Reconfiguring Second-order Dynamic Systems via P-D Feedback Eigenstructure Assignment: A Parametric Method

Guo-Sheng Wang, Bing Liang, and Guang-Ren Duan

Abstract: The design of reconfiguring a class of second-order dynamic systems via proportional plus derivative (P-D) feedback is considered. The aim is to resynthesize a P-D feedback controller such that the eigenvalues of the reconfigured closed-loop system can completely recover those of the original close-loop system, and make the corresponding eigenvectors of the former as close to those of the latter as possible. Based on a parametric result of P-D feedback eigenstructure assignment in second-order dynamic systems, parametric expressions for all the P-D feedback gains and all the closed-loop eigenvector matrices are established and a parametric algorithm for this reconfiguration design is proposed. The parametric algorithm offers all the degrees of design freedom, which can be further utilized to satisfy some additional performances in control system designs. This algorithm involves manipulations only on the original second-order system matrices, thus it is simple and convenient to use. An illustrative example and the simulation results show the simplicity and effect of the proposed parametric method.

Keywords: Second-order dynamic systems, eigenstructure assignment, P-D feedback, parametric method.

1. INTRODUCTION

Reconfigured control systems (RCS) posses the ability of accommodating system failures automatically with some prior assumptions. The research for RCS is largely motivated by the control problems encountered in designing the aircraft control systems. Its main aim is to achieve the so called "fault-tolerant" or "self-repairing" capability in flight control systems, so that these designed control systems can work properly or safely.

In recent years, RCS has drawn much attention of many researchers, and many new methods and schemes have been proposed (see e.g. [1-7] and their references). In addition to linear quadratic regulator method [1], pseudo inverse method [2], inverse component-mode synthesis method [3], Lyapunov method [4] and LMI method [5], eigenstructure assignment method [6,7] becomes more and more attractive. Based on the fact that the performances of a control system are mainly determined by their eigenvalues and the corresponding eigenvectors, thus eigenstructure assignment method is convenient to redesign a new gain matrix in order to recover the eigenvalues of the normal control system and make their corresponding eigenvectors of the reconfigured closed-loop systems as close to those of the normal closed-loop system as possible.

In this paper, we consider the design of reconfiguring a class of second-order dynamic system. Based on the result for parametric eigenstructure assignment via P-D feedback in second-order dynamic systems proposed by Duan [8-10], a parametric form of all the resynthesized gain matrices is derived and a corresponding algorithm for this reconfiguration is proposed.

This paper is organized as follows: in Section 2, the reconfiguration problem in a class of second-order dynamic systems via P-D feedback is formulated. Section 3 gives the preliminaries to solve this problem in study. The parametric expression of the resynthesized gain matrices and the corresponding algorithm is established in Section 4. To demonstrate the simplicity and effect of the proposed parametric method, an illustrative example and the simulation results are given in Section 5. Remarking conclusions are drawn in Section 6.

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Guo-Sheng Wang has finished his PhD degree from Harbin Institute of Technology, and is with the Department of Control Engineering, Academy of Armored Force Engineering, Beijing, 100072, China (e-mail: gswang@hit.edu.cn).

Bing Liang and Guang-Ren Duan are with the Center for Control Theory and Guidance Technology, Harbin Institute of Technology, P. O. Box 416, Harbin, 150001, China (e-mails: {liangbing, g.r.duan}@hit.edu.cn).

2. PROBLEM FORMULATION

Consider a class of second-order dynamic systems in the form of

$$E\ddot{q} - A\dot{q} - Cq = Bu, \quad (1)$$

where $q \in \mathbf{R}^n$ and $u \in \mathbf{R}^r$ are the state vector and the input vector, respectively; E, A, B and C are known matrices with appropriate dimensions, and satisfy the following assumptions:

Assumption A1: $\mathrm{rank}(E) = n$, $\mathrm{rank}(B) = r$;

Assumption A2: The matrix triple (E, A, B) is controllable, that is,

$$\mathrm{rank}[A - sE \quad B] = n, \quad \forall s \in \mathbf{C}. \quad (2)$$

Because of the outstanding variations, the system (1) becomes into the following form

$$E_f\ddot{q}_f - A_f\dot{q}_f - C_f q_f = B_f u_f, \quad (3)$$

where $q_f \in \mathbf{R}^n$ and $u \in \mathbf{R}^m$ are the state vector and the input vector, respectively; E_f, A_f, B_f and C_f are also known matrices with appropriate dimensions, which can be regarded as the disturbance matrices of E, A, B and C in the system (1), respectively, and satisfy the following assumptions:

Assumption A3: $\mathrm{rank}(E_f) = n$, $\mathrm{rank}(B_f) = r$;

Assumption A4: The matrix triple (E_f, A_f, B_f) is controllable, that is,

$$\mathrm{rank}[A_f - sE_f \quad B_f] = n, \quad \forall s \in \mathbf{C}. \quad (4)$$

For convenience, we call system (1) the normal second-order dynamic system and system (3) the fault second-order dynamic system.

For control applications, second-order dynamic systems are usually transformed into their first-order linear systems. Thus, the normal second-order dynamic system (1) is equivalent to the following first-order linear system:

$$E'\dot{x} = A'x + B'u, \quad (5)$$

where

$$E' = \begin{bmatrix} I & 0 \\ 0 & E \end{bmatrix}, \quad A' = \begin{bmatrix} 0 & I \\ C & A \end{bmatrix}, \quad B' = \begin{bmatrix} 0 \\ B \end{bmatrix}, x = \begin{bmatrix} q \\ \dot{q} \end{bmatrix}. \quad (6)$$

Similarly, the fault second-order dynamic system (3) is turned into the following first-order linear system

$$E'_f \dot{x}_f = A'_f x_f + B'_f u_f, \quad (7)$$

where

$$E'_f = \begin{bmatrix} I & 0 \\ 0 & E_f \end{bmatrix}, \quad A'_f = \begin{bmatrix} 0 & I_f \\ C_f & A_f \end{bmatrix}, \quad B'_f = \begin{bmatrix} 0 \\ B_f \end{bmatrix}, \quad x_f = \begin{bmatrix} q_f \\ \dot{q}_f \end{bmatrix}. \quad (8)$$

Applying a P-D feedback control law

$$u = K_0 q + K_1 \dot{q} = Kx, \quad K = [K_0 \quad K_1] \in \mathbf{R}^{r\times 2n}, \quad (9)$$

to system (5), yields the closed-loop system as

$$E'\dot{x} = A_c x, \quad A_c = A' + B'K. \quad (10)$$

Recall the fact that non-defective matrices possess eigenvalues which are less insensitive with respect to parameter perturbations, thus we only consider the eigenvalues of the closed-loop system (10) are distinct and self-conjugate. Denote

$$\sigma(E', A_c) = \{s_i, i = 1, 2, \cdots, 2n\},$$

where $s_i, i = 1, 2, \cdots, 2n$, are self-conjugate and distinct, and $\sigma(M, N)$ represents the set of finite eigenvalues of the matrix pair (M, N). Further, denoting the corresponding eigenvectors of the matrix pair (E', A_c) associated with $s_i \in \mathbf{C}$ by $v_i \in \mathbf{C}^{2n}$, produces

$$E' v_i s_i = A_c v_i, \quad i = 1, 2, \cdots, 2n. \quad (11)$$

Applying the following P-D feedback

$$u_f = K_f x_f, K_f = [K_{f0} \quad K_{f1}] \in \mathbf{R}^{m\times 2n}, \quad (12)$$

to system (7), we obtain the closed-loop system as follows:

$$E'_f \dot{x}_f = A_{fc} x_f, \quad A_{fc} = A'_f + B'_f K_f. \quad (13)$$

For simplicity, we call system (5) the normal system and system (10) the normal closed-loop system. Correspondingly, (7) is called the fault system and (13) is called the fault closed-loop system.

Based on the fact that the internal behaviors of a control system are determined by its eigenvalues together with the corresponding eigenvectors, and the performances of its closed-loop system can be improved by modifying the eigenvalues and the corresponding eigenvectors with some feedback control laws, then the problem of reconfiguring second-order dynamic system (1) via P-D feedback (12) to be solved in this paper can be stated as follows.

Problem RPD: Given matrices E, A, B and C satisfying Assumptions A1 and A2, matrices E_f, A_f, B_f and C_f satisfying Assumptions A3 and A4, and a set of self-conjugate distinct complex numbers $s_i, i=1,2,\cdots,2n$, then resynthesize a new P-D feedback gain matrix K_f in (12) such that

$$\sigma(E',A_c)=\sigma(E'_f,A_{fc})=\{s_i,\ i=1,\ 2,\cdots,\ 2n\}, \quad (14)$$

and

$$J_i=\left\|v_i-v_{f_i}\right\|^2,\ \ i=1,2,\cdots,2n, \quad (15)$$

are minimized, where $v_{fi}, v_i\in\mathbf{C}^{2n}$ are the eigenvectors of the matrix pairs (E'_f, A_{fc}) and (E',A_c) associated with s_i, respectively.

Remark 1: According to the description of Problem RPD, when the relation (14) is satisfied, there holds

$$E'_f\, v_{fi}s_i=A_{fc}v_{fi},\quad i=1,2,\cdots,2n. \quad (16)$$

3. PRELIMINARIES

Denote

$$\Lambda=\mathrm{diag}(s_1,s_2,\cdots,s_{2n}),\ \ V=[v_1\ v_2\ \cdots v_{2n}], \quad (17)$$

then equations in (11) can be written in the following compact form:

$$A'V+B'KV=E'V\Lambda. \quad (18)$$

Further, denote

$$W=KV, \quad (19)$$

Then (18) is changed into

$$A'V+B'W=E'V\Lambda. \quad (20)$$

Noticing (6) and letting

$$V=\begin{bmatrix}V_1\\V_2\end{bmatrix},\ \ V_1,V_2\in\mathbf{C}^{n\times 2n}, \quad (21)$$

we can obtain from (20) that

$$V_2=V_1\Lambda, \quad (22)$$

and

$$AV_2+BW=EV_2\Lambda-CV_1. \quad (23)$$

If the matrix triple (E,A,B) is controllable, applying some elementary matrix transformations to matrix $[A-sE\ \ B]$, we can obtain a pair of unimodular matrices $P(s)\in\mathbf{R}^{n\times n}[s]$ and $Q(s)\in\mathbf{R}^{(n+r)\times(n+r)}[s]$ satisfying

$$P(s)[A-sE\ \ B]Q(s)=[0\ \ I],\ \forall s\in\boldsymbol{C}. \quad (24)$$

Partition $Q(s)$ into the following form

$$Q(s)=\begin{bmatrix}Q_{11}(s) & Q_{12}(s)\\Q_{21}(s) & Q_{22}(s)\end{bmatrix},\ Q_{11}(s)\in\mathbf{R}^{n\times r}[s]. \quad (25)$$

Then we present the following theorem, which gives the result of parametric eigenstructure assignment for the second-order dynamic system (1) via P-D feedback (9) and utilizes the original system matrices in second-order dynamic system (1). The proof of the following theorem can be found in [9].

Theorem 1: Given matrices E, A, B and C satisfying Assumptions A1 and A2, and a group of distinct and self-conjugate scalars $s_i, i=1,2,\cdots,2n$, then

1) When the matrix triple (E, A, B) be controllable, the matrix triple (E',A',B') is also controllable if and only if there are a pair of unimodular matrices $H(s)\in\mathbf{R}^{n\times n}[s]$ and

$$L(s)=\begin{bmatrix}L_{11}(s) & L_{12}(s)\\L_{21}(s) & L_{22}(s)\end{bmatrix},L_{11}(s)\in\mathbf{R}^{n\times r}[s], \quad (26)$$

satisfying the following equation

$$H(s)[Q_{12}(s)P(s)C+sI\ \ -Q_{11}(s)]L(s)=\begin{bmatrix}0 & I\end{bmatrix}. \quad (27)$$

2) When the above condition is met, the parametric expressions for all the matrices V and W satisfying (22) and (23) are given by their column vectors, respectively,

$$v_i=\begin{bmatrix}v_{1i}\\v_{2i}\end{bmatrix}=\begin{bmatrix}L_{11}(s_i)\\s_iL_{11}(s_i)\end{bmatrix}g_i, \quad (28)$$

and

$$w_i = [Q_{21}(s_i)L_{21}(s_i) - Q_{22}(s_i)P(s_i)CL_{11}(s_i)]g_i, \quad (29)$$

where $v_{1i} \in \mathbf{C}^n$ and $v_{2i} \in \mathbf{C}^n$, are the column vectors of V_1 and V_2, respectively, and the corresponding P-D feedback gain matrix K satisfying (18) is determined by

$$K = WV^{-1}, \quad (30)$$

where $g_i \in \mathbf{C}^r, i = 1, 2, \cdots, 2n$, are a group of free parameter vectors, satisfying the following two constraints:

Constraint C1:

$$g_i = \overline{g_j} \Leftrightarrow s_i = \overline{s_j}, i, j = 1, 2, \cdots, 2n;$$

Constraint C2:

$$\det\begin{bmatrix} L_{11}(s_1)g_1 & \cdots & L_{11}(s_{2n})g_{2n} \\ s_1L_{11}(s_1)g_1 & \cdots & s_{2n}L(s_{2n})g_{2n} \end{bmatrix} \neq 0.$$

4. SOLUTION TO PROBLEM RPD

Due to Assumptions A3 and A4, and Theorem 1, we can know that if the matrix triples (E_f, A_f, B_f) and (E'_f, A'_f, B'_f) are both controllable, the 2n eigenvalues of the matrix pair (E'_f, A_{fc}) can be assigned arbitrarily via P-D feedback. Thus the eigenvalues s_i, $i = 1, 2, \cdots, 2n$, of the matrix pair (E', A_c) can be assigned to those of the matrix pair (E'_f, A_{fc}) via P-D feedback. Then the relation (14) in Problem RPD is satisfied and the main task left for the solution to Problem RPD is to design a P-D feedback such that (15) holds.

Clearly, denote

$$V_f = [v_{f1}\ v_{f2} \cdots v_{f2n}], \quad (31)$$

then equations in (16) can be written into the following compact form

$$E'_f V_f \Lambda = A'_f V_f + B'_f K_f V_f. \quad (32)$$

Further, denote

$$W_f = K_f V_f, \quad (33)$$

Then (32) is changed into the following form

$$E'_f V_f \Lambda = A'_f V_f + B'_f W_f. \quad (34)$$

Noticing (8) and letting

$$V_f = \begin{bmatrix} V_{f1} \\ V_{f2} \end{bmatrix}, \quad V_{f1}, V_{f2} \in \mathbf{C}^{n\times 2n}, \quad (35)$$

from (34), yields

$$V_{f2} = V_{f1}\Lambda, \quad (36)$$

and

$$A_f V_{f2} + B_f W_f = E_f V_{f2}\Lambda - C_f V_{f1}. \quad (37)$$

Due to Assumptions A3 and A4, applying a series of elementary matrix transformations to matrix $[A_f - sE_f \quad B_f]$, we can obtain a pair of unimodular matrices $P_f(s) \in \mathbf{R}^{n\times n}[s]$ and $Q_f(s) \in \mathbf{R}^{(n+m)\times(n+m)}[s]$ satisfying

$$P_f(s)[A_f - sE_f \quad B_f]Q_f(s) = [0 \quad I], \forall s \in \mathbf{C}. \quad (38)$$

Partition $Q_f(s)$ into the following form

$$Q_f(s) = \begin{bmatrix} Q_{11}^f(s) & Q_{12}^f(s) \\ Q_{21}^f(s) & Q_{22}^f(s) \end{bmatrix}, \quad Q_{11}^f(s) \in \mathbf{R}^{n\times m}[s]. \quad (39)$$

By utilizing the same method in Theorem 1, we can obtain the following theorem, which gives solutions to (36) and (37).

Theorem 2: Given matrices E_f, A_f, B_f and C_f satisfying Assumptions A3 and A4, and a group of distinct and self-conjugate scalars s_i, $i = 1, 2, \cdots, 2n$, then

1) When the matrix triple (E_f, A_f, B_f) is controllable, the matrix triple (E'_f, A'_f, B'_f) is also controllable if and only if there are a pair of unimodular matrices $H_f(s) \in \mathbf{R}^{n\times n}[s]$ and

$$L_f(s) = \begin{bmatrix} L_{11}^f(s) & L_{12}^f(s) \\ L_{21}^f(s) & L_{22}^f(s) \end{bmatrix}, L_{11}^f(s) \in \mathbf{R}^{n\times m}[s], \quad (40)$$

satisfying the following equation

$$H_f(s) \times [Q_{12}^f(s)P_f(s)C_f + sI - Q_{11}^f(s)]L_f(s) = [0 \quad I]. \quad (41)$$

2) When the above condition holds, the parametric

expressions of all the matrices V_f and W_f satisfying (36) and (37), are given by their column vectors, respectively,

$$v_{fi} = \begin{bmatrix} v_{f1i} \\ v_{f2i} \end{bmatrix} \begin{bmatrix} L_{11}^f(s_i) \\ s_i L_{11}^f(s_i) \end{bmatrix} g_{fi}, \quad (42)$$

and

$$w_{fi} = [Q_{21}^f(s_i) L_{21}^f(s_i) - Q_{22}^f(s_i) P_f(s_i) C_f L_{11}^f(s_i)] g_{fi} \quad (43)$$

where $v_{f1i} \in \mathbf{C}^n$ and $v_{f2i} \in \mathbf{C}^n$, $i = 1, 2, \cdots, 2n$, are the column vectors of V_{f1} and V_{f2}, respectively, and $g_{fi} \in \mathbf{C}^m$, $i = 1, 2, \cdots, 2n$, are a group of arbitrary parameter vectors.

Let

$$F(s_i) = \begin{bmatrix} L_{11}(s_i) \\ s_i L_{11}(s_i) \end{bmatrix}, \quad F_f(s_i) = \begin{bmatrix} L_{11}^f(s_i) \\ s_i L_{11}^f(s_i) \end{bmatrix}, \quad (44)$$

then (28) and (42) are equivalent with the following forms:

$$v_i = F(s_i) g_i, \quad i = 1, 2, \cdots, 2n, \quad (45)$$

and

$$v_{fi} = F_f(s_i) g_{fi}, \quad i = 1, 2, \cdots, 2n, \quad (46)$$

respectively. Substituting (45) and (46) into (15), then we can obtain

$$J_i = \left\| F_f(s_i) g_{fi} - F(s_i) g_i \right\|^2, \quad i = 1, 2, \cdots, 2n. \quad (47)$$

By using orthogonal projection, we can obtain

$$g_{fi} = \left[F_f^H(s_i) F_f(s_i) \right]^{-1} F_f^H(s_i) F(s_i) g_i, \quad (48)$$

which minimize the indexes in (47). Further, let

$$\Sigma_i = \left[F_f^H(s_i) F_f(s_i) \right]^{-1} F_f^H(s_i) F(s_i), \quad (49)$$

Then (48) is changed into

$$g_{fi} = \Sigma_i g_i, \quad i = 1, 2, \cdots, 2n. \quad (50)$$

Substituting (50) into (42) and (43), yields

$$v_{fi} = F_f(s_i) \Sigma_i g_i, \quad i = 1, 2, \cdots, 2n, \quad (51)$$

and

$$v_{fi} = F_f(s_i) \Sigma_i g_i, \quad i = 1, 2, \cdots, 2n, \quad (52)$$

where

$$\Psi_{fi}(s_i) = Q_{21}^f(s_i) L_{21}^f(s_i) - Q_{22}^f(s_i) P_f(s_i) C_f L_{11}^f(s_i). \quad (53)$$

Further, noticing that the eigenvectors v_{fi}, $i = 1, 2, \cdots, 2n$ of the matrix pair (E'_f, A_{fc}) are linearly independent, that is

$$\det(V_f) \neq 0. \quad (54)$$

From (33), we can obtain

$$K_f = W_f V_f^{-1}, \quad (55)$$

where

$$W_f = [w_{f1} \ w_{f2} \cdots w_{f2n}], V_f = [v_{f1} \ v_{f2} \cdots v_{f2n}]. \quad (56)$$

In order to guarantee the realness of the gain matrix K_f in (55), the following constraint must hold:

Constraint C3: $s_i = \overline{s_j} \Leftrightarrow \Sigma_i = \Sigma_j$, $g_i = \overline{g_j}$, $i, j = 1, 2, \cdots, 2n$.

Moreover, the condition (54) and Constraint C2 are clearly equivalent with the following two constraints, respectively,

Constraint C4:

$$\det[F_f(s_1)\Sigma_1 g_1 \ \cdots \ F_f(s_{2n})\Sigma_{2n} g_{2n}] \neq 0;$$

Constraint C5:

$$\det[F(s_1) g_1 \ \ F(s_2) g_2 \ \cdots \ F(s_{2n}) g_{2n}] \neq 0.$$

From Theorems 1, 2, and the above deductions, we can give the following theorem, which gives the solution to Problem RPD.

Theorem 3: Let matrices E, A, B and C satisfy Assumptions A1 and A2; matrices E_f, A_f, B_f and C_f satisfy Assumptions A3 and A4; $s_i, i = 1, 2, \cdots, 2n$, be a group of distinct and self-conjugate scalars. When the condition 1) in Theorem 1 and the condition 1) in Theorem 2 are satisfied, then all the desired solutions K_f in Problem RPD can be given by (55) with the parametric vectors $g_i \in \mathbf{C}^r, i = 1, 2, \cdots, 2n$, satisfying Constraints C3-C5.

According to Theorems 1-3 and the above

deductions, the following algorithm for Problem RPD can be proposed.

Algorithm RPD:

1. Calculate a pair of unimodular matrices $P(s)$ and $Q(s)$ satisfying (24), and partition $Q(s)$ as in (25);
2. Calculate a pair of unimodular matrices $H(s)$ and $L(s)$ satisfying (27), and partition $L(s)$ as in (26);
3. Calculate a pair of unimodular matrices $P_f(s)$ and $Q_f(s)$ satisfying (38), and partition $Q_f(s)$ as in (39);
4. Calculate a pair of unimodular matrices $H_f(s)$ and $L_f(s)$ satisfying (41), and partition $L_f(s)$ as in (40);
5. Find some parameters $g_i \in \mathbf{C}^r$, $i = 1, 2, \cdots, 2n$, satisfying Constraints C3-C5 and calculate the matrices V_f and W_f according to (50) and (51), respectively;
6. Calculate the P-D feedback gain matrix K_f according to (55).

5. AN ILLUSTRATIVE EXAMPLE

Consider a normal second-order dynamic system in the form of (1), with the following parameters:

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}, \quad A = \begin{bmatrix} -2.5 & 0.5 & 0 \\ 0.5 & -2.5 & 2 \\ 0 & 2 & -2 \end{bmatrix},$$

$$B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} -10 & 5 & 0 \\ 5 & -25 & 20 \\ 0 & 20 & -20 \end{bmatrix},$$

and its corresponding fault second-order dynamic system in the form of (3), with the following parameters:

$$E_f = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}, \quad A_f = \begin{bmatrix} -2.4 & 0.5 & 0 \\ 0.5 & -2.4 & 2 \\ 0 & 2 & -1 \end{bmatrix},$$

$$B_f = \begin{bmatrix} 0.9 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}, \quad C_f = \begin{bmatrix} -15 & 5 & 0 \\ 5 & -25 & 20 \\ 0 & 20 & -25 \end{bmatrix}.$$

Easily, we can find that the matrix triples (E, A, B), (E', A', B'), (E_f, A_f, B_f) and (E'_f, A'_f, B'_f) are controllable. In this example, we choose the eigenvalues of the normal closed-loop system as

$$s_1 = \overline{s_2} = -3.105 \pm 4.356i, \; s_3 = -4, \; s_4 = -5,$$
$$s_5 = \overline{s_6} = -6.458 \pm 1.356i.$$

Algorithm RPD is utilized to solve this reconfiguration problem. The results of each step are given as follows:

1. Calculate a pair of unimodular matrices satisfying (24) as $P(s) = \mathrm{diag}(1, 2, 1)$ and

$$Q(s) = \left[\begin{array}{cc|ccc} 2s+5 & -4 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ \hline 2(s+2.5)^2 - 0.5 & -4s-10 & 1 & s+2.5 & 0 \\ -2 & 2-s & 0 & 0 & 1 \end{array}\right].$$

2. Calculate a pair of unimodular matrices satisfying (27) as

$$H(s) = \begin{bmatrix} -0.005 & 0.005(2s+5) & -0.02 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix},$$

$$L(s) = \left[\begin{array}{cc|ccc} 2s^2 + 5s + 50 & -4 & 2s-15 & 0 & 0 \\ s+10 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ \hline s(s+10) & 0 & s & 1 & 0 \\ 0 & s & 0 & 0 & 1 \end{array}\right].$$

3. Calculate a pair of unimodular matrices satisfying (38) as $P_f(s) = I_3$ and

$$Q_f(s) = \left[\begin{array}{cc|ccc} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ -\frac{1}{4} & \frac{6}{5} + \frac{1}{2}s & 0 & \frac{1}{2} & 0 \\ \hline \frac{10}{9}s + \frac{8}{3} & -\frac{5}{9} & \frac{10}{9} & 0 & 0 \\ \frac{1}{4}s - \frac{3}{8} & -\frac{1}{5} - \frac{9}{20}s - \frac{1}{2}s^2 & 0 & \frac{3}{4} - \frac{1}{2}s & 1 \end{array}\right].$$

4. Calculate a pair of unimodular matrices satisfying (41) as

$$H_f(s) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \frac{1}{4} & -(\frac{6}{5} + \frac{1}{2}s) & 1 \end{bmatrix},$$

$$L_f(s) =$$
$$\left[\begin{array}{cc|ccc} 4 & 0 & 0 & 0 & 0 \\ 0 & \frac{20}{101}+\frac{2}{101}s & 0 & 0 & -\frac{2}{101} \\ -1 & \frac{25}{101}+\frac{12}{505}s+\frac{1}{101}s^2 & 0 & 0 & \frac{38}{505}-\frac{1}{101}s \\ \hline 4s & 0 & -1 & 0 & 0 \\ 0 & \frac{2}{101}(10+s)s & 0 & -1 & -\frac{2}{101}s \end{array}\right].$$

5. Specially choosing a group of parametric vectors as

$$\overline{g_1} = g_2 = g_5 = \overline{g_6} = \begin{bmatrix} 1+i \\ 1-i \end{bmatrix}, \quad g_3 = g_4 = \begin{bmatrix} -1 \\ 1 \end{bmatrix},$$

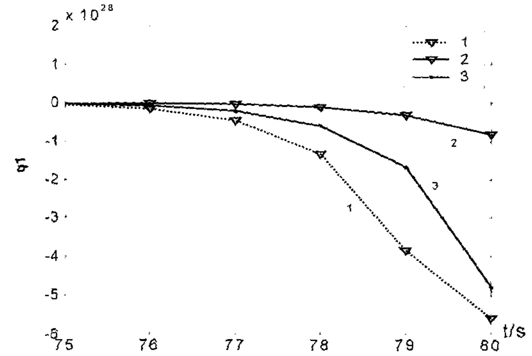

Fig. 1. Responses of the first state.

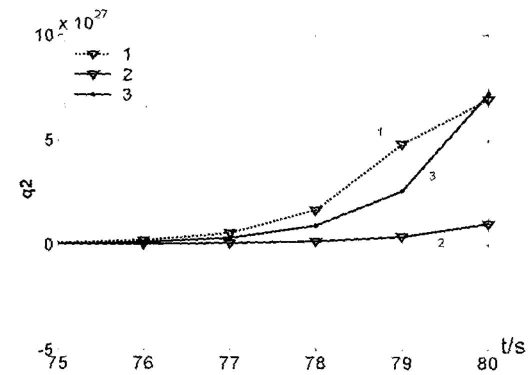

Fig. 2. Responses of the second state.

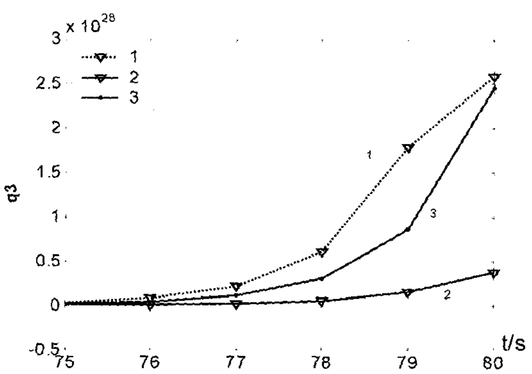

Fig. 3. Responses of the third state.

which satisfy Constraints C3-C5. From (50), we can easily obtain matrix V_f.

6. Form (55), we can obtain the desired feedback gain matrix $K_f = [K_{f0} \;\; K_{f1}]$, where

$$K_0 = \begin{bmatrix} -31.3839 & 67.6740 & -569.3502 \\ 0.7406 & -23.3790 & 36.5265 \end{bmatrix},$$
$$K_1 = \begin{bmatrix} -9.6161 & 0.8725 & -94.9544 \\ 0.0976 & -2.0302 & 9.2761 \end{bmatrix}.$$

Moreover, from (28) we can easily obtain V, and the corresponding gain matrix is obtained as $K = [K_0 \;\; K_1]$, where

$$K_0 = \begin{bmatrix} -31.3839 & 67.6740 & -569.3502 \\ 0.7406 & -23.3790 & 36.5265 \end{bmatrix},$$
$$K_1 = \begin{bmatrix} -9.6161 & 0.8725 & -94.9544 \\ 0.0976 & -2.0302 & 9.2761 \end{bmatrix}.$$

Then we can obtain

$$\min J_1 = \min J_2 = 0.8177, \quad \min J_3 = 0.0759,$$
$$\min J_4 = 0.0961, \quad \min J_5 = \min J_6 = 0.3695,$$

which show the effect of Algorithm RPD.

In order to further show the effect of Algorithm RPD, we give the simulation results of three states in this second-order dynamic system, where "1", "2" and "3" represent the response curves of the states of the normal closed-loop system under K, the fault closed-loop system under K, and the fault closed-loop system under K_f, respectively, in Figs. 1-3.

6. CONCLUSION

In this paper, we consider reconfiguring second-order dynamic systems via P-D feedback. By utilizing the freedom degrees offered by a parametric result of eigenstructure assignment in second-order dynamic systems, a parametric expression for all the P-D feedback gain matrices, which can recover the eigenvalues of the normal closed-loop system and make the eigenvectors of the fault closed-loop system as close to those of the normal closed-loop system as possible, is established and an algorithm for this design is proposed. The parametric method offers all the design degrees of freedom, which can be further utilized to satisfy certain specifications in control system designs, such as robustness etc. An illustrative example and the simulation figures show the effect of the proposed algorithm.

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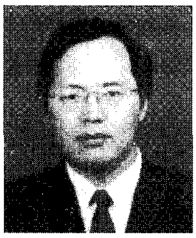
Guo-Sheng Wang received the B.S. and M.S. degrees in 1999 and 2001, respectively, and the Ph.D. degree in Control Theory and Control Engineering from Harbin Institute of Technology in 2004. He is currently a lecture of Department of Control Engineering at Academy of Armored Force Engineering. His research interests include robust control, eigenstructure assignment, and second-order linear systems.

Bing Liang received the B.S. and M.S. degrees in Mathematics from Hebei University in 1999 and 2002, respectively. She is currently working toward Ph.D. degree in Control Systems Theory of Harbin Institute of Technology. Her research interests include descriptor systems, fault-tolerant control and robust control.



Guang-Ren Duan received the B.S. degree in Applied Mathematics, and both the M.S. and Ph.D. degrees in Control Systems Theory from Harbin Institute of Technology. He is currently the Director of the Center for Control Systems Theory and Guidance Technology at Harbin Institute of Technology. His main research interests include robust control, eigenstructure assignment, descriptor systems, missile autopilot control and magnetic bearing control.