ECM and GLR Based Multiuser Detection with I-CSI

Antonio De Maio, Roberto Episcopo, and Marco Lops

Abstract: **This paper deals with the problem of multiuser detection over a direct-sequence code-division multiple access (DS-CDMA) channel with incomplete channel state informations (I-CSI). We devise and assess two novel recursive detectors based on the expectation conditional maximization (ECM) algorithm and the generalized likelihood ratio (GLR) principle, respectively. Both receivers entail an affordable computational complexity. Moreover, the performance assessment, conducted via Monte Carlo techniques, shows that they achieve satisfactory performance levels and outperform linear detectors.**

Index Terms: **DS-CDMA, ECM, GLR, I-CSI, multiuser detection.**

I. INTRODUCTION

The tremendous push towards the design and the assessment of multiuser detection schemes started as Verdù in his 1986 paper [1] showed that CDMA is *not* interference-limited, once co-operative detection between the active users is performed. The only apparent limitation to the complete exploitation of the potentials of CDMA appeared to be the computational burden required by fully cooperative detection, where optimum decision can be made at the cost of receivers which are exponentially complex in the user number. Thus, a first research track has been to devise new multiuser detection schemes, retaining some key characteristics of the optimum detector, while requiring an affordable computational complexity: The interest soon focused on linear receivers, including the decorrelating detector [2] and the minimum-mean square error (MMSE) receiver [3].

Linear detectors, however, have some major drawbacks, and in particular very poor performance in close-to-saturation—if not overloaded—scenarios, i.e., in the presence of a user-number close to the processing gain. Accordingly, a number of non-linear detection schemes have been devised and assessed (see [4, p. 344 ff.] and references therein), whose ultimate goal is to approximate the performance of the optimum maximum likelihood (ML) receiver. Among them, we find receivers relying on successive interference cancellation, multistage detectors [5], decision-driven [6], and decision-feedback receivers [7].

Most of these receivers share the need for complete channel state information (C-CSI), and indeed the values of the timing (one and the same for all of the users), the energies and the phases of the signals contributing the CDMA multiplex should be either known or perfectly estimated at the receiver end. Obviously, this situation is realistic in several applications, such as the downlink of a system with no multipath fading or a satellite haul, both with side channel informations. It is, however, understood that the availability of such a C-CSI calls for either a large transmitted power or very heavy signaling protocols, that may be far beyond the reach of the allowable system complexity. In particular, the assumption that the signal energies be exactly known at the receiver in a mobile terrestrial network may be endangered due to imperfect power control and/or shadowing as well as fading effects.

In the past several years, the EM algorithm has been also exploited in order to solve the problem of multiuser detection under different scenarios. For instance, in [8] the authors derive general iterative EM-based receivers for synchronous DS-CDMA systems achieving satisfactory performance. Moreover, in [9] the authors use EM-based algorithm to solve the problem of blind detection in a synchronous CDMA system with no *a priori* knowledge of the users' spreading sequences.

The present study represents a contribution to the problem of suboptimum multiuser detection in the presence of I-CSI, presenting two iterative receivers. Keeping the assumption of a known reference phase (the case of unknown phase can be handled by resorting to differential encoders and decoders), we relax the assumption that the signals energy be known. More precisely, we focus on the case that all of the user signals share one and the same unknown energy value—or, equivalently, that the energies are known but for a proportionality factor, common to all of the users, accounting for non-ideal control and/or for shadowing and/or fading effects: The channel state, which for the application at hand is merely the said common proportionality factor, is allowed to vary on a bit interval time basis. We explicitly highlight that the problem of simultaneously estimating bits and channel parameters is also addressed in [10] assuming a channel model with more unknown parameters than that considered in the present paper. The authors devise a two-step algorithm involving both the Gauss-Siedel as well as the EM algorithm. Specifically, in the first stage EM or least squares procedures are exploited for estimating the channel state while, in the latter, the multistage detector designed in [5] is applied for bit decoding. Moreover, in order to have sufficient data samples for estimating the unknown parameters, the authors assume that the channel is constant over a data packet duration and thus their procedure can not be applied with reference to fast fading channels, such as the one considered in the present paper, where the channel state may change on a bit interval time basis.

For the said scenario, we consider two possible strategies to cope with I-CSI. The former one, based on a Bayesian approach, consists of assigning the prior probability density function (pdf) of the channel state (CS). An ECM algorithm is thus designed around such a channel model, so as to take advantage of the inherent capability of the ECM approach to cope with incomplete data problems. The latter one, instead, assumes that not even the priors of the CS are known at the receiver end, thus resorting to

Manuscript received February 10, 2003; approved for publication by Kwang Bok Lee, Division II Editor, February 17, 2004.

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the GLR to remove the *a priori* uncertainty.

Even though for neither strategy convergence to the performance of the optimum multiuser detector can be analytically proven, reassuring arguments as to their asymptotic behavior can be in any case put forward. On the other hand, the performance assessment, conducted through Monte Carlo counting techniques, confirms that both receivers achieve visible gains with respect to linear receivers and that the reliability of the decisions rapidly increases as the iteration number increases, irrespective of the initial-stage decisions.

The paper is organized as follows. In Section II, we present the problem formulation and the design issues, Section III is devoted to the performance assessment, and finally, conclusions are drawn in Section IV.

II. PROBLEM FORMULATION AND DESIGN ISSUES

Let us consider a synchronous DS-CDMA network with K active users and assume that a known reference phase is available. We also suppose that all of the signal energies are known but for a proportionality factor, common to all of the users, accounting for non-ideal control and/or for shadowing and/or fading effects. Thus, adopting a BPSK modulation format, the baseband equivalent of the received signal in a symbol period, T, can be written as

$$r(t) = \sum_{i=1}^{K} A\sqrt{\mathcal{E}_i} b_i s_i(t) + n(t), \qquad (1)$$

where

- A is an unknown parameter, possibly random, accounting for the for I-CSI;
- $\sqrt{\mathcal{E}_i}$ is the square root of the i-th user energy;
- $b_i \in \{-1, 1\}$ is the bit transmitted by the i-th user;
- $s_i(t)$ is the signature waveform of duration T assigned to the i-th user, i.e., $s_i(t) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} \overline{s_i}(n) p(t - nT_c)$, with $\boldsymbol{s}_i = [\overline{s_i}(1), \cdots, \overline{s_i}(N)]^T$ the code sequence, $\overline{s_i}(n) \in \{-1, 1\}$ ($i = 1, \cdots, K$ and $n = 1, \cdots, N$), $p(t)$ a pulse of duration T_c, and N the spreading factor;
- $n(t)$ is additive zero-mean Gaussian noise with spectral density σ^2.

Notice that a more general model for the downlink channel would consider multipath fading implying that the overall received signal from the i-th user be $\sqrt{\mathcal{E}_i} b_i s_i(t) * h(t)$, where $*$ denotes the convolution operator and

$$h(t) = A \sum_{l=1}^{L} \alpha_l \delta(t - \tau_l),$$

is the channel impulsive response. In this case, assuming that the delays τ_l's and the amplitudes α_l's are known, the procedures, which will be derived in the sequel of the paper, can be easily generalized.

In the sequel, we model A as a Nakagami distributed random variable, i.e., the pdf of A can be written as

$$f_A(x) = 2\left(\frac{m}{\overline{A^2}}\right)^m \frac{x^{2m-1}}{\Gamma(m)} \exp\left(-\frac{m}{\overline{A^2}} x^2\right) u(x), \qquad (2)$$

wherein m is a positive integer number, $u(\cdot)$ is the unit step function, $\Gamma(\cdot)$ is the Eulerian gamma function, and $\overline{A^2}$ is the mean square value of the random variable A. We explicitly note that the pdf (2) is a family commonly employed in the design and the analysis of receivers for fading channels (in urban environment). In particular, it subsumes the Rayleigh distribution as the special case corresponding to $m = 1$, and for $m = 2$, it approximates the Rice probability law. Finally, for m approaching ∞, the random fluctuation vanishes and we end up with the AWGN channel model.

With reference to (1), the sampled matched filter outputs are

$$r_i = \int_0^T r(t) s_i(t) dt, \qquad i = 1, \cdots, K,$$

and the resulting discrete-time model is

$$\boldsymbol{r} = A\boldsymbol{R}\boldsymbol{\mathcal{E}}\boldsymbol{b} + \boldsymbol{n},$$

where $\boldsymbol{r} = [r_1, \cdots, r_K]^T$, $\boldsymbol{\mathcal{E}} = \mathrm{diag}(\sqrt{\mathcal{E}_1}, \cdots, \sqrt{\mathcal{E}_K})$, $\boldsymbol{R} = \{\rho_{i,j}\}$, $\rho_{i,j} = \mathbf{s}_i^T \mathbf{s}_j$, $\boldsymbol{b} = [b_1, \cdots, b_K]^T$, $\boldsymbol{n} = [n_1, \cdots, n_K]^T$, and $n_k = \int_0^T n(t) s_k(t) dt$. Moreover, the filtered noise $\boldsymbol{n}$ is a zero-mean Gaussian vector with covariance matrix $E[\boldsymbol{n}\boldsymbol{n}^T] = \sigma^2 \boldsymbol{R}$; with $E[\cdot]$ denoting statistical average and $(\cdot)^T$ the transpose operator.

The optimum (minimum error probability) detector for decoding $\boldsymbol{b}$ requires an exhaustive search over a set of cardinality 2^K which can be prohibitive for a real time implementation of the processor. It is thus of interest to envisage different design criteria, trading complexity for performance. To this end, in the sequel, we propose two novel decision rules based on different design criteria. More precisely, the former detector is devised according to the ECM algorithm and the latter according to the GLR strategy.

A. ECM-Based Receiver

The ECM algorithm, introduced by Meng and Rubin in [11], is a natural extension of the EM algorithm, which turns out advantageous as the M-step can be implemented through a chain of conditional maximizations, where conditioning is performed on suitably defined functions of the unknown parameters. The ECM algorithm therefore replaces the M-step of the plain EM by a number of computationally simpler conditional maximization (CM) steps. As a consequence, it typically converges more slowly than the EM algorithm in terms of number of iterations, but can be faster in total computer time. Finally, the ECM algorithm keeps the appealing convergence properties of the EM algorithm such as its monotone convergence.

Letting $\widehat{\boldsymbol{b}}^{(0)}$ an initial estimate of $\boldsymbol{b}$ it can be shown (see Appendix) that the ECM-based receiver, for the problem at hand, calculates the estimate of the transmitted sequence at the $(t+1)$-th iteration, $\widehat{\boldsymbol{b}}^{(t+1)}$ say, according to the following procedure

1. Compute $u^{(t)} = \frac{\widehat{\boldsymbol{b}}^{(t)T} \boldsymbol{\mathcal{E}}\boldsymbol{R}\boldsymbol{\mathcal{E}} \widehat{\boldsymbol{b}}^{(t)}}{2\sigma^2} + \frac{m}{\overline{A^2}}$ and $v^{(t)} = \frac{\boldsymbol{r}^T \boldsymbol{\mathcal{E}} \widehat{\boldsymbol{b}}^{(t)}}{\sigma^2}$;

2. Evaluate $\lambda_1^{(t)}$ and $\lambda_2^{(t)}$ through the following equations

$$\lambda_1^{(t)} = \sum_{i=0}^{2m} \binom{2m}{i} \left(\frac{v^{(t)}}{2u^{(t)}}\right)^{2m-i} \frac{\Gamma\left(\frac{i+1}{2}\right)}{2u^{(t)\frac{i+1}{2}}} \times \left[1 - [-\text{sign}(v^{(t)})]^{i+1} \mathcal{P}\left(\frac{i+1}{2}, \frac{v^{(t)2}}{4u^{(t)}}\right)\right],$$

$$\lambda_2^{(t)} = \sum_{i=0}^{2m+1} \binom{2m+1}{i} \left(\frac{v^{(t)}}{2u^{(t)}}\right)^{2m+1-i} \frac{\Gamma\left(\frac{i+1}{2}\right)}{2u^{(t)\frac{i+1}{2}}} \times \left[1 - [-\text{sign}(v^{(t)})]^{i+1} \mathcal{P}\left(\frac{i+1}{2}, \frac{v^{(t)2}}{4u^{(t)}}\right)\right],$$

where $\mathcal{P}(a,x) = \frac{\gamma(a,x)}{\Gamma(a)}$, and $\gamma(\cdot,\cdot)$ denotes the incomplete Gamma function [12].

3. Estimate $\widehat{\boldsymbol{b}}^{(t+1)} = [\widehat{b}_1^{(t+1)}, \cdots, \widehat{b}_K^{(t+1)}]$ according to the equation

$$\widehat{b}_i^{(t+1)} = \text{sign}\left(\lambda_1^{(t)} r_i - \lambda_2^{(t)} \sum_{j=1}^{i-1} \rho_{j,i} \sqrt{\mathcal{E}_j} \widehat{b}_j^{(t+1)} - + \lambda_2^{(t)} \sum_{j=i+1}^{K} \rho_{j,i} \sqrt{\mathcal{E}_j} \widehat{b}_j^{(t)}\right), \quad (3)$$

with $i = 1, \cdots, K$.

The above receiver deserves some comments. First of all, we notice that the ECM strategy exploits the *a priori* knowledge about the statistical characterization of A. As a consequence, it leads to a receiver which is tied up to the fading fluctuation law. Should another distribution be in force the coefficients $\lambda_1^{(t)}$ and $\lambda_2^{(t)}$ should be re-designed, even if step 3 remains unaltered.

We also observe that step 3 implies a weighted cancellation of the multiple access interference performed through the knowledge of the estimated bits and of the shape parameter m. Thus, step 3 can be also regarded as a generalization of the successive interference cancellation (SIC) receiver.

Finally, we point out that a possible generalization of the ECM-based receiver is the multicycle ECM-based receiver [11] where one E-step is performed before each CM-step or a few selected CM-steps. Also, this algorithm ensures an increasing likelihood function. However, an obvious disadvantage is the extra computation required at each iteration. Intuitively, as a tradeoff, one might expect it to result in larger increases in log-likelihood function per iteration.

B. GLR-Based Receiver

A possible alternative to the *Bayesian* approach proposed in the previous subsection relies on modelling, at the design stage, A as an unknown parameter. It follows that the detection of $\boldsymbol{b}$ is tantamount to solving a composite hypothesis test, which does not admit any uniformly minimum-error-probability solution [13]. However, a possible way to circumvent the *a priori* uncertainty as to A is to resort to the GLR criterion, wherein the unknown quantity is replaced by its maximum likelihood estimate. As a consequence the GLR receiver to demodulate the bit stream $\boldsymbol{b}$ is

$$\widehat{\boldsymbol{b}}_{GLR} = \arg \max_{\boldsymbol{b} \in \{-1,1\}^K, A \geq 0} \Psi_d(\boldsymbol{b}, A) = \arg \max_{\boldsymbol{b} \in \{-1,1\}^K, A \geq 0} \left[-\boldsymbol{b}^T \boldsymbol{\mathcal{E}} \boldsymbol{R} \boldsymbol{\mathcal{E}} \boldsymbol{b} A^2 + 2\boldsymbol{r}^T \boldsymbol{\mathcal{E}} \boldsymbol{b} A\right].$$

Performing the joint maximization we come up with the following GLR rule

$$\widehat{\boldsymbol{b}}_{GLR} = \arg \max_{\boldsymbol{b} \in \{-1,1\}^K} \left\{ -\boldsymbol{b}^T \boldsymbol{\mathcal{E}} \boldsymbol{R} \boldsymbol{\mathcal{E}} \boldsymbol{b} \left[\max\left(\frac{\boldsymbol{b}^T \boldsymbol{\mathcal{E}} \boldsymbol{r}}{\boldsymbol{b}^T \boldsymbol{\mathcal{E}} \boldsymbol{R} \boldsymbol{\mathcal{E}} \boldsymbol{b}}, 0\right)\right]^2 + 2\boldsymbol{r}^T \boldsymbol{\mathcal{E}} \boldsymbol{b} \max\left(\frac{\boldsymbol{b}^T \boldsymbol{\mathcal{E}} \boldsymbol{r}}{\boldsymbol{b}^T \boldsymbol{\mathcal{E}} \boldsymbol{R} \boldsymbol{\mathcal{E}} \boldsymbol{b}}, 0\right) \right\}.$$

Notice that an exhaustive search over all of the possible outcomes of $\boldsymbol{b}$ entails an exponential complexity, whereby iterative, reduced-complexity procedures should be envisaged instead.

To this end, let us assume again that an initial estimate, $\widehat{\boldsymbol{b}}^{(0)}$, of the transmitted bit sequence is available and that this estimate be achieved through a receiver whose structure does not depend on the receiver energies, such as the plain matched filter or the decorrelating. Replacing $\boldsymbol{b}$ by the available estimate $\widehat{\boldsymbol{b}}^{(t)}$ in $\Psi_d(\boldsymbol{b}, A)$ and maximizing with respect to $A \geq 0$ yields

$$\widehat{A}^{(t+1)} = \arg \max_{A \geq 0} \Psi_d(\widehat{\boldsymbol{b}}^{(t)}, A) = \max\left(\frac{\widehat{\boldsymbol{b}}^{(t)^T} \boldsymbol{\mathcal{E}} \boldsymbol{r}}{\widehat{\boldsymbol{b}}^{(t)^T} \boldsymbol{\mathcal{E}} \boldsymbol{R} \boldsymbol{\mathcal{E}} \widehat{\boldsymbol{b}}^{(t)}}, 0\right).$$

Hence, if $\widehat{A}^{(t+1)} > 0$, the next estimate $\widehat{\boldsymbol{b}}^{(t+1)}$ of the transmitted bit stream can be determined by applying the following successive maximizations procedure

$$\widehat{b}_i^{(t+1)} = \arg \max_{b_i \in \{-1,1\}} \Psi_d\left(\widehat{b}_1^{(t+1)}, \cdots, \widehat{b}_{i-1}^{(t+1)}, b_i, \widehat{b}_{i+1}^{(t)}, \cdots, \widehat{b}_K^{(t)}, \widehat{A}^{(t+1)}\right),$$

which, after some algebraic manipulations leads to the decisions

$$\widehat{b}_i^{(t+1)} = \text{sign}\left(r_i - \sum_{j=1}^{i-1} \rho_{j,i} \widehat{A}^{(t+1)} \sqrt{\mathcal{E}_j} \widehat{b}_j^{(t+1)} - + \sum_{j=i+1}^{K} \rho_{j,i} \widehat{A}^{(t+1)} \sqrt{\mathcal{E}_j} \widehat{b}_j^{(t)}\right), \quad (4)$$

with $i = 1, \cdots, K$.

If instead $\widehat{A}^{(t+1)} = 0$ the recursion should be halted and the value $\widehat{\boldsymbol{b}}^{(t+1)} = \widehat{\boldsymbol{b}}^{(t)}$ is delivered as final estimate. It can be shown that the above algorithm is convergent. Indeed, $\Psi_d(\cdot,\cdot)$ is a bounded function, in that

$$\Psi_d\left(\widehat{\boldsymbol{b}}^{(t)}, \widehat{A}^{(t)}\right) = -\left(\boldsymbol{r} - \widehat{A}^{(t)} \boldsymbol{R} \boldsymbol{\mathcal{E}} \widehat{\boldsymbol{b}}^{(t)}\right)^T \boldsymbol{R}^{-1} \left(\boldsymbol{r} - \widehat{A}^{(t)} \boldsymbol{R} \boldsymbol{\mathcal{E}} \widehat{\boldsymbol{b}}^{(t)}\right) + \boldsymbol{r}^T \boldsymbol{R}^{-1} \boldsymbol{r} \leq \boldsymbol{r}^T \boldsymbol{R}^{-1} \boldsymbol{r}, \quad \forall (t),$$

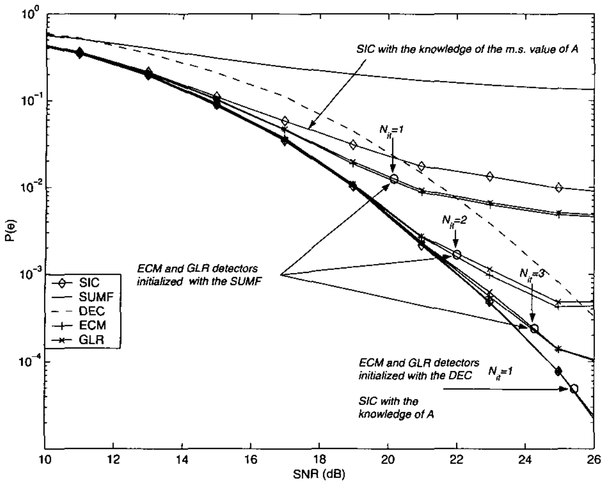

Fig. 1. $P(e)$ vs. SNR for $K = 15$, $m = 5$, several values of N_{it}, and two different initializations.

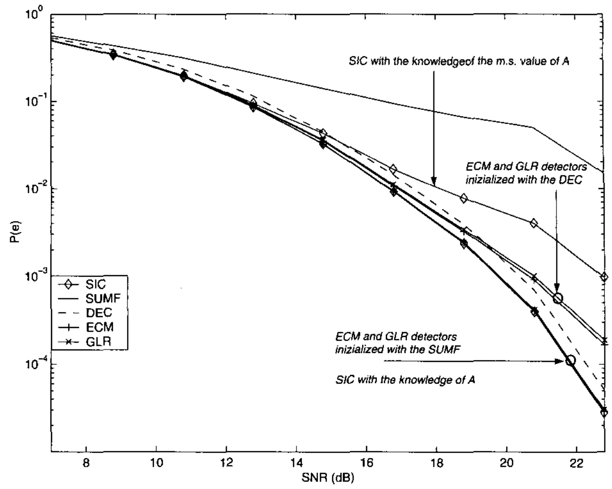

Fig. 2. $P(e)$ vs. SNR for $K = 12$, $m = 5$, $N_{it} = 1$, and two different initializations.

and complies with the following condition

$$\Psi_d(\widehat{\boldsymbol{b}}^{(t)}, \widehat{A}^{(t)}) \leq \Psi_d(\widehat{\boldsymbol{b}}^{(t+1)}, \widehat{A}^{(t+1)}).$$

In fact,

$$\Psi_d(\widehat{\boldsymbol{b}}^{(t)}, \widehat{A}^{(t)}) \leq \max_{A \geq 0} \Psi_d(\widehat{\boldsymbol{b}}^{(t)}, A) = \Psi_d(\widehat{\boldsymbol{b}}^{(t)}, \widehat{A}^{(t+1)}).$$

Moreover, the conditional maximizations over the b_i's are such that

$$\Psi_d(\widehat{\boldsymbol{b}}^{(t)}, \widehat{A}^{(t+1)}) \leq \Psi_d(\widehat{\boldsymbol{b}}^{(t+1)}, \widehat{A}^{(t+1)}).$$

The devised algorithms deserve some comment. First of all, we highlight that the GLR algorithm is simpler to implement than the ECM-based detector due to the presence of the weight coefficients $\lambda_1^{(t)}$ and $\lambda_2^{(t)}$ which involve the summation of $2m$ and $2m+1$ terms, respectively, as well as the computation of the incomplete Gamma function. We also point out that, when each user experiences an independent fading, the proposed algorithms can not be easily generalized as the size of the parameter space which are to be estimated becomes too large. In this case, a possible solution is to employ, for estimation purposes, data from more signaling intervals [10] where the fading can be assumed constant and hence re-devise the detection procedures.

III. PERFORMANCE ASSESSMENT

This section is devoted to the performance assessment of the proposed receivers also in comparison with the single user matched filter (SUMF), the decorrelating detector (DEC), and the successive interference cancellator (SIC). To this end, we consider a synchronous DS-CDMA system employing pseudo-noise spreading sequences of length $N = 15$. Moreover, we assume a very hostile near-far scenario where, upon sorting the active users in increasing power order, and assigning the power of the weakest, each user has a power advantage of $\frac{15}{K}$ dB on the previous one. The performance has been evaluated in terms of the error probability $P(e)$ (i.e., the probability that at least one demodulated bit is in error) resorting to Monte Carlo counting techniques based on $\frac{100}{P(e)}$ runs. Finally, for the proposed receivers and for the SIC, we give a top priority of the cancellation to the strongest interfering user.

In Fig. 1, the $P(e)$ is plotted versus the signal to noise ratio (SNR), i.e.,

$$SNR = \frac{\overline{A^2}}{\sigma^2} \sum_{k=0}^{K-1} \mathcal{E}_k, \tag{5}$$

for $m = 5$, $K = 15$, and several values of the number of iterations N_{it}. Therein, we also consider two different initializations of the receivers (3) and (4) with the SUMF and the DEC decisions. Finally, for comparison purposes, the performances of the SUMF, of the DEC, and of the SIC are reported, too. With reference to this last receiver two situations are considered. In the former, the SIC assumes the perfect knowledge of the CSI A for each bit interval. In the latter, it only assumes the knowledge of the mean square (ms) value of A.

The curves highlight that, for the parameters values chosen, the ECM-based receiver slightly outperforms the GLR-based detector especially when the SUMF decisions are adopted as initial conditions. Moreover, the number of iterations required in order to achieve convergence strongly depends on the initialization: Otherwise stated when the detectors are initialized with the DEC decisions one iteration is sufficient for ensuring convergence. On the other hand, when the SUMF decisions are employed as initial conditions, four iterations seem necessary for the convergence. We also notice that the performance of the SIC, which assumes a perfect CSI, is almost coincident with that of the GLR and ECM receiver which, remarkably, get rid of the quoted information. Finally, when the SIC knows only the mean square value of A, its performance is unsatisfactory.

In Fig. 2, we still consider $m = 5$, $N_{it} = 1$, and two different initializations of the receivers (3) and (4) with the SUMF and the DEC decisions, but therein the number of active users is $K = 12$. The curves show that the newly introduced receivers still outperform the plain SUMF and DEC, however the performance gain is smaller with respect to the case of Fig. 1. This

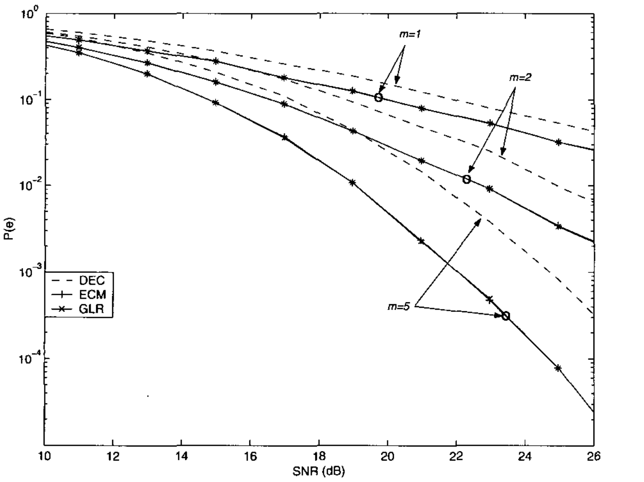

Fig. 3. $P(e)$ vs. SNR for $K = 15$, $N_{it} = 1$, DEC decisions as initial condition, and several values of m.

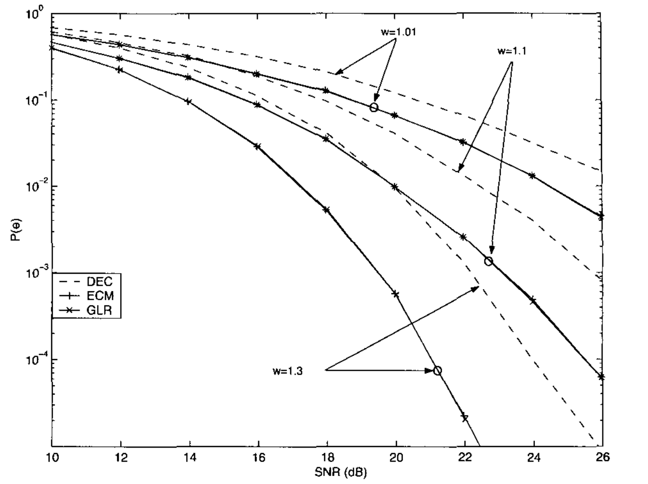

Fig. 4. $P(e)$ vs. SNR in the presence of distributional mismatch for $K = 15$, $N_{it} = 1$, DEC decisions as initial condition, and several values of w.

is not surprising since when the number of users is not very close to the processing gain the DEC and the SUMF ensure higher performance level. The convergence analysis deserves some comments, too; one iteration is still sufficient for ensuring convergence if the detectors are initialized with DEC; when, instead, detectors are initialized with SUMF two iterations seem necessary for the convergence (the curves corresponding to the second iteration are not plotted because they are indistinguishble of those corresponding to the receivers initialized with the DEC decisions). Finally, as to the SIC, the same considerations of Fig. 1 apply.

In Fig. 3, we analyze the effects on the receivers' performance of the shape parameter m. To this end, we assume again $N_{it} = 1$ and DEC decision as initial condition. It is clear from the curves that the depth of the fluctuation significantly affects the $P(e)$ of both receivers. Specifically, the lower the value of m the higher the $P(e)$.

We explicitly point out that all the described figures confirm the effectiveness of the GLR-based receiver which, without the knowledge of the shape parameter m, achieves the same performance level of the ECM-based receiver. This behavior can be explained in terms of the amount of information available for estimating the parameters. Precisely, in correspondence of the chosen value for the processing gain (and in general for large values of N), the matched filter outputs contain enough information for a reliable estimate of the parameter A and, thus, the incorporation of the prior knowledge regarding the pdf of A does not lead to a significant performance improvement. On the contrary, when small processing gains are considered, the availability of the prior information for the estimation of A becomes more important resulting in the ECM-based receiver's better performance than the GLR-based one. Indeed, other simulation results, not reported herein, have highlighted that the ECM technique may outperform the GLR receiver when small values of the processing gain ($N = 3$) and spreading codes different from the PN are adopted.

In the last part of this section, we study the robustness of the ECM-based receiver with respect to possible deviations between the nominal (i.e., assumed at the design level) and the actual pdf of A. To this end, assuming that the design pdf is given by (2) with $m = 1$, we have assessed the performance of the receiver (3) when A is distributed according to a lognormal law, i.e.,

$$f_A(x) = \frac{1}{\sqrt{2\pi \ln(w)}\, x} \exp\left\{ -\frac{[\ln(x) - \mu]^2}{2\ln(w)} \right\} u(x),$$

where w (≥ 1) and $\mu = -\ln(w)$ are parameters which rule the shape of the distribution. For comparison purposes, we have also analyzed the performance of the DEC and of the GLR-based receiver under this novel scenario. The results, reported in Fig. 4, highlight that the ECM-based receiver exhibits a strong robustness, in view of the closeness of the reported curve to the one corresponding to the GLR-based detector.

We finally point out that other simulations not reported herein have shown that the receiver (3) is also robust in the presence of different kind of mismatches. For instance, assuming that the design and the actual pdf of A is given by (2), the trend of the receiver performance is irrespective of discrepancies between the design and the actual value of the shape parameter m.

IV. CONCLUSIONS

In this paper, we have considered the problem of multiuser detection with I-CSI. To this end, we have devised and assessed two recursive detectors: The former is designed according to the ECM algorithm, a *Bayesian* statistical tool which is very effective to deal with problems with *missing data*. The latter relies on the GLR criterion which substitutes the ML estimates of the parameters in lieu of their unknown values. Interestingly both the receivers achieve satisfactory performance with an acceptable complexity. Moreover, even if the GLR-based receiver requires less *a priori knowledge* on A, it ensures, for reasonable values of the processing gain (namely, $N \geq 7$), a very close performance to the ECM-based detector. As a consequence, when N is enough high, the GLR technique is more convenient than the ECM approach for the present problem. On the contrary, for small processing gains, the prior information on the amplitude fluctuation becomes more important and, hence, the ECM approach leads to a detector more powerful than the GLR one.

We have also analyzed the performance of the receiver (3) in the presence of mismatches between the design and the operating conditions. The results have shown that the ECM-based detector enjoys an intrinsic robustness in the presence of both parameters and distributional mismatches.

Current authors' research is focused on the extension of the proposed framework to the case of multirate CDMA systems and more general scenarios accounting for I-CSI.

APPENDIX

In this appendix, we derive the ECM-based receiver for the problem at hand. To this end, we choose as complete-data the couple $(\boldsymbol{r}, A)$, which can be partitioned into the vector of observables or incomplete-data $\boldsymbol{r}$ and the missing data A. Hence, the complete-data log-likelihood function of $\boldsymbol{b}$ is

$$L(\boldsymbol{b}|A,\boldsymbol{r}) = \frac{1}{2\sigma^2}\left[2A\boldsymbol{r}^T\boldsymbol{\mathcal{E}}\boldsymbol{b} - A^2\boldsymbol{b}^T\boldsymbol{\mathcal{E}}\boldsymbol{R}\boldsymbol{\mathcal{E}}\boldsymbol{b} + c_1\right],$$

where c_1 is independent of $\boldsymbol{b}$.

E-step. The E-step of the ECM algorithm requires evaluating the conditional expectation of the complete-data log-likelihood with respect to the conditional distribution of the missing data given the observables and the current estimated parameter $\widehat{\boldsymbol{b}}^{(t)}$, i.e.,

$$E[L(\boldsymbol{b}|A,\boldsymbol{r})|\boldsymbol{r},\widehat{\boldsymbol{b}}^{(t)}] = \int_{-\infty}^{+\infty} L(\boldsymbol{b}|A,\boldsymbol{r})\, f(A|\boldsymbol{r},\boldsymbol{b}^{(t)})\, dA, \quad (6)$$

where $f(A|\boldsymbol{r},\boldsymbol{b}^{(t)})$ denotes the pdf of A given $\boldsymbol{r}$ and $\boldsymbol{b}^{(t)}$. This is tantamount to evaluating $E[A|\boldsymbol{r},\widehat{\boldsymbol{b}}^{(t)}]$ and $E[A^2|\boldsymbol{r},\widehat{\boldsymbol{b}}^{(t)}]$. It can be shown applying the Bayes' rule, and after some algebraic manipulations that

$$\begin{aligned} E[A|\boldsymbol{r},\widehat{\boldsymbol{b}}^{(t)}] &= c_2\,\lambda_1^{(t)} \\ &= c_2\int_0^{+\infty} A^{2m}\exp\left\{-u^{(t)}\left(A-\frac{v^{(t)}}{2u^{(t)}}\right)^2\right\}dA, \\ E[A^2|\boldsymbol{r},\widehat{\boldsymbol{b}}^{(t)}] &= c_2\,\lambda_2^{(t)} \\ &= c_2\int_0^{+\infty} A^{2m+1}\exp\left\{-u^{(t)}\left(A-\frac{v^{(t)}}{2u^{(t)}}\right)^2\right\}dA, \end{aligned} \quad (7)$$

where $v^{(t)} = \dfrac{\boldsymbol{r}^T\boldsymbol{\mathcal{E}}\boldsymbol{b}^{(t)}}{\sigma^2}$, $u^{(t)} = \dfrac{\boldsymbol{b}^{(t)T}\boldsymbol{\mathcal{E}}\boldsymbol{R}\boldsymbol{\mathcal{E}}\boldsymbol{b}^{(t)}}{2\sigma^2} + \dfrac{m}{\overline{A^2}}$.

Moreover, evaluating the integrals in (7), we can rewrite $\lambda_1^{(t)}$ and $\lambda_2^{(t)}$ as follows

$$\begin{aligned} \lambda_1^{(t)} &= \int_{-\frac{v^{(t)}}{2u^{(t)}}}^{+\infty}\left(A+\frac{v^{(t)}}{2u^{(t)}}\right)^{2m}\exp\left[-u^{(t)}A^2\right]dA \\ &= \sum_{j=0}^{2m}\binom{2m}{j}\left(\frac{v^{(t)}}{2u^{(t)}}\right)^{2m-j} \times\int_{-\frac{v^{(t)}}{2u^{(t)}}}^{+\infty} A^j\exp\left[-u^{(t)}A^2\right]dA \\ &= \sum_{i=0}^{2m}\binom{2m}{i}\left(\frac{v^{(t)}}{2u^{(t)}}\right)^{2m-i}\frac{\Gamma\left(\frac{i+1}{2}\right)}{2u^{(t)\frac{i+1}{2}}} \times\left[1-[-\mathrm{sign}(v^{(t)})]^{i+1}\mathcal{P}\left(\frac{i+1}{2},\frac{v^{(t)2}}{4u^{(t)}}\right)\right], \end{aligned}$$

$$\begin{aligned} \lambda_2^{(t)} &= \int_{-\frac{v^{(t)}}{2u^{(t)}}}^{+\infty}\left(A+\frac{v^{(t)}}{2u^{(t)}}\right)^{2m+1}\exp\left[-u^{(t)}A^2\right]dA \\ &= \sum_{j=0}^{2m+1}\binom{2m+1}{j}\left(\frac{v^{(t)}}{2u^{(t)}}\right)^{2m+1-j} \times\int_{-\frac{v^{(t)}}{2u^{(t)}}}^{+\infty} A^j\exp\left[-u^{(t)}A^2\right]dA \\ &= \sum_{i=0}^{2m+1}\binom{2m+1}{i}\left(\frac{v^{(t)}}{2u^{(t)}}\right)^{2m+1-i}\frac{\Gamma\left(\frac{i+1}{2}\right)}{2u^{(t)\frac{i+1}{2}}} \times\left[1-[-\mathrm{sign}(v^{(t)})]^{i+1}\mathcal{P}\left(\frac{i+1}{2},\frac{v^{(t)2}}{4u^{(t)}}\right)\right]. \end{aligned}$$

Finally, substituting (7) into (6) we get

$$E[L(\boldsymbol{b}|A,\boldsymbol{r})|\boldsymbol{r},\widehat{\boldsymbol{b}}^{(t)}] = \frac{1}{2\sigma^2}\left[2c_2\boldsymbol{r}^T\boldsymbol{\mathcal{E}}\boldsymbol{b}\,\lambda_1^{(t)} - c_2\boldsymbol{b}^T\boldsymbol{\mathcal{E}}\boldsymbol{R}\boldsymbol{\mathcal{E}}\boldsymbol{b}\,\lambda_2^{(t)} + c_1\right]. \quad (8)$$

CM-steps. Here is the main difference between the EM and the ECM algorithm in that the latter replaces the complicated maximization over $\boldsymbol{b}$ characteristic of the plain EM with K computationally simpler conditional maximizations over one dimensional spaces. Precisely, the i-th CM-step maximizes (8) subject to constraint $\boldsymbol{b} = (\widehat{b}_1^{(t+1)},\cdots,\widehat{b}_{i-1}^{(t+1)}, b_i, \widehat{b}_{i+1}^{(t)},\cdots,\widehat{b}_K^{(t)})$, i.e.,

$$\widehat{b}_i^{(t+1)} = \mathrm{sign}\left(\lambda_1^{(t)}r_i - \lambda_2^{(t)}\sum_{j=1}^{i-1}\rho_{j,i}\sqrt{\mathcal{E}_j}\widehat{b}_j^{(t+1)} - +\lambda_2^{(t)}\sum_{j=i+1}^{K}\rho_{j,i}\sqrt{\mathcal{E}_j}\widehat{b}_j^{(t)}\right),\ i=1,\cdots,K.$$

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