Multiple Access and Inter-Carrier Interference in OFDM-CDMA with Random Sequences

Won Mee Jang, Lim Nguyen, and Pooja Bidarkar

Abstract: **In this paper, we analyze the performance of code division multiple access (CDMA) systems with orthogonal frequency division multiplexing (OFDM) that employ random spreading sequences in an additive white Gaussian noise (AWGN) channel. We obtain the probability density function (pdf) of the multiple access interference and extend the results to OFDM-CDMA systems to determine the pdf of multiple access and inter-carrier interference in terms of the number of users, the spreading length, the number of sub-carriers, and the frequency offset. We consider the synchronous downlink of cellular multi-carrier CDMA and derive a Gaussian approximation of the multiple access and inter-carrier interference. Overall the effect of frequency offset is shown to vary with the system loading. The analysis in this paper is critical for further development into fading channels and frequency selective multipath channels.**

Index Terms: **CDMA, frequency offset, inter-carrier interference, OFDM, random sequences, spread spectrum.**

I. INTRODUCTION

The multiple access interference (MAI) in CDMA has been studied extensively in the literature [1]–[3]. CDMA systems with random spreading sequences have received special interest in [4]–[7], since random spread spectrum affords a convenient way of analysis due to its statistical symmetry, and can provide an upper bound on the MAI performance. In practice, random spread spectrum can be approximated in uplink asynchronous CDMA systems such as IS-95, Wideband CDMA, and CDMA 2000, that employ long codes as found in [8]–[10]. Random spreading sequences can also be employed in the analysis of synchronous downlink OFDM-CDMA systems to obtain an upper bound on the effect of frequency offset in practical systems. It has been shown that OFDM-CDMA outperforms direct sequence code division multiple access (DS-CDMA) in radio channels in terms of spectral efficiency [11]. However, inter-carrier interference (ICI) due to carrier frequency offset in OFDM can degrade the system performance significantly. The source of such frequency offset can be found in the frequency mismatch between the transmitter and receiver oscillators, a Doppler shift due to the vehicle motion, or the complex fading envelope and phase variations in the received signal due to multiple scatterings by buildings and other environmental structures. The main problem with the frequency offset is that it introduces interferences from other carriers since they are no longer orthogonal to the matched filter. Another problem is the reduction of the signal amplitude at the outputs of the filters that are matched to each of the carriers.

In this paper, we determine the pdf of the MAI in synchronous downlink CDMA systems with equal power and verify the analytical calculations with the simulation results. We employed random spreading sequences for analysis purpose that can provide an upper bound on the performance of the orthogonal downlink. We extend the analysis to synchronous OFDM-CDMA systems with random spreading sequences in AWGN channels. Such applications may be found in the downlink of a cellular OFDM-CDMA systems in open area where fading is not a critical issue. In particular, we examine the impact of the carrier frequency offset on the system performance. We obtain the pdf of the MAI and ICI in OFDM-CDMA systems in terms of the number of users, the spreading length, number of sub-carriers, and the frequency offset. To reduce the computational complexity in calculating the bit error rate (BER) using the pdf of the MAI and ICI, we derive a Gaussian approximation that can provide an accurate estimation of the BER in a rather convenient way.

In summary, our principle contribution is the detailed analysis of the effect of frequency offset in OFDM-CDMA with random sequences in AWGN channels. The theoretical results presented in this paper are critical for further development into fading channels as well as frequency selective channels. In Section II, we propose the system model and then analyze the MAI and ICI in Section III. Section IV compares the analytical results to the numerical results and Section V concludes the paper.

II. SYSTEM MODEL

We consider a multiuser system with $K+1$ users (K interferers) sharing the channel, with the received signal given by

$$r(t) = x(t) + n(t), \tag{1}$$

where $n(t)$ represents AWGN with two-sided power spectral density σ^2. The transmitted signal is specified by

$$x(t) = \sum_{i=-\infty}^{\infty} \sum_{k=1}^{K+1} A_k s_i^k(t) b_i^k, \tag{2}$$

where T_b is the bit duration. The signature waveform of the k-th user, $s_i^k(t)$ is a binary random sequence defined in $iT_b \le t < (i+1)T_b$ and consists of $n = T_b/T_c$ antipodal random binary chips where T_c is the chip duration. A_k is the amplitude of the k-th user, b_i^k represents the data bit of the k-th user during the i-th bit interval, and $b_i^k \in [-1, 1]$, $\forall k, i$. It is assumed that signature

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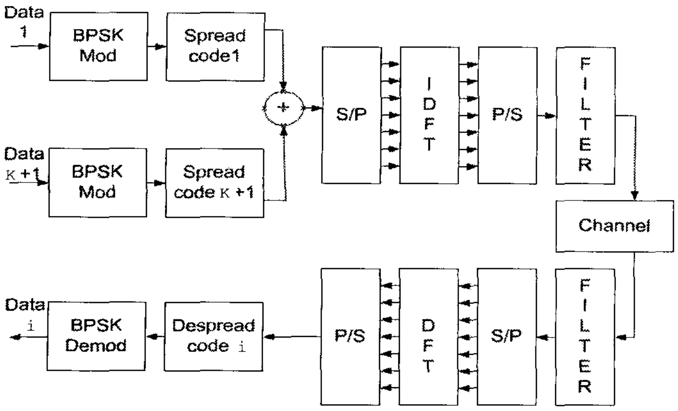

Fig. 1. OFDM-CDMA transmitter and receiver.

waveforms have unit energy

$$\int_{iT_b}^{(i+1)T_b} |s_i^k(t)|^2 dt = 1, \quad \forall\ k, i. \tag{3}$$

The output of the matched filters (MF) matched to the k-th user signature waveform during the i-th bit interval is

$$y_i^k = \int_{iT_b}^{(i+1)T_b} r(t) s_i^k dt, \tag{4}$$

and

$$y_i^k = A_k b_i^k + \sum_{j=1, j\neq k}^{K+1} A_j R_i^{k,j} b_i^j + \eta, \tag{5}$$

where η is an independent Gaussian random variable with variance equal to σ^2. The crosscorrelation of user k and user j during the i-th bit interval is

$$R_i^{k,j} \;=\; \int_{iT_b}^{(i+1)T_b} s_i^k(t) s_i^j(t) dt. \tag{6}$$

$R_i^{k,j}$ are independent for all j for given k, and the $K(K+1)/2$ crosscorrelations of $R_i^{k,j}$ are pairwise independent but not jointly independent [12]. The receiver under consideration in this paper is suboptimal because it treats the multiuser interference as noise without attempting to exploit possible knowledge of the codebooks of the interfering users. The multiuser detections of direct sequence spread spectrum multiple access (DS/SSMA) systems with random sequences are detailed in [13].

Fig. 1 shows the block diagram of OFDM-CDMA. The $K+1$ sequences of symbols obtained by BPSK modulation, are spread and added together. We have used random spreading sequences under assumption of perfect synchronization. v_m is the summation of $K+1$ users' m-th chip during the i-th bit interval

$$v_m = \sum_{k=1}^{K+1} b_i^k c_{i,m}^k, \qquad 0 \leq m \leq M-1, \tag{7}$$

where $c_{i,m}^k$ is the k-th user's m-th chip in the i-th bit interval. Then the sequence is serial-to-parallel (S/P) converted and its inverse discrete Fourier transform (IDFT) is taken. The IDFT of v_m corresponding to a block of M symbols is

$$w_h = \frac{1}{M} \sum_{m=0}^{M-1} v_m \exp\left\{j2\pi m \frac{h}{M}\right\}, \qquad 0 \leq h \leq M-1. \tag{8}$$

The P/S conversion and zero-th order interpolation give the continuous-time signal

$$w(t) = \sum_{h=0}^{M-1} w_h q(t - hT_c), \tag{9}$$

where $q(t)$ is the unit rectangular pulse over a chip interval. Let the normalized frequency offset ϵ be $f_o/\Delta f$, where f_o is a frequency offset and $\Delta f = 1/MT_c$. At the receiver input, the received signal impaired by the frequency offset is

$$r(t) = \frac{1}{M} \sum_{h=0}^{M-1} \sum_{m=0}^{M-1} v_m q(t - hT_c) \exp\left\{j2\pi(m+\epsilon)\frac{h}{M}\right\}. \tag{10}$$

The sampled values of the noiseless component of the received signal at the output of the receiver filter are

$$y_h = \frac{1}{T_c} \int_{hT_c}^{(h+1)T_c} r(t) c_{i,m}^d q(t - hT_c) dt, \qquad 0 \leq h \leq M-1, \tag{11}$$

where $c_{i,m}^d$ is the desired user's m-th chip in the i-th bit interval. The frequency offset included in (10) produces ICI. Thus, the h-th noiseless DFT input is

$$y_h = \frac{1}{M} \sum_{m=0}^{M-1} \nu_m \exp\left\{j2\pi h \frac{m+\epsilon}{M}\right\}, \tag{12}$$

where $\nu_m = \sum_{k=1}^{K+1} c_{i,m}^d b_i^k c_{i,m}^k$. The M samples of y_h corrupted by the AWGN samples are fed to the discrete Fourier transform (DFT). Thus, the decision variable z_g is

$$z_g = \sum_{h=0}^{M-1} y_h \exp\left\{-j2\pi h \frac{g}{M}\right\} + \eta_g, \quad 0 \leq g \leq M-1, \tag{13}$$

where η_g is the noise variable that is still Gaussian since a phase rotation due to frequency offset does not change the statistics of a complex Gaussian process [14]. It is easy to see that $z_g = v_g + \eta_g$ without the frequency offset. The DFT output can be written as

$$z_g = \sum_{h=0}^{M-1} \sum_{m=0}^{M-1} \nu_m \frac{1}{M} \exp\left\{j\frac{2\pi}{M}(m+\epsilon-g)h\right\} + \eta_g. \tag{14}$$

III. MULTIPLE ACCESS AND INTER-CARRIER INTERFERENCE

Random variables characterizing various aspects of the MAI are described using the random spreading sequences, the data bits, and the correlation functions of the spreading sequence. The MAI is discussed conditioning on an appropriately selected set of random variables. Conditioning on the correlation parameter for the specific spreading sequence is important in the analysis since it provides conditional independence for a set of random variables [12]. The conditional characteristic function of the MAI is the basis of our derivation. In (5), the MAI term is $\sum_{j=1,j\neq k}^{K+1} A_j R_i^{k,j} b_i^j$. We begin with the two-user system. Then the crosscorrelation in (6) can be written as

$$R_i^{k,j} = \int_{iT_b}^{(i+1)T_b} s_i^k(t) s_i^j(t) dt = \sum_{m=1}^{n} c_{i,m}^k c_{i,m}^j, \tag{15}$$

where $c_{i,m}^k$ is independent identically distributed (i.i.d.) with $Pr(c_{i,m}^k = 1/\sqrt{n}) = Pr(c_{i,m}^k = -1/\sqrt{n}) = 1/2, \forall i,k,m.$ Here we assume that the random sequence is dynamically changing every bit interval though the result can be applied to static random sequences. Then the crosscorrelation is

$$R_i^{k,j} = (n-2l)/n, \quad l = 0, \cdots, n, \tag{16}$$

where $Pr(l=i) = \binom{n}{i}(\frac{1}{2})^i(\frac{1}{2})^{n-i} = \binom{n}{i}(\frac{1}{2})^n$. l denotes the number of chip positions that differ: l=0 if the two spreading sequences are the same and $l = n$ if they are exactly opposite. The pdf of the sum of MAI and noise in CDMA is given in Appendix A as

$$f_{CDMA}(x) = \frac{1}{2^{nK}} \sum_{i=0}^{nK} \binom{nK}{i} \frac{1}{\sqrt{2\pi\sigma^2}} \times \exp\{-[x - A(K - 2i/n)]^2/(2\sigma^2)\}. \tag{17}$$

We compare with Gaussian approximation with the same variance derived in Appendix B as

$$\sigma_{CDMA}^2 = \sigma^2 + KA^2/n. \tag{18}$$

This result agrees with the result in [15].

The MAI and noise variable at the g-th sub-carrier can be written as

$$\gamma_g = \sum_{m=0}^{M-1} u_m S_{m-g} + \eta_g = u_g S_0 + \sum_{\substack{m=0 \\ m\neq g}}^{M-1} u_m S_{m-g} + \eta_g, \tag{19}$$

where [16], [17]

$$S_{m-g} = \frac{\sin(\pi(m-g+\epsilon))}{M\sin(\frac{\pi}{M}(m-g+\epsilon))} \times \exp\left(j\{\pi(1-\frac{1}{M})(m-g+\epsilon)\}\right), \tag{20}$$

and u_m is defined in (37). The pdf of the sum of MAI, ICI, and noise in OFDM-CDMA is given in Appendix C as

$$f_{OFDM-CDMA}(x) = \frac{1}{2^{nKM}} \sum_{i_0=0}^{nK} \cdots \sum_{i_{M-1}=0}^{nK} \times \binom{nK}{i_0} \cdots \binom{nK}{i_{M-1}} \frac{1}{\sqrt{2\pi\sigma^2 \sum_{k=0}^{M-1} S_k^2}} \times \exp\left(-([x - A\{K\sum_{k=0}^{M-1} S_k - 2(\sum_{k=0}^{M-1} i_k S_k)/n\}]^2) /(2\sigma^2 \sum_{k=0}^{M-1} S_k^2)\right). \tag{21}$$

It is easy to see that (21) is reduced to (17) for zero frequency offset. The computational complexity of (21) can be reduced using a Gaussian approximation with the same variance derived in Appendix D as

$$\sigma_{OFDM-CDMA}^2 = \sigma^2 \sum_{k=0}^{M-1} S_k^2 + A^2\Big\{-(K\sum_{k=0}^{M-1} S_k)^2 + K^2 \sum_{h=0}^{M-1}\sum_{g\neq h}^{M-1} S_h S_g + \frac{K}{n}(nK+1)\sum_{k=0}^{M-1} S_k^2\Big\}. \tag{22}$$

We can see that (22) is reduced to (18) for zero frequency offset. The pdf of the OFDM-CDMA is the weighted sum of nKM Gaussian pdfs. Heuristically, the effect of frequency offset is to increase the number of Gaussian pdfs with different means. With zero frequency offset, the number of Gaussian pdfs is reduced to nK in (18).

In fact, (21) does not include the self-interference from other sub-carriers. The self-interference is introduced from its own data in other chip intervals. We can easily modify (21) to include the self-interference as

$$\hat{f}_{OFDM-CDMA}(x) = \frac{1}{2^{nK}} \frac{1}{2^{n(K+1)(M-1)}} \times \sum_{i_0=0}^{nK} \sum_{i_1=0}^{n(K+1)} \cdots \sum_{i_{M-1}=0}^{n(K+1)} \binom{nK}{i_0}\binom{n(K+1)}{i_1} \cdots \binom{n(K+1)}{i_{M-1}} \frac{1}{\sqrt{2\pi\sigma^2 \sum_{k=0}^{M-1} S_k^2}} \times \exp\left(-([x - A\{(K+1)\sum_{k=0}^{M-1} S_k - S_0 - 2(\sum_{k=0}^{M-1} i_k S_k)/n\}]^2)/(2\sigma^2 \sum_{k=0}^{M-1} S_k^2)\right), \tag{23}$$

and the corresponding variance is

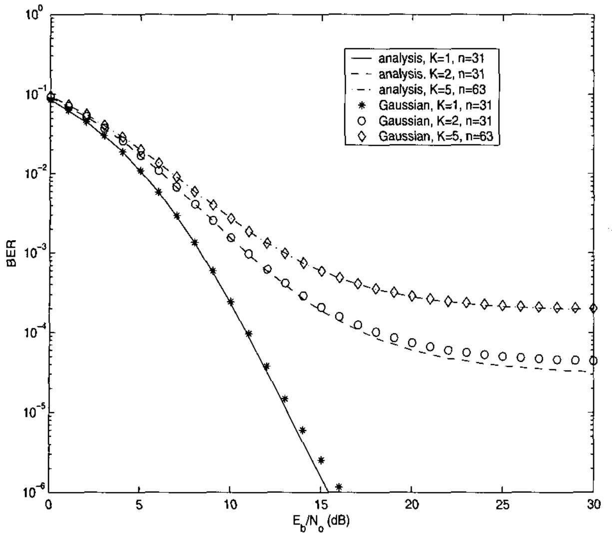

Fig. 2. Example BER calculation of CDMA for one, two, and five interferers ($n = 31, 63$).

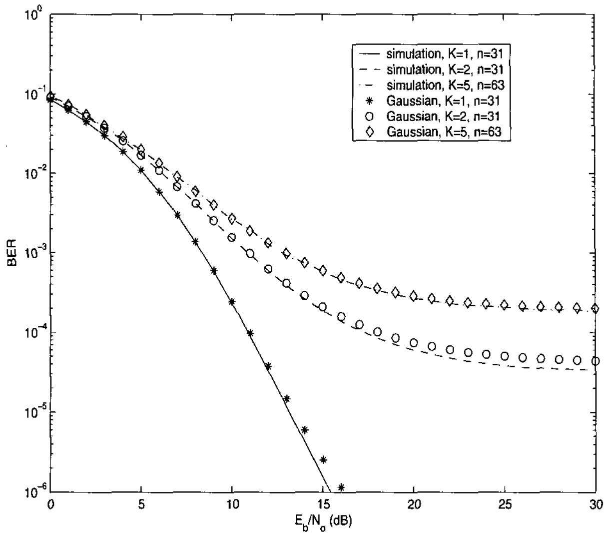

Fig. 3. Comparison between example calculation and simulation of CDMA for one, two, and five interferers ($n = 31, 63$).

$$\begin{aligned}\hat{\sigma}^2_{OFDM-CDMA} &= \sigma^2 \sum_{k=0}^{M-1} S_k^2 \\ &+A^2\Big\{-((K+1)\sum_{k=0}^{M-1} S_k)^2 + (K+1)^2 \sum_{h=0}^{M-1}\sum_{g\neq h}^{M-1} S_h S_g \\ &+\frac{K+1}{n}(n(K+1)+1)\sum_{k=0}^{M-1} S_k^2 - 2(K+1)\sum_{k=1}^{M-1} S_0 S_k \\ &-2(K+1)S_0^2 - \frac{1}{n}S_0^2 + 2(K+1)S_0 \sum_{k=0}^{M-1} S_k\Big\} \qquad (24)\\ &= \sigma^2 \Xi + A^2 \Pi, \qquad (25)\end{aligned}$$

where $\Xi = \sum_{k=0}^{M-1} S_k^2$ and

$$\begin{aligned}\Pi &= -((K+1)\sum_{k=0}^{M-1} S_k)^2 + (K+1)^2 \sum_{h=0}^{M-1}\sum_{g\neq h}^{M-1} S_h S_g \\ &+\frac{K+1}{n}(n(K+1)+1)\sum_{k=0}^{M-1} S_k^2 \\ &-2(K+1)\sum_{k=1}^{M-1} S_0 S_k - 2(K+1)S_0^2 - \frac{1}{n}S_0^2 \\ &+2(K+1)S_0 \sum_{k=0}^{M-1} S_k. \qquad (26)\end{aligned}$$

Then, the Gaussian approximation BER of the OFDM-CDMA can be written as

$$\begin{aligned}P_b &= Q\left(\sqrt{\frac{S_0^2 A^2}{\sigma^2(\Xi + \frac{A^2}{\sigma^2}\Pi)}}\right) \\ &= Q\left(\sqrt{\frac{2S_0^2 E_b}{N_o}\frac{1}{(\Xi + \frac{2E_b}{N_o}\Pi)}}\right), \qquad (27)\end{aligned}$$

where the Q function is $Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty e^{-\alpha^2/2} d\alpha$.

IV. NUMERICAL RESULTS

Fig. 2 shows the example probability of bit error calculation from (17) for two or three users with 31 chips/bit and six users with 63 chips/bit. Fig. 2 shows that Gaussian approximation becomes poorer at high SNR [4], [6]. The Gaussian approximation is close to the probability of error calculated from (17) for $E_b/N_o \leq 10$ dB for all three cases. Fig. 3 shows the simulation results for two or three users with 31 chips/bit and six users with 63 chips/bit. We can see that the simulation results agree well with the analytical results.

Fig. 4 displays the Gaussian approximation of OFDM-CDMA channel for two users with 31 chips/bit. The number of sub-carriers is equal to the spreading length. Notice that the performance degrades as frequency offset, ϵ, increases as expected. At 10 dB SNR, the BER with $\epsilon = 0, 0.1, 0.2, 0.3$ are $2.4 \times 10^{-4}, 4.2 \times 10^{-4}, 0.002, 0.0173$, respectively. We can see similar results for the three and six-user cases shown in Figs. 5–7. The effect of frequency offset becomes less significant as the MAI increases as shown in Fig. 6 with six users and the same spreading length. In the comparison of Figs. 5 and 7 that have the same ratio of the number of users to the spreading length, the effect of frequency offset is shown to be similar. The effect of frequency offset is more considerable for small number of users. For a larger number of users, MAI is dominant and frequency offset has smaller effect on the performance. Overall the effect of frequency offset is proportional to the ratio of the number of users to the spreading length (K/n) that is the measure of the MAI in a random spread spectrum system. Fig. 8 shows the simulation results of OFDM-CDMA for two users and 31 chips/bit and 31 sub-carriers for $\epsilon = 0, 0.05, 0.1, 0.2$. The Gaussian approximation is precise for $\epsilon = 0, 0.05, 0.1$, and 0.2. Thus, Gaussian approximation represents the performance of OFDM-CDMA accurately for the range of interest in practical applications.

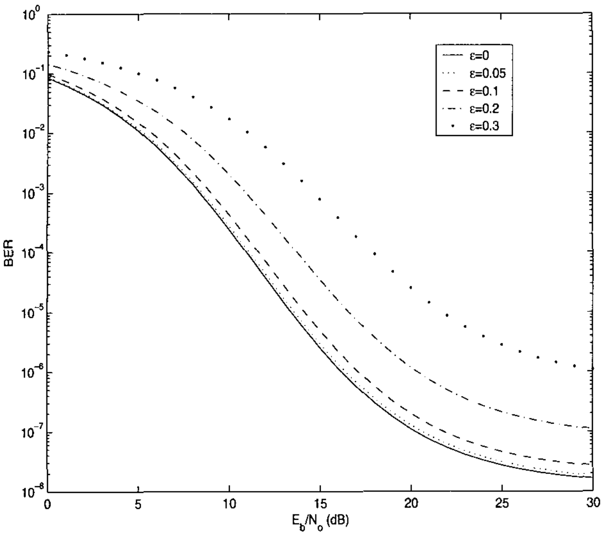

Fig. 4. Example BER calculation of OFDM-CDMA for one interferers ($n = 31$), frequency offset= $0, 0.05, 0.1, 0.2, 0.3$.

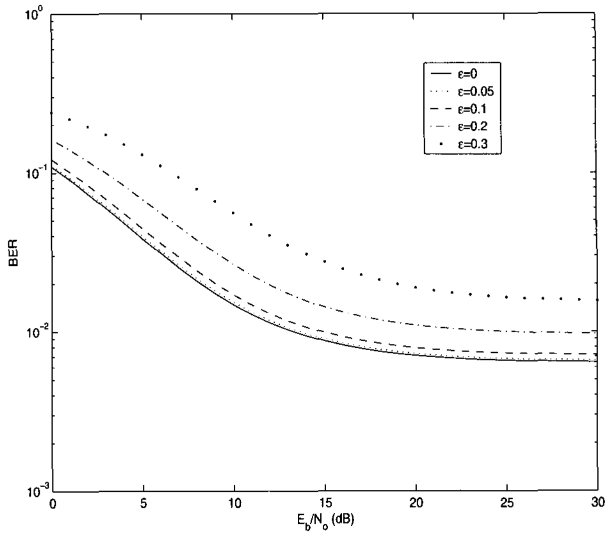

Fig. 6. Example BER calculation of OFDM-CDMA for five interferers ($n = 31$), frequency offset= $0, 0.05, 0.1, 0.2, 0.3$.

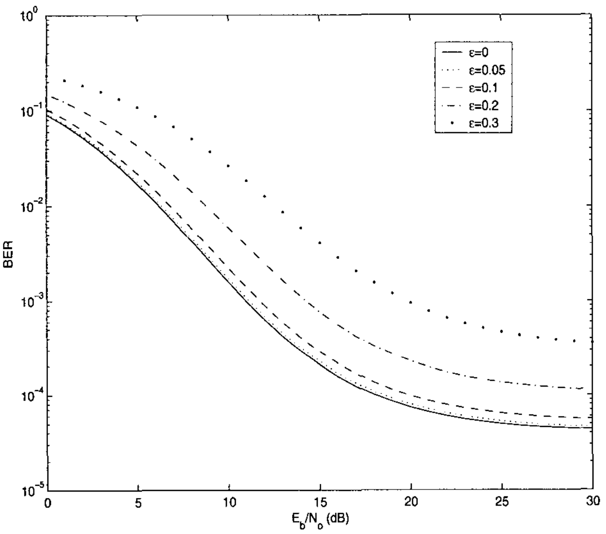

Fig. 5. Example BER calculation of OFDM-CDMA for two interferers ($n = 31$), frequency offset= $0, 0.05, 0.1, 0.2, 0.3$.

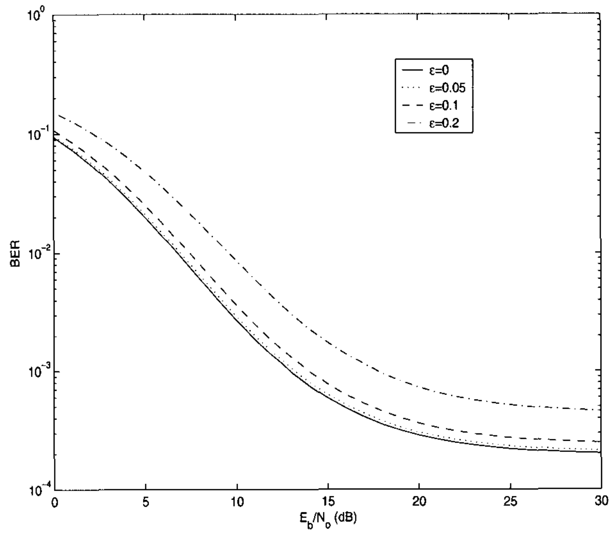

Fig. 7. Example BER calculation of OFDM-CDMA for five interferers ($n = 63$), frequency offset= $0, 0.05, 0.1, 0.2$.

V. CONCLUSIONS

In this paper, we have analyzed the BER performance of OFDM-CDMA systems that employ random spreading sequences in an AWGN channel. The pdf of the MAI in down link CDMA was obtained and the BER calculations were verified with the simulation results. We extended our result to the pdf of the MAI and ICI in a downlink OFDM-CDMA to determine the BER performance in terms of the number of users, the spreading length, the number of sub-carriers, and the frequency offset. To reduce the computational complexity involved in calculating the BER, we have derived a Gaussian approximation which can provide the approximate BER that compares well with the simulation. The effect of frequency offset has been shown to vary with the ratio of the number of users to the spreading length. The analysis in this paper is critical for further development into fading channels and frequency selective multipath channels.

APPENDIX A

Pdf of MAI and noise in CDMA.

The MAI conditioning on the specific spreading sequence is characterized by a binomial distribution. Conditioning the decision statistic on the specific spreading sequence such as the first user's spreading sequence is necessary for conditional independence of each pairwise interference. Assuming a two-user system with spreading factor n and equal power A^2, the discrete characteristic function is

$$\Phi_1(\omega) = \sum_{i=0}^{n} \binom{n}{i} p^i q^{n-i} \exp\{j\omega x_i\}, \tag{28}$$

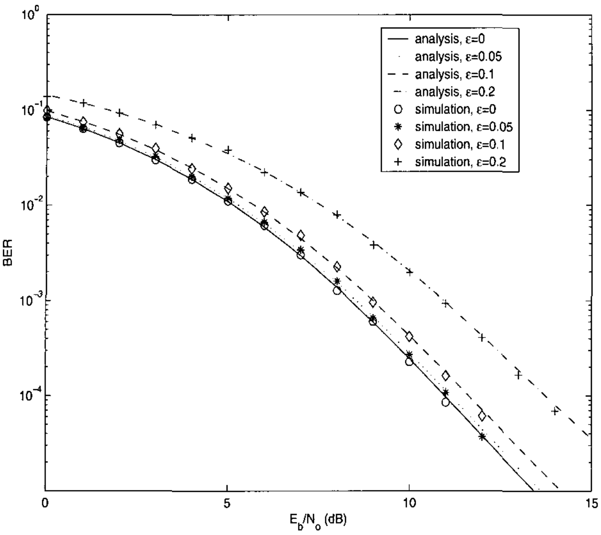

Fig. 8. Comparison between example calculation and simulation of OFDM-CDMA for one interferers ($n = 31$), frequency offset$= 0, 0.05, 0.1, 0.2$.

where $p = q = 1/2$ and $x_i = (n - 2i)A/n$. Notice that the random variable x_i represents the correlation between spreading signal of the interferer and the spreading sequence of the desired user. Therefore, x_i^2 is the MAI power at the output of the matched filter. Then,

$$\Phi_1(\omega) = \exp\{j\omega A\}\Big(p\ \exp\{-2j\omega A/n\} + q\Big)^n. \quad (29)$$

For K interferers, the characteristic function is

$$\Phi_K(\omega) = \Big\{\Phi_1(\omega)\Big\}^K = \sum_{i=0}^{nK}\binom{nK}{i}p^i q^{nK-i}\exp\{j\omega A(K - 2i/n)\}. \quad (30)$$

The probability of the crosscorrelation value $A(K - 2i/n)$ is $(1/2^{nK})\binom{nK}{i}$ for $i = 1, \cdots, nK$. In an AWGN channel, the characteristic function of MAI and noise is

$$\Phi(\omega) = \Phi_K(\omega) \times \Phi_{AWGN}(\omega) = \sum_{i=0}^{nK}\binom{nK}{i}p^i q^{nK-i} \times \exp\{j\omega A(K - 2i/n)\}\exp\{-\sigma^2\omega^2/2\}. \quad (31)$$

The corresponding probability density function can be obtained by its inverse Fourier transform

$$f_{CDMA}(x) = \frac{1}{2^{nK}}\sum_{i=0}^{nK}\binom{nK}{i}\frac{1}{\sqrt{2\pi\sigma^2}} \times \exp\{-(x - A(K - 2i/n))^2/(2\sigma^2)\}. \quad (32)$$

The pdf of the MAI and noise is the sum of Gaussian pdf with the mean equal to $A(K - 2i/n)$ weighted by $(2^{-nK})\binom{nK}{i}$ for $i = 0, \cdots, nK$.

APPENDIX B

Variance of MAI and noise in CDMA.

The variance of the MAI and noise can be written as the second moment,

$$E[X^2] = \frac{1}{2^{nK}}\sum_{i=0}^{nK}\binom{nK}{i}\int_{-\infty}^{\infty}x^2\frac{1}{\sqrt{2\pi\sigma^2}} \times \exp\{-(x - A(K - 2i/n))^2/2\sigma^2\}dx. \quad (33)$$

By a change of variable, $y = x - z$ where $z = A(K - 2i/n)$, the variance is

$$\sigma^2_{CDMA} = \frac{1}{2^{nK}}\sum_{i=0}^{nK}\binom{nK}{i}\int_{-\infty}^{\infty}(y+z)^2\frac{1}{\sqrt{2\pi\sigma^2}} \times \exp\{-y^2/2\sigma^2\}dy \quad (34)$$

$$= \sigma^2 + \frac{1}{2^{nK}}\sum_{i=0}^{nK}\binom{nK}{i}\{A(K - 2i/n)\}^2 = \sigma^2 + \frac{KA^2}{n}. \quad (35)$$

APPENDIX C

Pdf of MAI, ICI, and noise in OFDM-CDMA.

Without loss of generality, consider the MAI, ICI, and noise at the zero-th sub-carrier. The MAI and noise variable γ_0 can be written as

$$\gamma_0 = u_0 S_0 + \sum_{m=1}^{M-1}u_m S_m + n_0, \quad (36)$$

where

$$u_m = \sum_{\substack{k=1 \\ k\neq d}}^{K+1} c_{i,m}^d b_i^k c_{i,m}^k, \qquad 0 \le m \le M-1, \quad (37)$$

where subscript d denotes the desired user. u_m is the summation of K interferers' m-th chip during the i-th bit interval matched to the desired user's m-th chip in the i-th bit interval. S_m are normalized with respect to the receiver signal to noise ratio in the following analysis. The real part of S_m have been taken since we assume real signature waveforms and real binary data. The pdf of γ_0 is

$$f_{OFDM-CDMA}(x) = \frac{1}{|S_0|}f\Big(\frac{x}{S_0}\Big) * \frac{1}{|S_1|}f\Big(\frac{x}{S_1}\Big) * \cdots * \frac{1}{|S_{M-1}|}f\Big(\frac{x}{S_{M-1}}\Big), \quad (38)$$

where $f(\cdot)$ is the pdf defined in (32) and $*$ indicates the convolution. $|S_m|$ is the magnitude of S_m. Substituting (32) into

(38),

$$f_{OFDM-CDMA}(x) = \frac{1}{2^{nKM}} \times \sum_{i_0=0}^{nK} \cdots \sum_{i_{M-1}=0}^{nK} \binom{nK}{i_0} \cdots \binom{nK}{i_{M-1}} \Big\{ \frac{1}{\sqrt{2\pi(S_0\sigma)^2}} \times \exp\{-(x - AS_0[K - 2i_0/n])^2/2(S_0\sigma)^2\} * \cdots * \frac{1}{\sqrt{2\pi(S_{M-1}\sigma)^2}} \exp\Big\{ -(x - AS_{M-1} \times [K - 2i_{M-1}/n])^2/2(S_{M-1}\sigma)^2\Big\}\Big\} \quad (39)$$

$$= \frac{1}{2^{nKM}} \sum_{i_0=0}^{nK} \cdots \sum_{i_{M-1}=0}^{nK} \binom{nK}{i_0} \cdots \binom{nK}{i_{M-1}} \times \frac{1}{\sqrt{2\pi\sigma^2 \sum_{k=0}^{M-1} S_k^2}} \exp\Big\{-\{x - A[K \sum_{k=0}^{M-1} S_k - 2(\sum_{k=0}^{M-1} i_k S_k)/n]\}^2/(2\sigma^2 \sum_{k=0}^{M-1} S_k^2)\Big\}, \quad (40)$$

that is the weighted sum of Gaussian pdf with the mean equal to $A[K\sum_{k=0}^{M-1} S_k - 2(\sum_{k=0}^{M-1} i_k S_k)/n]$ and the variance equal to $\sigma^2 \sum_{k=0}^{M-1} S_k^2$.

APPENDIX D

Variance of MAI, ICI, and noise in OFDM-CDMA.

The variance of the MAI, ICI and noise can be written as the second moment,

$$E[X^2] = \frac{1}{2^{nKM}} \sum_{i_0=0}^{nK} \cdots \sum_{i_{M-1}=0}^{nK} \binom{nK}{i_0} \cdots \binom{nK}{i_{M-1}} \times \int_{-\infty}^{\infty} x^2 \frac{1}{\sqrt{2\pi\sigma^2 \sum_{k=0}^{M-1} S_k^2}} \exp\Big\{ -\{x - A[K \sum_{k=0}^{M-1} S_k - 2(\sum_{k=0}^{M-1} i_k S_k)/n]\}^2/(2\sigma^2 \sum_{k=0}^{M-1} S_k^2)\Big\} dx. \quad (41)$$

Applying the same procedure in Appendix B, the variance is

$$\sigma^2_{OFDM-CDMA} = \sigma^2 \sum_{k=0}^{M-1} S_k^2 + 2^{-nKM} \sum_{i_0=0}^{nK} \sum_{i_{M-1}=0}^{nK} \cdots \binom{nK}{i_0} \cdots \binom{nK}{i_{M-1}} \times \Big\{A[K \sum_{k=1}^{M-1} S_k - 2(\sum_{k=0}^{M-1} i_k S_k)/n]\Big\}^2 \quad (42)$$

$$= \sigma^2 \sum_{k=0}^{M-1} S_k^2 + A^2\Big\{-(K \sum_{k=0}^{M-1} S_k)^2 + K^2 \sum_{h=0}^{M-1} \sum_{g\neq h}^{M-1} S_h S_g + \frac{K}{n}(nK+1) \sum_{k=0}^{M-1} S_k^2\Big\}. \quad (43)$$

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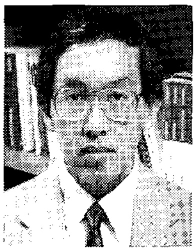

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