z,\alpha) = \frac{(1-q)^{-1}}{(q;q)_\infty} = \int_0^\infty t^{z-1}(tq;q)_\infty T_{1c}(0,at)d_qt.$$

$$U_{1c}(q^z,\alpha) = \frac{(1-q)^{-1}}{(q;q)_\infty} = \int_0^\infty t^{z-1}(tq;q)_\infty U_{1c}(0,at)d_qt.$$

$$V_{1c}(q^z,\alpha) = \frac{(1-q)^{-1}}{(q;q)_\infty} = \int_0^\infty t^{z-1}(tq;q)_\infty V_{1c}(0,at)d_qt.$$

*Proof.* Writing $q^z$ for $z$ and $a$ for $q^\alpha$ in (1), we have

$$\begin{aligned}
S_{0c}(q^z,\alpha) &= \frac{1}{(q^z;q)_\infty}\sum_{n=-\infty}^{\infty}\frac{a^n(q^z;q)_n(-q;q^2)_n q^{n^2-n}}{(-q^2;q^2)_n}\\
&= \sum_{n=-\infty}^{\infty}\frac{a^n(-q;q^2)_n q^{n^2-n}}{(-q^2;q^2)_n(q^{n+z};q)_\infty}\\
&= \sum_{n=-\infty}^{\infty}\frac{a^n q^{n^2-n}(-q;q^2)_n}{(-q^2;q^2)_n}\int_0^\infty t^{n+z-1}(tq;q)_\infty d_qt\\
&= \frac{(1-q)^{-1}}{(q,q)_\infty}\int_0^\infty t^{z-1}(tq;q)_\infty\sum_{n=-\infty}^{\infty}\frac{q^{n^2-n}(-q;q^2)_n(at)^n}{(-q^2;q^2)_n}d_qt. \qquad (11)
\end{aligned}$$

But

$$S_{0c}(0,\alpha) = \sum_{n=-\infty}^{\infty}\frac{(-q;q^2)_n q^{n^2+n\alpha-n}}{(-q^2;q^2)_n}.$$

Writing $a$ for $q^\alpha$, we have

$$S_{0c}(0,\alpha) = \sum_{n=-\infty}^{\infty}\frac{a^n(-q;q^2)_n q^{n^2-n}}{(-q^2;q^2)_n}.$$

Hence

$$S_{0c}(q^z,\alpha) = \frac{(1-q)^{-1}}{(q;q)_\infty}\int_0^\infty t^{z-1}(tq;q)_\infty S_{0c}(0,at)d_qt,$$

which proves the first relation.

The proofs of the others are exactly on similar lines. □

For $a = q$, $S_{0c}(0, at)$ reduces to the bilateral mock theta function $S_{0c}(q)$. Similarly, one can obtain the results for other functions.

## 7. Relation between bilateral mock theta functions and mock theta functions of order eight

$$\begin{aligned}
S_{0c}(q) &= \sum_{n=-\infty}^{\infty} \frac{q^{n^2}(-q;q^2)_n}{(-q^2;q^2)_n} \\
&= \sum_{n=0}^{\infty} \frac{q^{n^2}(-q;q^2)_n}{(-q^2;q^2)_n} + \sum_{n=1}^{\infty} \frac{q^{n^2}(-q;q^2)_{-n}}{(-q^2;q^2)_{-n}} \\
&= S_0(q) + \sum_{n=1}^{\infty} \frac{q^{n^2+n}(-1;q^2)_n}{(-q;q^2)_n} \\
&= S_0(q) + 2\sum_{n=1}^{\infty} \frac{q^{n^2+n}(-q^2;q^2)_{n-1}}{(-q;q^2)_n} \\
&= S_0(q) + 2\sum_{n=0}^{\infty} \frac{q^{n^2+3n+2}(-q^2;q^2)_n}{(-q;q^2)_{n+1}} \\
&= S_0(q) + 2T_0(q). \qquad (12)
\end{aligned}$$

Similarly we have the following relations:

$$T_{0c}(q) = T_0(q) + \frac{1}{2}S_0(q) \qquad (13)$$

$$V_{0c}(q) = V_0(q) + V_0(-q) \qquad (14)$$

$$S_{1c}(q) = S_1(q) + 2T_1(q) \qquad (15)$$

$$T_{1c}(q) = T_1(q) + \frac{1}{2}S_1(q) \qquad (16)$$

$$V_{1c}(q) = V_1(q) + V_1(-q). \qquad (17)$$

We have not been able to give a corresponding relation for $U_0(q)$ and $U_1(q)$.

## 8. Transformation of series for bilateral mock theta functions

We shall use the following bilateral transformation of Bailey [1, p. 137, (5.20)]

$$ {}_2\psi_2\left[\begin{matrix} a, b \\ c, d \end{matrix}; q; z\right] = \frac{(az, bz, cq/abz, dq/abz; q)_\infty}{(q/a, q/b, c/d; q)_\infty} \tag{18} $$

$$ \times {}_2\psi_2\left[\begin{matrix} abz/c.abz/d \\ az.bz \end{matrix}; q; cd/abz\right] $$

Making $q \to q^2$, $a \to \infty$, $b = -q$, $c = -q^2$, $d = 0$, $z = -q/a$ in (18), we get after a little simplification

$$ S_{0c}(q) = 2T_{0c}(q). $$

Similarly we have the following relations. We have given in brackets the value of the parameters taken in each case.

$$ V_{0c}(q) = V_{0c}(-q). $$

$$ (q \to q^2, a \to \infty, b = -q, c = q, d = 0, z = -q/a) $$

$$ S_{1c}(q) = 2T_{1c}(q). $$

$$ (q \to q^2, a \to \infty, b = -q, c = -q^2, d = 0, z = -q^3/a) $$

$$ V_{1c}(q) = V_{1c}(-q). $$

$$ (q \to q^2, a \to \infty, b = -q, c = q^3, d = 0, z = -q^3/a) $$

## 9. Another definition of bilateral mock theta functions of order eight

Using the general transformation of Slater [1, p. 129, (5.4.3)], we give another definition of bilateral mock theta functions. The advantage of using this transformation is that the $c's$ are absent on the left hand side and we can choose them in any convenient way. For $r = 2$, we have the transformation

$$\frac{(b_1, b_2, q/a_1, q/a_2, dz, q/dz; q)_\infty}{(c_1, c_2, q/c_1, q/c_2; q)_\infty} {}_2\psi_2 \left[ \begin{matrix} a_1, a_2 \\ b_1, b_2 \end{matrix} ; q; z \right]$$

$$= \frac{q}{c_1} \frac{(c_1/a_1, c_1/a_2, qb_1/c_1, qb_2/c_1, dc_1z/q, q^2/dc_1z; q)_\infty}{(c_1, q/c_1, c_1/c_2, qc_2/c_1; q)_\infty} {}_2\psi_2 \left[ \begin{matrix} qa_1/c_1, qa_2/c_1 \\ qb_1/c_1, qb_2/c_1 \end{matrix} ; q; z \right]$$

$$+idem(c_1; c_2), \tag{19}$$

where $d = a_1a_2/c_1c_2$, $|b_1b_2/a_1a_2| < |z < 1$, and $idem(c_1; c_2)$ after the expression means that the proceeding expression is repeated with $c_1$ and $c_2$ interchanged.

(i) Making $q \to q^2$, $a_1 \to \infty$ and taking $a_2 = -q$, $b_1 = -q^2$, $b_2 = 0$, $z = -q/a_1$ in (19), we have another form of $S_{0c}(q)$:

$$\frac{(-q, -q^2, q^2/c_1c_2, c_1c_2; q^2)_\infty}{(c_1, c_2, q^2/c_1, q^2/c_2; q^2)_\infty} S_{0c}(q)$$

$$= \frac{q^2}{c_1} \frac{(-c_1/q, -q^4/c_1, 1/c_2, c_2q^2; q^2)_\infty}{(c_1, q^2/c_1, c_1/c_2, q^2c_2/c_1; q^2)_\infty} \sum_{n=-\infty}^{\infty} \frac{q^{n^2+2n}(-q^3/c_1; q^2)_n}{c_1^n(-q^4/c_1; q^2)_n}$$

$$+idem(c_1; c_2). \tag{20}$$

(ii) Similarly, we obtain the following relations, the values of the parameters are given in the brackets

$$\frac{(-q, -q^2, q^4/c_1c_2, c_1c_2/q^2; q^2)_\infty}{(c_1, c_2, q^2/c_1, q^2/c_2; q^2)_\infty} S_{1c}(q)$$

$$= \frac{q^2}{c_1} \frac{(-c_1/q, -q^4/c_1, q^2/c_2, c_2; q^2)_\infty}{(c_1, q^2/c_1, c_1/c_2, q^2c_2/c_1; q^2)_\infty} \sum_{n=-\infty}^{\infty} \frac{q^{n^2+4n}(-q^3/c_1; q^2)_n}{c_1^n(-q^4/c_1; q^2)_n}$$

$$+idem(c_1; c_2). \tag{21}$$

$$(q \to q^2, a_1 \to \infty, a_2 = -q, b_1 = -q^2, b_2 = 0, z = -q^3/a_1).$$

(iii)

$$\frac{(q,-q,q^2/c_1c_2,c_1c_2;q^2)_\infty}{(c_1,c_2,q^2/c_1,q^2/c_2;q^2)_\infty}(V_{0c}(q)+1)$$

$$=2\frac{q^2}{c_1}\frac{(-c_1/q,q^3/c_1,1/c_2,q^2c_2;q^2)_\infty}{(c_1,q^2/c_1,c_1/c_2,q^2c_2/c_1;q^2)_\infty}\sum_{n=-\infty}^{\infty}\frac{q^{n^2+2n}(-q^3/c_1;q^2)_n}{c_1^n(q^3/c_1;q^2)_n}$$

$$+idem(c_1;c_2). \tag{22}$$

$$(q\to q^2, a_1\to\infty, a_2=-q, b_1=q, b_2=0, z=-q/a_1).$$

(iv)

$$\frac{1}{q}\frac{(q,-q,q^4/c_1c_2,c_1c_2/q^2;q^2)_\infty}{(c_1,c_2,q^2/c_1,q^2/c_2;q^2)_\infty}V_{1c}(q)$$

$$=\frac{q^2}{c_1}\frac{(-c_1/q,q^5/c_1,q^2/c_2,c_2;q^2)_\infty}{(c_1,q^2/c_1,c_1/c_2,q^2c_2/c_1;q^2)_\infty}\sum_{n=-\infty}^{\infty}\frac{q^{n^2+4n}(-q^3;q^2)_n}{c_1^n(q^5/c_1;q^2)_n}$$

$$+idem(c_1;c_2). \tag{23}$$

$$(q\to q^2, a_1\to\infty, a_2=-q, b_1=q^3, b_2=0, z=-q^3/a_1).$$

## 10. Relation between $S_{0c}(q)$ and $T_{1c}(q)$

Taking $c_1=q$ in (20), we have

$$\frac{(-q,-q^2,q/c_2,qc_2;q^2)_\infty}{(q,c_2,q,q^2/c_2;q^2)_\infty}S_{0c}(q)=\frac{2q(-q^2,-q,1/c_2,q^2c_2;q^2)_\infty}{(q,q,q/c_2,q/c_2;q^2)_\infty}T_{1c}(q)$$

$$+idem(c_1;c_2). \tag{24}$$

**Relation between $S_{1c}(q)$ and $T_{0c}(q)$.** Taking $c_1 = q$ in (21), we have

$$\frac{(-q,-q^2,q^3/c_2,c_2/q;q^2)_\infty}{(q,c_2,q,q^2/c_2;q^2)_\infty}S_{1c}(q) = \frac{2}{q}\frac{(-q^2,-q,q^2/c_2,c_2;q^2)_\infty}{(q,q,q/c_2,qc_2;q^2)_\infty}T_{0c}(q)$$

$$+idem(c_1;c_2).$$

## References

1. G. Gasper and M. Rahman, *Basic Hypergeometric Series,* Cambridge University Press, Cambridge, 1990.
2. B. Gordon and R. J. McIntosh, *Some eight order mock theta functions,* J. London Math. Soc. **62** (2000), 321–335.
3. A. Gupta, *On certain Ramanujan mock theta functions,* Proc. Indian Acad. Sci. **103** (1993), 257–267.
4. Bhaskar Srivastava, *Certain bilateral basic hypergeometric transformations and mock theta functions,* Hiroshima Math. J. **29** (1999), 19–26.
5. C. Truesdell, *An Essay toward a Unified Theory of Special Functions,* Princeton University Press, Princeton, 1948.
6. G.N. Watson, *The final problem: An account of the mock theta functions,* J. London Math. Soc. **11** (1936), 55–80.
7. G.N. Watson, *The mock theta functions (2),* Proc. London Math. Soc. **42** (1937), 274–304.

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