A SENSITIVITY STUDY OF THE DISTORTED INLET FLOW IN AXIAL TURBOMACHINERY WITH NOVEL INTEGRAL SCHEME

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For proper installation, operation and performance of axial flow jet engines in aircrafts, the impacts and effects of inlet flow distortion in axial compressors have to be understood. Inlet distortion conditions may cause component-mismatch and instability problems known as rotating stall, and severe oscillations of mass flow rate called surge or a combination of both. Typical effects of this phenomenon include stresses and wear on the compressor blading, destruction of entire jet engines due to the failure of airfoil and mechanical failure or interruption of the combustion process. Therefore, it is important to study inlet flow distortion and its propagation effects to minimize and hence to prevent the occurrence of such calamity.

The current novel integral method with parametric analysis signifies its validity to this field of research and offers much potential for further improvements. The present effort further indicates that this simple method may be flourishing in the problems of strongly distorted flow and propagating stall in axial compressor. It is therefore believe that using a more realistic and flexible velocity and pressure profiles could develop this approach further.

Keywords: Improved Integral Scheme, Taguchi Method, Inlet Distortion, Axial Turbomachine.

NOMENCLATURE

k_l	lift coefficient
k_d	drag coefficient
$F_{x,0}$	*x*-direction force in the undistorted region
$F_{y,0}$	*y*-direction force in the undistorted region
F_x	*x*-direction force in the distorted region
F_y	*y*-direction force in the distorted region
K_0	constant, $K_0 = K_1 + [1 - K_1/\alpha]\,\alpha_0$
K_1	constant, $K_1 = \dfrac{\delta\alpha}{\pi R}$
K_2	velocity parameter, $K_2 = \dfrac{K_1(K_1 - K_0)}{(\alpha - K_1)^2}$
K_3	velocity parameter, $K_3 = 1 + \dfrac{K_2(K_1 - K_0)}{\alpha - K_1}$
R	compressor rotor mean radius
U_0	x-component of referenced inlet velocity
V_0	y-component of referenced inlet velocity
u	x-component of non-dimensional distorted velocity
v	y-component of non-dimensional distorted velocity
u_0	x-component of non-dimensional undistorted velocity
v_0	y-component of non-dimensional undistorted velocity
x	axial coordinate
y	circumferential coordinate
$Y(x)$	function expression of a symmetric line of distorted region
α	distorted velocity coefficient in x-direction
α_0	undistorted velocity coefficient in x-direction
$\tilde{\alpha}$	wing section angle of attack
β	distorted velocity coefficient in y-direction
β_0	undistorted velocity coefficient in y-direction
γ	tangent of incidence angle in inlet, $\gamma = \tan\theta_0 = V_0/U_0$
δ	circumferential extension of distorted flow
θ_0	incidence angle at inlet

1. INTRODUCTION

Taguchi off-line quality control method[1] is a very efficient tool for analyzing and developing high quality products at a low cost. Instead the real experiment that could be simulated by using various analytical tools or numerical techniques, the Taguchi method is implemented using a series of combinations from the different factors and levels involved. The grouping of these iterations is called

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an orthogonal array. Each parameter is referred as a factor and each different value for each factor is termed as a level. This method is applied to investigate on the effects of major parameters influencing the flow through the compressor.

Kim et al.[2] previously deduced that the two most important parameters to control the growth of the distortion propagation are the ratio of drag to lift coefficients of the blade and the angle of incidence of the undistorted upstream flow. The authors' earlier work of improved integral scheme based on the actual airfoil characteristics[3] suggested that the inlet distortion level and the inlet incidence angle have noticeable effects upon the downstream flow features, especially in the level of distortion propagation. In the present effort, a parametric study is conducted using the Taguchi off-line quality method[1] to investigate on the degree of influence of the two selected inlet parameters identified from the current prediction of inlet distortion. The axial inlet distorted velocity coefficient and the angle of inlet incidence are identified here based on their effects on various flow characteristics as analyzed in the results reported earlier.[3] The results of the analysis contradict that of Kim et al.[2] These differences may be explained by the fact that the ratio of drag-to-lift coefficient needs not to be considered in the present analysis. Also, the development of the improved integral method may have altered the effects of inlet parameters.

2. PARAMETRIC ANALYSIS USING TAGUCHI METHOD

Using Taguchi methods for problem solving will:

(i) provide a strategy for dealing with multiple and interrelated problems,
(ii) give a process that will provide a better understanding of the products and processes,
(iii) provide techniques for rational decision-making for prioritizing problems, allowing one to better focus the engineering resources, and
(iv) provide a tool for optimizing functioning processes.

Traditionally, one tends to change only one variable of an experiment at a time. The strength of the Taguchi technique is that one can change many variables at the same time and still retain control of the experiment. In present research, we solve the improved integral equations 1(a-e), (see eqns. 24 of [1]), by using orthogonal arrays[3] to identify the variable that has the most influence on the distortion so as to minimize it in the actual functioning of the axial compressor and provide relevant suggestions to the designers for improvement in compressor operation and performance.

$$\alpha\frac{d\alpha}{dx}=\frac{1}{K_3}\left(\frac{F_x-F_{x,0}}{U_0^2}\right) \tag{1a}$$

$$\alpha\frac{d\beta}{dx}=\frac{1}{\gamma}\left(\frac{F_y}{U_0^2}\right) \tag{1b}$$

$$\frac{d\alpha_0}{dx}=K_2\frac{d\alpha}{dx} \tag{1c}$$

$$\alpha_0\frac{d\beta_0}{dx}=\frac{1}{\gamma}\left(\frac{F_{y,0}}{U_0^2}\right) \tag{1d}$$

$$\frac{d}{dx}\left(\frac{p}{\rho}\right)=U_0{}^2\left(\frac{F_x}{U_0^2}-\alpha\frac{d\alpha}{dx}\right) \tag{1e}$$

Investigations of distortion propagation conditions in axial compressors have been carried out and analyzed in.[1] The main factors affecting the behavior of inlet distortion propagation are identified: axial inlet distorted velocity coefficient, $\alpha(0)$ and angle of inlet incidence, θ_0. These parameters are ranked according to their influence on the distortion using Taguchi table. Three values are selected for the axial inlet distorted velocity coefficient in the distorted region as 0.3, 0.5 and 0.7. The values chosen for the angle of inlet incidence are 15^0, 20^0 and 25^0. These values are chosen based on the research work done by Kim *et al.*[2] and Ng *et al.*[4,5] Therefore, by retaining these values in this work, the results obtained by Kim *et al.* can be verified and confirmed. Nevertheless, the ratio of drag-to-lift coefficient k_D/k_L has been redundant in this analysis as this factor is avoided after the application of airfoil characteristics in the recently coded novel integral scheme.[1]

The two factors to be analyzed are arranged as shown in Table. 1 The two factors highlighted in Table. 1 are arranged into an orthogonal array using two columns of $L_9(3^2)$, as shown on the left side of Table. 2 The experiments were carried out in 9 possible combinations. Column 1 of the table represents experimental numbers. For experiment

Table. 1 Factors and Levels

Factors	1	2	3
Axial inlet distorted velocity coefficient, $\alpha\ (0)$	$\alpha\ (0)_1 =$ 0.3[A_1]	$\alpha\ (0)_2 =$ 0.5[A_2]	$\alpha\ (0)_3 =$ 0.7[A_3]
Angle of inlet incidence, θ_0	$\theta_{01} =$ 15° [B_1]	$\theta_{02} =$ 20° [B_2]	$\theta_{03} =$ 25° [B_3]

Table. 2 Layout on Orthogonal Array

	Factors		Parameters		Distortion Region Size Y at X =1
No	A	B	$\alpha\ (0)$	θ_0	
1	1	1	0.3	15	0.497564
2	1	2	0.3	20	0.497346
3	1	3	0.3	25	0.497863
4	2	1	0.5	15	0.499567
5	2	2	0.5	20	0.499719
6	2	3	0.5	25	0.500398
7	3	1	0.7	15	0.500211
8	3	2	0.7	20	0.500407
9	3	3	0.7	25	0.500898

No.1 (row 1), the number "1" that appears in the orthogonal array for each of the factors A and B, implies that the experiment is calculated using A_1 and B_1. Likewise, for experiment No.8 (row 8), calculation is done using factors A_3 and B_2.

The nine experiments provided a fair comparison of A_1, A_2 and A_3; B_1, B_2 and B_3. Furthermore, they offer a good comparison between the two factors α (0) and θ_0 that are important inlet parameters of an axial compressor. Y is obtained from executing the in-house developed program.[1] In the program, the values of $\alpha\ (0)$ and θ_0 are changed according to the requirements of different experiments. The specifications of the single low speed experimental test cases[6] with the rotor blade (mid-span) speed is $\omega R = 36.6 m/s$, 510 rpm, $\sigma = 2$ (ratio of rotor blade speed to y-component of referenced inlet velocity), and the airfoil blade section is NASA 65-series, number of blades = 28, true chord and pitch = 6 inch respectively, solidity = 0.99 at mid span and hub/tip diameter ratio = 0.8. Fig. 1 displays the circumferential size at the inlet of a compressor.

The value of X at the inlet and outlet is 0 and 1 respectively. The inlet size of distortion (at X = 0) is Y = 0.5 and it is used as a reference to measure the variation of Y at the outlet. The outlet size of distortion, Y (at X = 1), is tabulated in the rightmost column of Table. 2 For instance, if the result is 0.48, it implies that the circumferential size of the distorted region starts with Y = 0.50 at the inlet and ends with Y = 0.48 at the outlet. A decrease in distortion can be calculated as $(\Delta Y) = 0.50 - 0.48 = 0.02$.

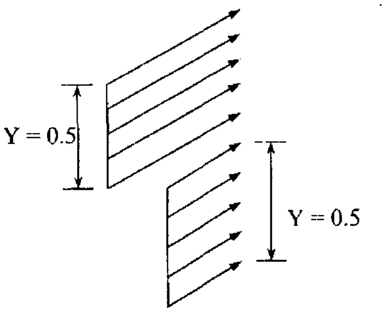

Fig. 1 Circumferential size at inlet of a compressor

The values of A_1, A_2 and A_3 are computed by the average value of Y over the experiments using level A_1 (No. 1-3), A_2 (No. 4-6) and A_3 (No. 7-9). These totals are:

$$A_1 = 0.497564 + 0.497346 + 0.497863 = 1.492774$$
$$A_2 = 0.499567 + 0.499719 + 0.500398 = 1.499685$$
$$A_3 = 0.500211 + 0.500407 + 0.500898 = 1.501516$$

The average value of Y for each set of the experiments was obtained by dividing the above figures by three that yield:

$$\overline{A_1} = 1.492774 / 3 = 0.497591$$
$$\overline{A_2} = 1.499685 / 3 = 0.499895$$
$$\overline{A_3} = 1.501516 / 3 = 0.500505$$

Similarity B_1, B_2 and B_3 are calculated by the average value of Y over the experiments using level B_1 (No. 1, 4 and 7), B_2 (No. 2, 5 and 8) and B_3 (No. 3, 6 and 9).

These totals are:

$$B_1 = 0.497564 + 0.499567 + 0.500211 = 1.497343$$
$$B_2 = 0.497346 + 0.499719 + 0.500407 = 1.497472$$
$$B_3 = 0.497863 + 0.500398 + 0.500898 = 1.499160$$

Their averages are:

$\overline{B_1}$ = 1.497343 / 3= 0.499114

$\overline{B_2}$ = 1.497472 / 3= 0.499157

$\overline{B_3}$ = 1.499160 / 3= 0.499720

Table. 3 Summarized Results of Parameters

Parameter	Value	Average Y at X =1	Range (%)
$\alpha (0)_1$	0.3	0.497591	0.291403
$\alpha (0)_2$	0.5	0.499895	
$\alpha (0)_3$	0.7	0.500505	
θ_{01}	15	0.499114	0.060555
θ_{02}	20	0.499157	
θ_{03}	25	0.499720	

3. RESULTS

Table. 3 presents the calculated average and the percentage of distortion range for different combinations of the values of the two parameters. The results indicate that the axial inlet distorted velocity coefficient, $\alpha (0)$ has a higher percentage of distortion range than that of the angle of inlet incidence, θ_0. Hence, it can be deduced that variation of axial inlet distorted velocity coefficient will have much more effect on the distortion.

In the previous research, Kim *et al.*[2] concluded that the two most important parameters that determine the growth of the distortion are the ratio of drag to lift coefficients of the blade, followed by the angle of inlet incidence in the distorted upstream flow. However, present results based on the novel integral scheme with actual airfoil characteristics (Table. 4 summarizes the collated data of the coefficients of lift and drag for NACA 65 series airfoil (65-006, 65-009, 65-206, 65-210 and 65-410 wing sections) suggest otherwise. Besides the fact that the ratio of lift to drag coefficients is not considered in the current analysis, the axial inlet distorted velocity coefficient has emerged as a crucial parameter that affects the distortion propagation rather than the angle of inlet incidence as highlighted in Kim *et al.*'s results.

In general, the Taguchi off-line quality control method facilitates the study of inlet parameters that affect the growth of the distortion propagation by differentiating their contributions to the inlet flow distortion in terms of their degree of influence.

4. CONCLUDING REMARKS

A parametric study is conducted using the Taguchi off-line quality method to investigate on the degree of influence on the distortion. The axial inlet distorted velocity coefficient and the angle of inlet incidence are selected based on their effects on various flow characteristics. The results of this analysis contradict that of Kim *et al.* These differences may be explained by the fact that the ratio of drag-to-lift coefficient needs not to be considered in present analysis. Furthermore, the development of the new integral method based on the actual airfoil characteristics may have altered

Table. 4 Collated data of the coefficients of lift and drag for other series of wing sections

NACA Wing Section Series	Collated Results	Range
65-006	$k_L = 0.1014\tilde{\alpha} - 0.02155$ $k_D = 0.61599\times10^{-2} - 0.23661k_L + 0.17057\times10^{-2}k_L^{\,2} + 1.23835k_L^{\,3}$ $k_D = 0.0037$	$\left(-8^0 \le \tilde{\alpha} \le 8^0\right)$ $\left(\tilde{\alpha} < -0.97^0,\ \tilde{\alpha} > 1.49^0\right) or \left(k_L < -0.12,\ k_L > 0.13\right)$ $\left(-0.97^0 \le \tilde{\alpha} \le 1.49^0\right) or \left(-0.12 \le k_L \le 0.13\right)$
65-009	$k_L = 0.1075\tilde{\alpha} + 0.01$ $k_D = 0.76134\times10^{-2} + 0.11655k_L + 0.14921\times10^{-2}k_L^{\,2} - 0.21389k_L^{\,3}$ $k_D = 0.0043$	$\left(-8^0 \le \tilde{\alpha} \le 10^0\right)$ $\left(\tilde{\alpha} < -1.49^0,\ \tilde{\alpha} > 1.67^0\right) or \left(k_L < -0.15,\ k_L > 0.19\right)$ $\left(-1.49^0 \le \tilde{\alpha} \le 1.67^0\right) or \left(-0.15 \le k_L \le 0.19\right)$
65-206	$k_L = 0.1013\tilde{\alpha} + 0.1811$ $k_D = 0.70780\times10^{-2} - 0.11038k_L - 0.50325\times10^{-2}k_L^{\,2} + 1.06401k_L^{\,3}$ $k_D = 0.0037$	$\left(-8^0 \le \tilde{\alpha} \le 8^0\right)$ $\left(\tilde{\alpha} < -0.90^0,\ \tilde{\alpha} > 0.88^0\right) or \left(k_L < 0.09,\ k_L > 0.27\right)$ $\left(-0.90^0 \le \tilde{\alpha} \le 0.88^0\right) or \left(0.09 \le k_L \le 0.27\right)$
65-210	$k_L = 0.1068\tilde{\alpha} + 0.1521$ $k_D = 0.84375\times10^{-2} - 0.38021k_L + 0.66460\times10^{-1}k_L^{\,2} + 1.61309k_L^{\,3}$ $k_D = 0.0043$	$\left(-8^0 \le \tilde{\alpha} \le 10^0\right)$ $\left(\tilde{\alpha} < -1.33^0,\ \tilde{\alpha} > 2.50^0\right) or \left(k_L < 0.01,\ k_L > 0.42\right)$ $\left(-1.33^0 \le \tilde{\alpha} \le 2.50^0\right) or \left(0.01 \le k_L \le 0.42\right)$
65-410	$k_L = 0.1053\tilde{\alpha} + 0.246$ $k_D = 0.75306\times10^{-2} - 0.10253\times10^{-1}k_L + 0.47345\times10^{-2}k_L^{\,2} + 0.32964\times10^{-1}k_L^{\,3}$ $k_D = 0.0043$	$\left(-8^0 \le \tilde{\alpha} \le 10^0\right)$ $\left(\tilde{\alpha} < 0.51^0,\ \tilde{\alpha} > 3.36^0\right) or \left(k_L < 0.30,\ k_L > 0.60\right)$ $\left(0.51^0 \le \tilde{\alpha} \le 3.36^0\right) or \left(0.30 \le k_L \le 0.60\right)$

the effects of inlet parameters. In all, this integral method signifies its validity to this field of research and offers much potential for further improvements.

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