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On Fuzzy M-Sets and Fuzzy M-Continuity

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Abstract

In this paper, we introduce the concept of fuzzy m-sets induced by a given fuzzy supratopology and investigate general properties of the new class consisted of fuzzy m-sets. We also introduce notions of fuzzy m-continuity, fuzzy m-open(closed) maps and study some properties of them.

Key Words : fuzzy m-set, fuzzy m-continuous, fuzzy m-open map, fuzzy m-closed map

1. Introduction

Fuzzy topological spaces were first introduced by Chang [2] who studied a number of the basic concepts including fuzzy continuous maps and fuzzy compactness. And fuzzy topological spaces are a very natural generalization of topological spaces. In 1983, A. S. Mashhour et al [3] introduced supratopological spaces and studied s-continuous functions and s*-continuous functions. In [1], M. E. Abd El-Monsef et al. [1] introduced fuzzy suprtopological spaces which are a generalization of supratopological spaces, and studied fuzzy supracontinuous functions and a number of basic concepts.

In this paper, we introduce the notions of fuzzy m-sets, fuzzy m-continuity, fuzzy m-open maps and fuzzy m-closed maps. We also establish some characterizations of such notions in terms of fuzzy m-closure and fuzzy m-interior of fuzzy sets.

2. Preliminaries

Let X be a set and $I=[0,1]$. Let I^X denote the set of all mapping $a: X \to I$. A member of I^X is called a fuzzy subset of X. And unions and intersections of fuzzy sets are denoted by $\vee$ and $\wedge$, respectively, and defined by

$$\vee a_i = \sup\{a_i(x) \mid i \in J \text{ and } x \in X\},$$
$$\wedge a_i = \inf\{a_i(x) \mid i \in J \text{ and } x \in X\}.$$

Definition 2.1([2]). A fuzzy topology τ on X is a collection of fuzzy sets of X such that

(i) $0_X, 1_X \in \tau$.

(ii) If $a, b \in \tau$, then $a \wedge b \in \tau$.

(iii) If $a_i \in \tau$, for all $i \in J$, then $\vee a_i \in \tau$.

(X, τ) is called a fuzzy topological space. Members of τ are called fuzzy open sets in (X, τ) and complement of a fuzzy open set is called a fuzzy closed set.

Definition 2.2 ([6]). Let f be a mapping from a set X into a set Y. Let a and b be respectively the fuzzy sets of X and Y. Then $f(a)$ is a fuzzy set in Y, defined by

$$f(a)(y) = \begin{cases} \sup a(z)_{z \in f^{-1}(y)}, & \text{if } f^{-1}(y) \neq 0,\ y \in Y \\ 0, & \text{otherwise}, \end{cases}$$

and $f^{-1}(b)$ is a fuzzy set in X, defined by $f^{-1}(b)(x) = b(f(x))$, $x \in X$.

Definition 2.3 ([1]). A subfamily τ^* of I^X is said to be a fuzzy supratopology on X if

(1) $0_X, 1_X \in \tau^*$,

(2) if $a_i \in \tau^*$ for all $i \in J$, then $\vee a_i \in \tau^*$.

(X, τ^*) is called a fuzzy supratopological space. The elements of τ^* are called fuzzy supraopen sets in (X, τ^*). And a fuzzy set u is supraclosed on X iff $co(u) = 1 - u$ is a fuzzy supraopen set.

Definition 2.4 ([1]). Let (X, τ^*) be a fuzzy supratopological space. The supraclosure of a fuzzy set u is, denoted by $scl(u)$, given by

$$scl(u) = \wedge\{s \mid s \text{ is a fuzzy supraclosed set and } u \leq s\}.$$

The suprainterior of a fuzzy set u is, denoted by $si(u)$, given by

$$si(u) = \vee\{t \mid t \text{ is a fuzzy supraopen set and } t \leq u\}.$$

Definition 2.5. ([4]). Let (X, τ) be a fuzzy topological space and τ^* be a fuzzy supratopology on X. We call τ^* a fuzzy supratopology associated with τ if $\tau \subseteq \tau^*$.

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Definition 2.6 ([1], [4]). Let $f:(X, \tau^*) \to (Y, \mu^*)$ be a mapping between two fuzzy supratopological spaces. Then f is called a fuzzy supracontinuous function if $f^{-1}(\mu^*) \subseteq \tau^*$.

Definition 2.7 ([4]). Let (X, τ) and (Y, μ) be fuzzy topological spaces and let τ^* be an associated fuzzy supratopology with τ. A function $f: X \to Y$ is a fuzzy s-continuous function if the inverse image of each fuzzy open set in Y is fuzzy supraopen in X.

Definition 2.8 ([1], [4]). Let (X, τ) and (Y, μ) be fuzzy topological spaces, τ^* and μ^* be two associated fuzzy supratopological spaces with τ and μ, respectively. Let $f: X \to Y$ be a function. f is said to be fuzzy supracontinuous if the inverse image of each fuzzy supraopen set is fuzzy supraopen.

Definition 2.9 ([4]). A function $f:(X, \tau) \to (Y, \mu)$ is called fuzzy s-open (resp., fuzzy s-closed) map if the image of each fuzzy open (resp., fuzzy closed) set in (X, τ), is fuzzy supraopen (resp., fuzzy supraclosed) in (Y, μ^*).

Clearly, every fuzzy open (fuzzy closed) map is a fuzzy s-open map (fuzzy s-closed map). And every fuzzy supraopen map is a fuzzy s-open map.

3. Fuzzy m-sets induced by fuzzy supratopologies

Definition 3.1. Let (X, τ^*) be a fuzzy supratopological space. A fuzzy subset u of X is called a fuzzy m-set with τ^* if $u \wedge v \in \tau^*$ for all $v \in \tau^*$. The class of all fuzzy m-sets with τ^* will be denoted by $Fm(X)$.

Lemma 3.2. Let (X, τ^*) be a fuzzy supratopological space. If a fuzzy set u in X is a fuzzy m-set, then u is fuzzy supraopen.

Proof. Since $1_X \in \tau^*$, it is obvious.

We obtain the following theorem from Definition 3.1.

Theorem 3.3. Let (X, τ^*) be a fuzzy supratopo- logical space. Then the class $Fm(X)$ of all fuzzy m-sets of X is a fuzzy topology on X.

Proof. For all $u \in \tau^*$, since $0_X \wedge u = 0_X \in \tau^*$ and $1_X \wedge u = u \in \tau^*$, we get $0_X, 1_X \in Fm(X)$.
Let $u, v \in Fm(X)$. By definition of fuzzy m-set, we obtain $v \wedge w \in \tau^*$ and $u \wedge (v \wedge w) \in \tau^*$ for all $w \in \tau^*$. Thus $(u \wedge v) \in Fm(X)$.
Let $\{u_i : i \in I\}$ be a class of members of $Fm(X)$. By definitions of fuzzy m-sets and fuzzy supratopology, it follows $(\vee u_i) \wedge w = \vee(u_i \wedge w) \in \tau^*$ for all $w \in \tau^*$. Thus the union $\vee_{i \in I} u_i$ also belongs to $Fm(X)$.

We will call the class $Fm(X)$ a fuzzy m-topology with τ^* and the members of $Fm(X)$ fuzzy m-open sets, simply fuzzy m-sets. A fuzzy set u of I^X is called a fuzzy m-closed set if the complement of u is a fuzzy m-set. Thus the intersection of any family of fuzzy m-closed sets is a fuzzy m-closed set and the union of finitely many fuzzy m-closed sets is a fuzzy m-closed set.

Remark. Given a fuzzy topological space (X, τ) with an associated fuzzy supratopology τ^*, if u is a fuzzy supraopen set in X and v is a fuzzy open set, then $u \wedge v$ may not be a fuzzy supraopen set. In the following example, we can show that there is no any relation between fuzzy m-sets and fuzzy open sets. But we can say τ^* is an associated fuzzy supratopology with $Fm(X)$ by Lemma 3.2 and Theorem 3.3.

Example 3.4. Let $X = I$. Consider the fuzzy sets;

$$a(x) = x, \qquad 0 \le x \le 1,$$

$$b(x) = 1 - x, \qquad 0 \le x \le 1,$$

$$c(x) = \begin{cases} 1 - x, & 0 \le x \le \frac{1}{2}, \\ x, & \frac{1}{2} \le x \le 1. \end{cases}$$

Let $\tau = \{0_X, a, 1_X\}$ be a fuzzy topology on X and let $\tau^* = \{0_X, a, b, a \vee b, 1_X\}$ be a fuzzy supratopology with τ. Then $Fm(X) = \{0_X, c, 1_X\}$. Therefore we can say the fuzzy $c = a \vee b$ is a fuzzy m-set but not fuzzy open, and the fuzzy set a is fuzzy open but not a fuzzy m-set in X.

Definition 3.5. Let (X, τ) be a fuzzy topological space and let τ^* be an associated fuzzy supratopology with τ. Then the fuzzy supratopology τ^* is called a fuzzy m-supratopology with τ if every fuzzy open set is also a fuzzy m-set with respect to τ^*. The fuzzy m-supratopology τ^* with τ will be denoted by $\tau_m{}^*$.

It is clear that $\tau \subseteq Fm(X) \subseteq \tau_m{}^*$

Definition 3.6. Let (X, τ) be a fuzzy supratopological space and $u \in I^X$. The fuzzy m-interior of u is, denoted by $mi(u)$, given by

$$mi(u) = \vee\{t \in I^X \mid t \text{ is a fuzzy m-set and } t \le u\}.$$

The fuzzy m-closure of u is, denoted by $mcl(u)$, given by

$$mcl(u) = \wedge\{s \in I^X \mid s \text{ is fuzzy m-closed and } u \le s\}.$$

By Definitions 3.5 and 3.6, we obtain the following properties.

Theorem 3.7. Let (X, τ) be a fuzzy topological space and let τ^* be an associated fuzzy m-supratopology with τ. For each fuzzy set u of X,

(1) $u \leq scl(u) \leq mcl(u) \leq scl(u)$.

(2) $u \leq int(u) \leq mi(u) \leq sci(u)$.

Theorem 3.8. Let (X, τ^*) be a fuzzy supratopological space and let u, v be fuzzy subsets of X.

(1) u is fuzzy m-open if and only if $u = mi(u)$.

(2) u is fuzzy m-closed if and only if $u = mcl(u)$.

(3) $mcl(mcl(u)) = mcl(u)$ and $mi(mi(u)) = mi(u)$.

(4) $u \leq v$ implies $mcl(u) \leq mcl(v)$.

(5) $mcl(u) \vee mcl(v) = mcl(u \vee v)$.

4. Fuzzy m-continuity

Definition 4.1. Let (X, τ) and (Y, μ) be fuzzy topological spaces and let τ^* be an associated fuzzy supratopology with τ. A function $f: X \to Y$ is called fuzzy m-continuous if the inverse image of each fuzzy open set of Y is a fuzzy m-set in X.

Remark. In general, there is no relation between fuzzy continuity and fuzzy m-continuity.

Example 4.2. Let $X = I$. Consider the fuzzy sets;

$$a(x) = x, \quad 0 \leq x \leq 1,$$

$$b(x) = 1 - x, \quad 0 \leq x \leq 1,$$

$$c(x) = \begin{cases} 1 - x, & 0 \leq x \leq \frac{1}{2} \\ \frac{1}{2}, & \frac{1}{2} \leq x \leq 1, \end{cases}$$

$$d(x) = \begin{cases} x, & 0 \leq x \leq \frac{1}{2} \\ 1 - x, & \frac{1}{2} \leq x \leq 1. \end{cases}$$

Let $\tau = \{0_X, a, 1_X\}$ be a fuzzy topology on X and let $\tau^* = \{0_X, a, b, a \vee b, 1_X\}$ be an associated fuzzy supratopology with τ. Let $f: X \to Y$ be defined as $f(x) = 1 - x$. In case $\mu = \{0_1, a, 1_X\}$, the function f is fuzzy continuous but not fuzzy m-continuous. In case $\mu = \{0_1, c, 1_X\}$, f is fuzzy m-continuous but not fuzzy continuous.

From the definitions of fuzzy m-continuity and fuzzy m-supratopology, we get the following:

Theorem 4.3. Let $f: X \to Y$ be a function on fuzzy topological spaces (X, τ) and (Y, μ), and let τ^* be an associated fuzzy m-supratopology with τ.

(1) If f is fuzzy continuous, then it is fuzzy m-continuous.

(2) $f: (X, Fm(X)) \to (Y, \mu)$ is fuzzy continuous iff it is fuzzy m-continuous.

Theorem 4.4. Let $f: X \to Y$ be a function on fuzzy topological spaces (X, τ) and (Y, μ), and let τ^* be an associated fuzzy supratopology with τ. Then the following are equivalent:

(1) f is fuzzy m-continuous.

(2) The inverse image of each fuzzy closed set in Y is fuzzy m-closed in X.

(3) $mcl(f^{-1}(v)) \leq f^{-1}(cl(v))$ for every fuzzy set v in Y.

(4) $f(mcl(u)) \leq cl(f(u))$ for every fuzzy set u in X.

(5) $f^{-1}(int(v)) \leq mi(f^{-1}(v))$ for every fuzzy set v in Y.

(6) For each fuzzy set u in X and each fuzzy neighborhood v of $f(u)$, there exists a fuzzy m-neighborhood w of u such that $f(w) \leq v$.

Proof. (1)$\Rightarrow$(2) Let a fuzzy set v be fuzzy closed in Y; then since f is a fuzzy m-continuous function, $f^{-1}(1-v) = 1 - f^{-1}(v)$ is a fuzzy m-set in X. Therefore $f^{-1}(v)$ is a fuzzy m-closed set in X.

(2)$\Rightarrow$(3) Since $f^{-1}(cl(v))$ is a fuzzy m-closed set. $f^{-1}(cl(v)) = mcl(f^{-1}(cl(v))) \geq mcl(f^{-1}(v))$.

(3)$\Rightarrow$(4) Let u be a fuzzy set in X and $f(u) = v$; then $f^{-1}(mcl(v)) \geq mcl(f^{-1}(v))$. From (3), it follows $f^{-1}(cl(f(u))) \geq mcl(f^{-1}f(u)) \geq mcl(u)$, and so $cl(f(u)) \geq f(mcl(u))$.

(4)$\Rightarrow$(2) Let v be a fuzzy closed set in Y and $u = f^{-1}(v)$. Then $f(mcl(u)) \leq cl(f(f^{-1}(v))) \leq v$. Since $mcl(u) \leq f^{-1}(f(mcl(u)) \leq f^{-1}(v) = u$, it follows the fuzzy set u is fuzzy m-closed.

(2)$\Rightarrow$(1) Obvious.

(1)$\Rightarrow$(5) Let v be a fuzzy set in Y; then by (1), $f^{-1}(int(v))$ is a fuzzy m-set. From Theorem 3.8, $f^{-1}(int(v)) \leq mi(f^{-1}(int(v))) \leq mi(f^{-1}(v))$.

(5)$\Rightarrow$(1) Let v be a fuzzy open set in Y. From $f^{-1}(v) \leq mi(f^{-1}(v)) \leq f^{-1}(v)$, it follows $f^{-1}(v)$ is a fuzzy m-set.

(6)$\Rightarrow$(1) Let a fuzzy set v be fuzzy open in Y and $f^{-1}(v) = u$. Then the fuzzy set v is a fuzzy neighborhood of $f(u) = f(f^{-1}(v))$ and there is a fuzzy m-neighborhood w of $u = f^{-1}(v)$ such that $f(w) \leq v$. Thus we have $w \leq f^{-1}(f(w)) \leq f^{-1}(v)$ and $f^{-1}(v)$ is a fuzzy m-set in X.

(1)$\Rightarrow$(6) It is obvious.

Remark. Let (X, τ), (Y, μ) and (Z, ν) be fuzzy topological spaces and let μ^* be a fuzzy supratopology with μ. If $f: X \to Y$ is fuzzy m-continuous and $g: Y \to Z$ is fuzzy continuous, then $g \circ f$ is fuzzy m-continuous.

Definition 4.5. Let (X, τ^*) and (Y, μ^*) be fuzzy supratopological spaces. A function $f: X \to Y$ is said to be fuzzy mS-continuous function if the inverse image of each fuzzy m-set in Y is fuzzy supraopen in X.

We recall that a fuzzy set u in a fuzzy supratopological space (X, τ^*) is a fuzzy supra s-neighborhood of a fuzzy point p if there is $v \in \tau^*$ with $p \in v \leq u$ [1].

Theorem 4.6. Let $f: X \to Y$ be a function on fuzzy supratopological spaces (X, τ^*) and (Y, μ^*). Then the following are equivalent:

(1) f is fuzzy mS-continuous.
(2) The inverse image of each fuzzy m-closed set in Y is fuzzy supraclosed in X.
(3) $scl(f^{-1}(v)) \leq f^{-1}(mcl(v))$ for every fuzzy set v in Y.
(4) $f(scl(u)) \leq mcl(f(u))$ for every fuzzy set u in X.
(5) For each fuzzy set u in X and each fuzzy m-neighborhood v of $f(u)$, there is a fuzzy supra s-neighborhood w of u such that $f(w) \leq v$.

Remark. Let $f: X \to Y$ be a function on fuzzy topological spaces (X, τ) and (Y, μ), and let τ^* and μ^* be associated fuzzy supratopologies with τ and μ, respectively. Then, in case $\tau^* = \tau_m^*$, we can get the following diagrams:

fuzzy continuity $\Rightarrow$ fuzzy m-continuity $\Rightarrow$
fuzzy s-continuity $\Leftarrow$ fuzzy mS-continuity
$\Leftarrow$ fuzzy s^*-continuity

5. Fuzzy m-open maps and Fuzzy m-closed maps

Definition 5.1 Let (X, τ) and (Y, μ) be fuzzy topological spaces and let μ^* be an associated fuzzy supratopology with μ. A function $f: (X, \tau) \to (Y, \mu)$ is called fuzzy m-open (resp., fuzzy m-closed) map if the image of each fuzzy open (resp., fuzzy closed) set in X is a fuzzy m-set (resp., fuzzy m-closed set).

Example 5.2. Let $X = I$. Consider the fuzzy sets;

$$a(x) = \begin{cases} 2x, & 0 \leq x \leq \frac{1}{2} \\ \frac{1}{2}, & \frac{1}{2} < x \leq 1, \end{cases}$$

$$b(x) = \begin{cases} \frac{1}{2}, & 0 \leq x < \frac{1}{4} \\ 2x, & \frac{1}{4} \leq x \leq \frac{1}{2} \\ 0, & \frac{1}{2} < x \leq 1, \end{cases}$$

$$c(x) = \begin{cases} 1, & 0 \leq x \leq \frac{1}{2} \\ 0, & \frac{1}{2} < x \leq 1. \end{cases}$$

Let $\tau = \{0_X, a, 1_X\}$ and $\mu = \{0_X, b, c, 1_X\}$ be fuzzy topologies on X. Consider an associated fuzzy supratopology $\mu^* = \{0_X, a, b, c, a \vee c, a \vee b, 1_X\}$ with μ. Then $\mu^*_m = \{0_X, a \vee c, a \vee b, 1_X\}$ which is the associated fuzzy m-topology with μ^*. Now we consider a function $f: (X, \tau) \to (Y, \mu)$ defined as the following:

$$f(x) = \begin{cases} x, & 0 \leq x \leq \frac{1}{2} \\ 1-x, & \frac{1}{2} < x \leq 1. \end{cases}$$

Then $f(a) = b$ and $f(1_X) = c$. Since both b and c are fuzzy open sets in (X, μ), f is a fuzzy open map. But f is not fuzzy m-open, since b is not a fuzzy m-set in X. If $\tau_1 = \{0_X, a \vee b, 1_X\}$ is a fuzzy topology and $g: (X, \tau_1) \to (X, \mu)$ is the identity function, then g is fuzzy m-open with respect to μ^* but not fuzzy open.

Theorem 5.3. Let (X, τ) and (Y, μ) be fuzzy topological spaces and let μ^* be an associated fuzzy m-supratopology with μ. Then a fuzzy open map $f: (X, \tau) \to (Y, \mu)$ is also fuzzy m-open.

Remark. Let (X, τ) and (Y, μ) be fuzzy topological spaces and $f: X \to Y$. If τ^* and μ^* be associated fuzzy m-supratopologies with τ and μ, respectively, then we get the following diagrams:

fuzzy open $\Rightarrow$ fuzzy m-open $\Rightarrow$ fuzzy s-open

Theorem 5.4. Let (X, τ) and (Y, μ) be fuzzy topological spaces and let μ^* be an associated fuzzy supratopology with μ. Let $f: (X, \tau) \to (Y, \mu)$ be a fuzzy function. Then the following are equivalent:

(1) f is a fuzzy m-open map.
(2) $f(int(a)) \leq mi(f(a))$ for each fuzzy set a in X.

Proof. (1)$\Rightarrow$(2) It is obvious $f(int(a)) \leq f(a)$. From (1), it follows $f(int(a))$ is a fuzzy m-set, and so $f(int(a)) \leq mi(f(a))$.
(2)$\Rightarrow$(1) Let a be a fuzzy open set in X; then we get $f(a) \leq mi(f(a))$ by (2). Thus $f(a)$ is a fuzzy m-set in Y.

Theorem 5.5. Let (X, τ) and (Y, μ) be fuzzy topological spaces and let μ^* be an associated fuzzy supratopology with μ. Then $f: (X, \tau) \to (Y, \mu)$ is a fuzzy m-open map if and only if for each fuzzy point p in X and each fuzzy open set u of X containing p, there exists a fuzzy m-set w containing $f(p)$ such that $w \leq f(u)$.

Proof. Suppose f is a fuzzy m-open map. For each fuzzy point p in X and each fuzzy open set u of X containing p, $f(u)$ is a fuzzy m-set in Y containing $f(p)$. Set $w = f(u)$. Then w is a fuzzy m-set containing $f(p)$ such that $w \leq f(u)$.
Conversely, it is obvious.

Theorem 5.6. Let (X, τ) and (Y, μ) be fuzzy topological spaces and let μ^* be an associated fuzzy supratopology with μ. Then $f: X \to Y$ is a fuzzy m-closed map if and only if $mcl(f(u)) \le f(cl(u))$ for each fuzzy set u in X.

Proof. Suppose that f is a fuzzy m-closed map. For each fuzzy set u in X, since $f(cl(u))$ is a fuzzy m-closed set, we have

$$f(cl(u)) = mcl(f(cl(u))) \ge mcl(f(u)).$$

Conversely, let u be a fuzzy closed set in X. Since $mcl(f(u)) \le f(cl(u)) = f(u)$, it follows $f(u)$ is fuzzy m-closed, and so f is a fuzzy m-closed map.

Theorem 5.7. Let (X, τ) and (Y, μ) be fuzzy topological spaces and let μ^* be an associated fuzzy supratopology with μ, and let a function $f: X \to Y$ be fuzzy m-open. If w is a fuzzy set in Y and v is a fuzzy closed set containing $f^{-1}(w)$, then there exists a fuzzy m-closed set u in Y containing w such that $f^{-1}(u) \le v$.

Proof. Set $u = 1_Y - f(1_X - v)$. Then u is a fuzzy m-closed set, $f^{-1}(u) \le v$ and $w \le u$.

Theorem 5.8. Let (X, τ), (Y, μ) and (Z, ν) be fuzzy topological spaces and let μ^* and ν^* be an associated fuzzy supratopologies with μ and ν, respectively. Let $f: X \to Y$ and $g: Y \to Z$.

(1) If $g \circ f$ is a fuzzy m-open map and f is a fuzzy continuous surjection, then g is also a fuzzy m-open map.

(2) If $g \circ f$ is a fuzzy open map and g is a fuzzy m-continuous injection, then f is fuzzy m-open.

Proof. (1) Let a be any fuzzy open set in Y. Then $f^{-1}(a)$ is a fuzzy open set in X. Since $g \circ f$ is a fuzzy m-open map, $(g \circ f)(f^{-1}(a))$ is a fuzzy m-set in Z. It is obvious $(g \circ f)(a \circ f) = g(a)$, since f is a surjective function. Thus the map g is fuzzy m-open.
(2) Let a be any fuzzy open set in X. Then $(g \circ f)(a) = g(f(a))$ is a fuzzy open set in Z. Since g is a fuzzy m-continuous and injective function, $g^{-1}(g(f(a))) = g(f(a)) \circ g = f(a)$ is a fuzzy m-set. Therefore f is a fuzzy m-open map.

Theorem 5.9. Let (X, τ) and (Y, μ) be fuzzy topological spaces and let μ^* be an associated fuzzy supratopology with μ. If $f: X \to Y$ is a fuzzy bijection, then following statements are equivalent:

(1) f is a fuzzy m-open map.
(2) f is a fuzzy m-closed map.
(3) f^{-1} is fuzzy m-continuous.

Proof. (1) $\Rightarrow$ (2) Let a be a fuzzy closed set in X. Then $f(1-a) = 1 - f(a)$ is a fuzzy m-set in Y, since f is a fuzzy m-open map. Hence $f(a)$ is a fuzzy m-closed set in Y.
(2) $\Rightarrow$ (3) Let a be a fuzzy closed set in X. Since f is a fuzzy m-closed map, the fuzzy set $f(a)$ is fuzzy m-closed in Y. Thus we can say f^{-1} is fuzzy m-continuous.
(3) $\Rightarrow$ (1) Let a be a fuzzy open set in X. Since f^{-1} is fuzzy m-continuous, it is obvious that f is a fuzzy m-open map.

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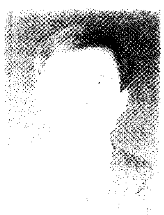

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