Worst Closed-Loop Controlled Bulk Distributions of Stochastic Arrival Processes for Queue Performance

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Abstract: **This paper presents basic queueing analysis contributing to teletraffc theory, with commonly accessible mathematical tools. This paper studies queueing systems with bulk arrivals. It is assumed that the number of arrivals and the expected number of arrivals in each bulk are bounded by some constraints B and λ, respectively. Subject to these constraints, convexity argument is used to show that the bulk-size probability distribution that results in the worst mean queue performance is an extremal distribution with support $\{1, B\}$ and mean equal to λ. Furthermore, from the viewpoint of security against denial-of-service attacks, this distribution remains the worst even if an adversary were allowed to choose the bulk-size distribution at each arrival instant as a function of past queue lengths; that is, the adversary can produce as bad queueing performance with an open-loop strategy as with any closed-loop strategy. These results are proven for an arbitrary arrival process with bulk arrivals and a general service model.**

Index Terms: **Performance tools and methodology, queueing analysis, teletraffic.**

I. INTRODUCTION

As diverse network services are integrated, traffic considerations become very important for quality of service and cost effectiveness. A protocol data unit (PDU) arriving at a service point of communication networks have a random size with the constraint on the maximum size (e.g., maximum transfer unit [1]). The distribution on the finite support, however, is unknown a priori and may well be non-static. The distribution of the PDU size is a possible application of the results presented in this paper.

We consider queueing systems with bulk arrivals, described in terms of three stochastic processes: a) An arrival process that specifies the times at which items arrive; b) a bulk-size process that describes the number of items (e.g., bytes in PDU) arriving at each arrival time; c) a service process that determines the service completion times of the items in queue. The bulk size can model the lengths of PDUs arriving, and the randomness of the service process can possibly model the randomly time-varying wireless links as the transmission speed may adapt to the channel condition or as ARQ and/or the adaptive error correction coding schemes may be employed. We assume that the joint statistics of the arrival and the service processes are given. (The arrival process and the service process are allowed to be statistically dependent.) We assume that the bulk-size process and the vector process having the arrival and service processes as its components are statistically independent of each other. Regarding the bulk-size process, we assume that the bulk-sizes at different arrival times are statistically independent and that the bulk size at the n-th arrival time is a random variable U_n described by a probability mass function f_n. Subject to the constraints $E[U_n] \leq \lambda$ and $U_n \in \{1, \cdots, B\}$, we are interested in finding a sequence $\{f_n\}_{n=1}^{\infty}$ of bulk-size distributions that leads to the worst possible values for certain natural performance measures such as the expected number of items waiting in queue. Let $\mathcal{F}$ be the set of all probability mass functions satisfying the above two constraints, and let $\mathcal{F}_\lambda$ be the subset of $\mathcal{F}$ in which the constraint $E[U_n] \leq \lambda$ is satisfied with equality. Throughout this paper, we assume that $\lambda \leq B$ so that $\mathcal{F}_\lambda$ and $\mathcal{F}$ are nonempty. Let $f^* \in \mathcal{F}$ be the "extremal" distribution defined by

$$f^*(i) = \begin{cases} \frac{B-\lambda}{B-1}, & \text{if } \ i = 1, \\ 0, & \text{if } \ 1 < i < B, \\ \frac{\lambda-1}{B-1}, & \text{if } \ i = B. \end{cases}$$

The results of this paper establish that for a wide class of systems and performance measures, the worst case sequence of bulk-size distributions is the sequence $\pi^* = (f^*, f^*, \cdots)$. The set of systems to be considered includes the queueing systems with highly correlated interarrival times with bulk arrivals and/or non-exponential service times.

It will be seen that, in fact, the results hold in an even stronger sense. Let us introduce an adversary who at any arrival time, is allowed to choose the distribution of the current bulk-size based on a fair amount of information on the realization of the arrival and service processes. It will be shown that even under such circumstances, the sequence π^* remains the worst-case choice of bulk-size distributions. In other words, it makes no difference if we allow the adversary to use "closed-loop" strategies. Furthermore, statistical dependence between the bulk-sizes at different arrival times cannot worsen the value of the performance measures under consideration.

It is fair to view f^* as the "most bursty" element of $\mathcal{F}$. In that respect, this paper establishes that out of all bulk-size processes with given mean and support, the most bursty one leads to the worst queueing delay. This result can be viewed as the opposite extreme of the results on the stochastic processes that minimize the average queueing delay [2]–[4]. The methodology of the present paper can also be viewed as an application of the stochastic ordering concept [5]–[7].

Performance under the worst case arrival pattern presents a new field in teletraffic theory [8]. The studies in this field can be categorized by the combination of two criteria; the performance measure with which to decide what is the worst and the set of traffic patterns among which to decide the worst one. Much of the existing literature uses the cell (packet) loss rate as

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the performance measure and considers the set of traffic passing through the leaky bucket regulation [9]–[21]. Reference [8] considers the cell loss ratio as the performance measure of a multiplexer with deterministic bandwidth, wherein different traffic sources are multiplexed. Under the assumption that the cell arrival patterns are stationary and ergodic, upper bounds of the cell loss ratio ("conservative CLR estimation") are elaborately defined, and then the worst case cell arrival patterns for a tightly bounding conservative CLR (cell loss ratio) estimation were identified from the set of patterns conforming the leaky bucket constraint. References [22]–[24] again consider cell loss ratio (CLR) as a performance measure of the multiplexer, wherein input traffic streams are all from the leaky-bucket-based sources. Various cases were discussed; for example, in some cases the traffic sources input to the multiplexer are constrained to have identical patterns (homogeneous traffic), and in other cases without such constraint. In particular, much of the literature discussed whether the on-off process is the worst pattern. In some cases (e.g, homogeneous sources and unbuffered multiplexer), the on-off process was proven to be the worst [23]. However, [22] and [23] provide cases wherein the three-state source causes worse cell loss ratio than the on-off sources. Reference [25] employs loss rate, but for the application of multimedia communication any traffic failing to meet a certain delay requirement is counted as loss. (This performance measure has been also used in [26] under a completely deterministic formalism.)

References [24], [27], and [28] consider as a performance measure the queue length distribution of the multiplexer with an infinite buffer. Again, with this performance measure is discussed the issue of whether the on-off pattern is the worst one passing through the leaky bucket. Reference [27] studies the traffic pattern consisting of periodic bursts of a maximum length under the cell delay variation constraint at the peak rate followed by a silence period. This is an on-off pattern, and the papers study the queue length distribution for the cases that multiple sources with such traffic patterns are multiplexed. Reference [28] provides through simulation a traffic pattern that results in the queue length's survival function (complementary cumulative distribution) worse than the on-off pattern. In fact, [24] considers both the queue length distribution and the cell loss rate as performance measures and compares the on-off pattern and the pattern presented in [28]. The simulation results indicate that the on-off pattern exhibits the worse cell loss rate yet better queue length distribution than the pattern presented in [28]. In [29], the performance measure in determining the "worst" is the variation of the interarrival times. With this measure, the worst traffic passing through leaky bucket regulations is evaluated.

Most literature mentioned above views the traffic as a stationary stochastic process. In deterministic setup, the studies relating to obtaining tight bounds on the worst delay of traffic regulated by leaky bucket have been elegantly presented in various contexts in different forms [20], [26], [30]–[33]. This approach can be also viewed as bounding performance of the worst traffic where the performance measure is the maximum delay experienced by traffic.

Study of the worst arrival traffic in the present paper is different from the above literature in that we focus on the traffic described by a stochastic bulk arrival process and identify the worst sequence of bulk distributions. In addition, the results in the present paper hold true even under the assumption that the adversary generates the worst-case traffic, in a closed-loop manner, by observing the queue status. As a performance measure, this paper considers the expected value of a general increasing and convex function of the queue length.

II. PRELIMINARIES

The following simple property of the distribution f^* is useful in establishing the results of this paper.

Lemma 1: Let U be a random variable with probability mass function $f \in \mathcal{F}$ and let $g : \mathcal{R} \to \mathcal{R}$ be convex. Then, the value of $E[g(U)]$ is maximized over all $f \in \mathcal{F}_\lambda$ if $f = f^*$. Furthermore, if g is also nondecreasing, then f^* maximizes $E[g(U)]$ over the set $\mathcal{F}$ as well.

Proof: Consider some $f \in \mathcal{F}_\lambda$ such that $f(v) = \delta > 0$ for some v satisfying $1 < v < B$. We can construct another probability mass function $\hat{f} \in \mathcal{F}_\lambda$ by letting $\hat{f}(v) = 0$, $\hat{f}(B) = f(B) + (v-1)\delta/(B-1)$, $\hat{f}(1) = f(1) + (B-v)\delta/(B-1)$, and $\hat{f}(u) = f(u)$ if $u \notin \{1, v, B\}$. It is easily seen that $\hat{f} \in \mathcal{F}_\lambda$. Let

$$\Delta = \sum_{u=1}^{B} \hat{f}(u)g(u) - \sum_{u=1}^{B} f(u)g(u).$$

Then,

$$\begin{aligned}
\Delta &= \frac{v-1}{B-1}\delta[g(B)-g(v)] + \frac{B-v}{B-1}\delta[g(1)-g(v)] \\
&= \frac{v-1}{B-1}\delta \sum_{i=v+1}^{B} [g(i)-g(i-1)] \\
&\quad - \frac{B-v}{B-1}\delta \sum_{i=2}^{v} [g(i)-g(i-1)] \\
&\geq (B-v)\frac{v-1}{B-1}\delta[g(v+1)-g(v)] \\
&\quad -(v-1)\frac{B-v}{B-1}\delta[g(v+1)-g(v)] \\
&= 0.
\end{aligned}$$

The inequality above follows from the convexity of g. By repeating this process up to $B-2$ times, we end up with a probability mass function which is zero outside $\{1, B\}$ and which belongs to $\mathcal{F}_\lambda$. Such a probability mass function can only be equal to f^*. Furthermore, throughout this process, the value of the objective function cannot decrease, and this shows that f^* maximizes the objective function over the set $\mathcal{F}_\lambda$. Let us now assume that g is nondecreasing. Then, it is clear that by increasing the mean of U, we can increase $E[g(U)]$, and this implies that the maximum of $E[g(U)]$ over $\mathcal{F}_\lambda$ is the same as the maximum over $\mathcal{F}$. □

Lemma 2: If the functions $f : \mathcal{R} \to \mathcal{R}$ and $g : Z^+ \to \mathcal{R}$ are convex and f is nondecreasing, the composite function, $f \circ g : Z^+ \to \mathcal{R}$ is convex.

Proof:

$$\begin{aligned}
f(\, g(\lambda x + (1-\lambda)y)\,) &\leq f(\, \lambda g(x) + (1-\lambda)g(y)\,) \\
&\leq \lambda f(g(x)) + (1-\lambda)f(g(y)).
\end{aligned}$$

The first inequality holds because g is convex, and f is nondecreasing. The second inequality holds because f is convex. □

III. A SIMPLE DISCRETE-TIME MODEL WITH DETERMINISTIC INTERARRIVAL TIMES

In this section, we consider a simple discrete-time queueing system. The arrival process is deterministic with arrivals occurring at each integer time. The service process is specified in terms of a sequence $\{Q_n\}$ of random variables as follows: The number of items served during the time interval $[n, n+1)$ is equal to Q_n unless we run out of items in the queue. More precisely, let $X(t)$ be the number of items in the queue at time t, assumed to be a right-continuous process. Then, $X(t)$ changes only at integer times and evolves according to the equation

$$X(n+1) = [X(n) - Q_n]^+ + U_{n+1}, \quad (1)$$

where $[a]^+ \equiv \max\{a, 0\}$.

Theorem 1: The sequence of bulk-size distributions π^* maximizes $E[g(X(n))]$ for every nonnegative integer n and for every convex and nondecreasing function $g : \mathcal{R} \to \mathcal{R}$.

Proof: Fix some n and let $m \le n$. We will show that the worst-case bulk-size distribution f_m at time m is equal to f^*. Let us fix a sample path of the service process $\{Q_n\}$ and let us also condition on the values of $\{U_k | k \neq m\}$. Using Lemma 2, an easy inductive argument based on (1) shows that $X(n)$ is a convex nondecreasing function of U_m. Using Lemma 2, $g(X(n))$ is also a convex nondecreasing function of U_m. Then, Lemma 1 implies that

$$E[\, g(X(n)) \mid \{Q_n\}, \{U_k, k \neq m\}\,], \quad (2)$$

is maximized by letting $f_m = f^*$. It follows that $E[g(X(n))]$ is also maximized by letting $f_m = f^*$. Since this argument is valid for every m, the result is proved. □

As a corollary of Theorem 1, the sequence π^* maximizes $E[X(n)]$ for all $n \ge 0$. In particular, it maximizes the infinite-horizon average and the infinite horizon discounted expected number of items in the system.

IV. GENERAL ARRIVAL PROCESS

In this section, we extend the results of the previous section to general arrival processes with bulk arrivals. The arrival process is defined here in terms of an infinite sequence of arrival times. No further assumptions will be needed on the statistics of this process; e.g., interarrival times can be correlated in any way. Let $\mathcal{N} = \{N(t) | t \ge 0\}$ be a right-continuous counting process. The process $N(t)$ models virtual service completion times: at each time that $N(t)$ jumps by 1, service is completed for the item currently being served, if any; if no item is currently served, nothing happens. If, as a special case, $N(t)$ is a Poisson counting process, this model is equivalent to the standard model of a server with exponentially distributed service times due to the memoryless property of the Poisson process. Let $X(t)$ be the queue size at time t, assumed to be a right-continuous function of time.

Theorem 2: The sequence of bulk-size distributions π^* maximizes $E[g(X(t))]$ for every $t \ge 0$ and for every convex and nondecreasing function $g : \mathcal{R} \to \mathcal{R}$.

Proof: Let $\mathcal{A} = \{T_n | n = 1, 2, \cdots\}$ denote the arrival process, where T_n are the arrival times. Let $X_n = X(T_n)$ be the queue size immediately after the n-th bulk arrival. We notice that X_n evolves according to

$$X_{n+1} = [\, X_n - \{\, N(T_{n+1}) - N(T_n)\,\}\,]^+ + U_{n+1}. \quad (3)$$

Let us consider $g(X(t))$ at an arbitrary time t. Let us fix, by conditioning, a particular sample path of the process $\mathcal{A}$. Conditioned on $\mathcal{A}$, there exists an integer $n(t, \mathcal{A})$ such that $T_{n(t,\mathcal{A})} \le t < T_{n(t,\mathcal{A})+1}$. Furthermore, $X(t) = [\, X_{n(t,\mathcal{A})} - N(t) + N(T_{n(t,\mathcal{A})})\,]^+$. Let us consider the problem of choosing f_m so as to maximize

$$\begin{aligned} E[\, g(X(t)) | \mathcal{N}, \mathcal{A}, \{U_k, k \neq m\}\,] \\ = E\Big[\, g\left([X_{n(t,\mathcal{A})} - N(t) + N(T_{n(t,\mathcal{A})})]^+\right) \Big| \\ \mathcal{N}, \mathcal{A}, \{U_k, k \neq m\}\Big], \end{aligned} \quad (4)$$

for an arbitrary set of bulk distributions $\{f_k, k \neq m\}$. Using (3), we see that $X_{n(t,\mathcal{A})}$ is a convex and nondecreasing function of U_m. Using Lemma 2 twice, we conclude that $g([X_{n(t,\mathcal{A})} - N(t) + N(T_{n(t,\mathcal{A})})]^+)$ is also a convex and nondecreasing function of U_m. Lemma 1 then implies that the maximum of (4) is achieved by letting $f_m = f^*$ for each realization of $\mathcal{N}$, $\mathcal{A}$, $\{U_k, k \neq m\}$. Therefore, $f_m = f^*$ maximizes $E[g(X(t))]$ for an arbitrary set of bulk distributions $\{f_k, k \neq m\}$. Therefore, the sequence π^* maximizes $E[g(X(t))]$. □

Comments on modeling

If the process $N(t)$ is a Poisson counting process, the queueing model is reduced to the one with an exponential service time distribution. Thus, the queueing model presented in this section is a generalization of the single-server queueing system with an exponential server—e.g., $G/M/1$ with bulk arrivals. For another queueing model, which is more intuitive, Appendix will prove that the statement in Theorem 2 is still true.

V. ADVERSARY'S STRATEGY

The fact that $f_m = f^*$ maximizes expression (2) for each realization of $\{Q_n\}$ and $\{U_k, k \neq m\}$ generalizes the result of Theorem 1 further. Even if an adversary were given knowledge of the sample path of the service process $\{Q_n\}$, and were allowed to choose the bulk-size distributions based on such information, the sequence π^* would be still chosen for the purpose of maximizing $E[g(X(n))]$ for the following reason. Since $f_m = f^*$ maximizes the expression (2) for each sample path of $\{Q_n\}$ and $\{U_k, k \neq m\}, f_m = f^*$ maximizes $E[g(X(n))|\{Q_n\}]$ without regard to the choice of $\{f_k, k \neq m\}$. As a result, π^* maximizes $E[g(X(n))|\{Q_n\}]$ for each sample path of $\{Q_n\}$. Therefore, π^* maximizes $E[g(X(n))]$.

Moreover, consider a case where an adversary is given knowledge of the realization of $\{U_k, k < m\}$ at each decision making time m as well as the sample path of the service process $\{Q_n\}$, and is allowed to choose the bulk-size distributions based on such information. In this case, an adversary is allowed to exercise a closed-loop strategy, where a closed-loop strategy is defined as a set of mappings

$$\mu_m : \{\{Q_n\}\} \times \{\{U_k, k < m\}\} \to \mathcal{F}, \quad \text{integer } m.$$

Even in this case, an open-loop strategy that uses the sequence π^* maximizes $E[g(X(n))]$ for the following reason. At each time $m, f_m = f^*$ maximizes expression (2) for each realization of $\{Q_n\}, \{U_k, k < m\}$, and $\{U_k, k > m\}$. Therefore, without regard to mappings $\{\mu_k, k \neq m\}, f_m = f^*$ maximizes $E[g(X(n))|\{Q_n\}, \{U_k, k < m\}]$ for each realization of $\{Q_n\}$ and $\{U_k, k < m\}$. Therefore, a constant mapping $\mu_m(\{Q_n\}, \{U_k, k < m\}) = f^*$ maximizes $E[g(X(n))]$ for any $\{\mu_k, k \neq m\}$. Hence, the open-loop strategy that uses the sequence π^* maximizes $E[g(X(n))]$. Since the open-loop strategy is maximal in the case that the information of $\{Q_n\}$ and $\{U_k, k < m\}$ is available to an adversary at each decision making time m, the open-loop strategy is also maximal for the case that any less amount of information (e.g., only intermediate information of $\{Q_k, k < m\}$ and $\{U_k, k < m\}$) is given to an adversary at each time m. As a side remark, we can alternatively prove this result, using convexity and backward induction from the dynamic programming equation.

Again, it is clear from the proof of Theorem 2 that open-loop strategy π^* would remain the worst-case sequence of bulk-size distributions even if an adversary were given full knowledge of the realization of the processes $\mathcal{A}$, $\mathcal{N}$, and the past bulk sizes $\{U_k, k < m\}$ prior to choosing f_m. (The argument follows the same pattern as the case of the discrete time model.) The sequence π^* would also remain the worst-case sequence if the adversary were only given some intermediate (less) amount of information. For example, at any arrival instant, the adversary might be allowed to use the knowledge of past arrival times and of the current number of items in queue: The result would still be the same.

VI. DISCUSSIONS

The proofs in the present paper explicitly establish Theorems 1 and 2 in the queueing system described by bulk process as well as the arrival and service processes, even under the assumption that allows arrival and service processes to be statistically dependent with each other. The present paper also addresses the strategy setting wherein an adversary is given certain degrees of knowledge in the sample paths of the bulk process, the service process, and the arrival process in choosing the bulk size distributions for the denial-of-service attack. The proofs provided by the present paper explicitly show that the open-loop control choosing the extremal distribution is the worst for the queue, even in the case the adversary is given various degrees knowledge on the sample paths.

We also note that these results in the present paper extend to the fluid queue wherein the random bulk size U_n takes values in continuum $[a, b]$ and the virtual service process $N(t)$ is generalized to have nondecreasing sample functions (not necessarily discrete counting process). For this fluid queueing system, the adversary's open-loop choice of extremal probability distribution

$$F^*_{U_n}(u) \equiv P^*(U_n \le u) \quad \equiv \quad \begin{cases} 0, & \text{if } u < a, \\ \frac{b-\lambda}{b-a}, & \text{if } a \le u < b, \\ 1, & \text{if } u \ge b, \end{cases}$$

which is a generalization of the extremal probability mass function f^*, results in the worst queue performance under the assumption of the adversary's various degrees of knowledge discussed before. The proof of Theorem 2 remains valid in this fluid queue as well.

APPENDIX

This appendix will augment the discussion in Section IV with a different queueing model. Instead of using the virtual service completion counting process, $N(t)$, defined in Section IV, we now consider a nonnegative random process $\gamma(t)$ to model the service of items. An item that begins receiving its service at time t_0 departs at $\inf\{\tau | \int_{t_0}^{\tau} \gamma(t)dt = 1\}$. In this appendix, we assume that the queueing discipline is the first-in-first-out (FIFO). Random process $\gamma(t)$ can be interpreted as the instantaneous service rate available to serve an item. In this appendix, we show that the statement in Theorem 2 is still true with this model of the queueing system.

Let us fix, by conditioning, a particular sample path of the arrival process $\mathcal{A}$. Then, we have

$$X(t) \quad = \quad X_0 + \sum_{k=1}^{n(t,\mathcal{A})} U_k - D(t), \tag{5}$$

where X_0 is the number of items in the queue size at some time T_0 prior to the first bulk arrival, and $D(t)$ is the number of departures in interval $[T_o, t]$. Let Γ denote the service rate process $\gamma(t), -\infty < t < \infty$. Let us also fix, by conditioning, a particular sample path of the process Γ and a particular sample path of $\{U_k, k \neq m\}$. We now consider the problem of choosing f_m so as to maximize

$$E[\, g(X(t)) \,|\Gamma, \mathcal{A}, \{U_k, k \neq m\}\,]. \tag{6}$$

Lemma 3: $X(t)$ at an arbitrary time t is a convex and nondecreasing function of U_m, given fixed sample paths, $\Gamma, \mathcal{A}, \{U_k, k \neq m\}$.

Proof: We consider two cases: $T_m > t$ and $T_m \le t$.

Case 1. $T_m > t$:

The queue size at t is not affected by the bulk size of its future arrival U_m. Therefore, $X(t)$ is a constant function of U_m, and thus is a convex and nondecreasing function of U_m.

Case 2. $T_m \le t$:

Equation (5) can be re-written as

$$X(t) \quad = \quad X_0 + \sum_{k \neq m, k=1}^{n(t,\mathcal{A})} U_k + U_m - D(t). \tag{7}$$

We note that $D(t)$ at time $t > T_m$ is a function of U_m for fixed sample paths of $\Gamma, \mathcal{A}, \{U_k, k \neq m\}$ and a fixed value of X_0. To make this explicit, we will also denote $D(t)$ as $D(t, U_m)$. We consider the following three subcases defined by fixed sample paths, $\Gamma, \mathcal{A}, \{U_k, k \neq m\}$.

Case 2.a. $X(\tau) \geq 1, \forall \tau \in [T_m, t]$ for $U_m = 1$:
In this case, the server of the queueing system never runs out of an item to serve in $[T_m, t]$; even for $U_m = 1$, the service rate $\gamma(\tau)$ is fully utilized at each moment $\tau \in [T_m, t]$. Thus, the number of departures in $[T_m, t]$ cannot be increased by increasing the bulk size U_m, and we have $D(t,1) = D(t,2) = \cdots = D(t,B)$. Therefore, $U_m - D(t, U_m)$ is a linearly increasing function of U_m, and this implies that $X(t)$, in accordance with (7), is a convex and nondecreasing function of U_m.

Case 2.b. For $U_m = B$, $X(\tilde{t}) = 0$ for some $\tilde{t} \in [T_m, t)$:
In this case, all U_m items entering the queueing system depart prior to time t no matter what the bulk size U_m is. Therefore, $U_m - D(t, U_m)$ is a constant function of U_m. This implies that $X(t)$, in accordance with (7), is a convex and nondecreasing function of U_m.

Case 2.c. For $U_m = 1$, $X(\tilde{t}) = 0$ for some $\tilde{t} \in [T_m, t)$, and for $U_m = B$, $X(\tau) \geq 1, \forall \tau \in [T_m, t]$:
In this case, there is the unique integer $l \leq B$ such that i) the queueing system becomes empty at some time in $[T_m, t)$ if $1 \leq U_m \leq l-1$, and ii) $X(\tau) \geq 1, \forall \tau \in [T_m, t]$ if $l \leq U_m$. For $U_m = 1, 2, \cdots, l-1$, $U_m - D(t, U_m)$ is constant. With regard to the value $U_m = l$, we have

$$U_m - D(t, U_m) = l - D(t,l) = \begin{cases} (l-1) - D(t, l-1), \\ (l-1) - D(t, l-1) + 1, \end{cases} \tag{8}$$

because $D(t,l)$ is either $D(t, l-1)$ or $D(t, l-1)+1$. With regard to $U_m > l$, we have

$$(l+i) - D(t, l+i) = l - D(t,l) + i, \quad \text{for } i = 1, 2, \cdots. \tag{9}$$

Therefore, $U_m - D(t, U_m)$ is a convex and nondecreasing function of U_m.

In each of the cases 2.a, 2.b, and 2.c, $U_m - D(t, U_m)$ is a convex and nondecreasing function of U_m. This and (7) implies that $X(t)$ is a convex and nondecreasing function of U_m for case 2 ($T_m \leq t$). Due to the results in cases 1 and 2, $X(t)$ is a convex and nondecreasing function of U_m. □

Lemmas 2 and 3 imply that $g(X(t))$ is a convex and nondecreasing function of U_m, given an arbitrary set of sample path realizations $\Gamma, \mathcal{A}, \{U_k, k \neq m\}$. Lemma 1 then implies that the maximum of (6) is achieved by letting $f_m = f^*$ for each realization of Γ, $\mathcal{A}$, $\{U_k, k \neq m\}$. Therefore, $f_m = f^*$ maximizes $E[g(X(t))]$ for an arbitrary set of bulk distributions $\{f_k, k \neq m\}$. Therefore, the sequence π^* maximizes $E[g(X(t))]$.

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