Reconfiguration of Redundant Thrusters by Allocation Method

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Abstract

Thrusters are important actuators where air is rare. Since the maintenance or replacement of thrusters is not easy in such an environment, a thrusting system must be highly reliable. Redundant thrusters are used to meet the reliability requirement. In this paper, a reconfiguration problem for those redundant thrusters is discussed, especially the management or distribution logic of redundant thrusters is focused on. The logic has to be changed if faults occur at thrusters. Reconfiguration is to change the distribution logic to accommodate thrusters' faults. The authors propose a reconfiguration algorithm based on the linear programming method. The authors define the reconfiguration problem as an optimization problem. The performance index is a quantity related with total fuel consumption by thrusters. This algorithm can accommodate multiple faults. Numerical examples are given to show the advantage of the proposed algorithm over existing methods.

Key Word : reconfiguration, redundant thrusters, thruster allocation, linear programming

Introduction

A thruster for reaction control is a unique actuator. It is usually fixed and one directional. The control method of a thruster is not to adjust the magnitude of thrust but to adjust the on-time of thrust. Since a thruster can operate where the air density is very low, it is an important actuator for attitude control of a satellite[1] or a launch vehicle's upper stage[2].

Redundant thrusters generally increase the reliability of a control system. In spite of a thruster's failure, the remaining ones can maintain normal operations of a control system. For this purpose, a geostationary satellite has 12 thrusters[1] and a launch vehicle's upper stage has 8 thrusters[2] even though 6 thrusters are sufficient for controlling 3 degrees of freedom. And it is more important to manage redundant thrusters in a reasonable way than to design and install redundant thrusters[3].

Reconfiguration of redundant thrusters is also one of management problems. When some thrusters are out of order, management logic has to be changed properly to accommodate faults. A list of possible sets may be prepared in advance and one of the sets is selected if faults occur [3]. Even though this is simple and in use, the list can not consider all combinations of possible

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faults, especially multiple faults. A pseudo inverse method is also simple and practical [2]. A right pseudo inverse has been used as a distributor when the control degrees of freedom are greater than the motion degrees of freedom. However, a pseudo inverse solution can not consider the inequality constraints of control inputs. Some elements of the solution may violate the limits of control inputs. Hence additional treatment to the solution is needed.

In this paper, the authors propose a reconfiguration algorithm based on the linear programming method. A reconfiguration problem is defined as an optimization problem. The performance index is a quantity related with total fuel consumption by thrusters. This is a linear cost function. Limitations of on-times are expressed as inequality constraints and a desired momentum change is expressed as an equality constraints. The proposed algorithm can accommodate multiple faults. Numerical examples are given to show the advantage of the proposed algorithm over existing methods.

System Modeling and Attitude Control

Thruster Control Model

It is assumed that a target object is a rigid body. Then, the dynamic equation for the attitude control of the target object is represented as

$$\begin{aligned} \dot{\theta} &= \omega \\ J\dot{\omega} + \omega \times J\omega &= B_c \tau_c. \end{aligned} \tag{1}$$

J is the inertia matrix, θ and ω are the rotational angle and angular velocity vector, B_c is the input matrix, and τ_c is the control torque vector. Disturbance has been neglected. Since the reaction forces of thrusters are fixed, the control torques are also fixed. Hence the momentum instead of torque magnitude is controlled by adjusting a thruster's on-time as shown in Fig. 1.

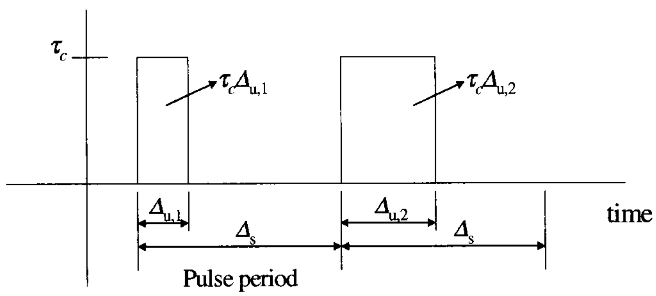

Fig. 1. Momentum control by adjusting a thruster's on-time

Then, Eq. (1) can be rewritten as

$$\begin{aligned} \theta(k+1) &= \theta(k) + \omega(k)\Delta_s \\ J\omega(k+1) &= J\omega(k) - \omega(k) \times J\omega(k)\Delta_s + B_T\Delta_u(k), \\ 0 \le \Delta_{u_i}(k) &\le \Delta_s \ . \end{aligned} \tag{2}$$

Δ_s is the control update period or pulse period. $\Delta_u(k)$ is a vector whose element represents the on-time of each thruster. The subscript i means the i-th element of a vector. B_T is another input matrix such that the torque magnitude of thrusters has been reflected. The change of J due to fuel consumption has been neglected during maneuvers. The ratio of the on-time to the pulse period, $u(k) \equiv \Delta_u(k)/\Delta_s$, is defined as a new control input variable and Eq. (2) is rewritten as

$$\begin{aligned} &\theta(k+1)=\theta(k)+\omega(k)\Delta_s \\ &J\omega(k+1)=J\omega(k)-\omega(k)\times J\omega(k)\Delta_s+Bu(k), \\ &0 \leqq u_i(k) \leqq 1\ . \end{aligned} \tag{3}$$

$B(\equiv B_T\Delta_s)$ is a new control input matrix. The control inputs are bounded between 0 and 1, and they are assumed to be continuous values.

Redundant Thrusters Allocation and Definition of Reconfiguration Problem

The procedure to determine the control input u is shown in Fig. 2. If a desired attitude or maneuver is given, control logic calculates a momentum change to follow a desired state. It is a desired momentum change and represented as m_d. Then, a distributor block calculates each thruster's on-time to realize the desired momentum change. This is the thrust allocation problem. Finally, thrusters actuates in accordance with the on-time commands.

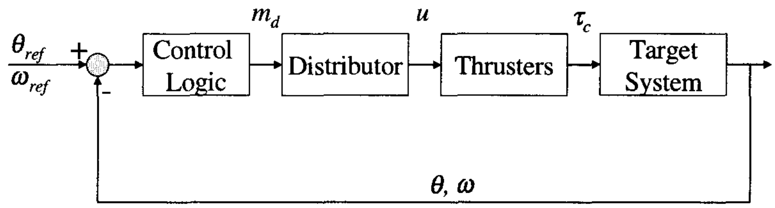

Fig. 2. Schematic diagram of an attitude control system

There are many kinds of control logic and the design of control logic is not considered here. It is assumed that proper control logic is operating and the control logic calculates m_d which can be realized in one pulse period.

The distribution logic is to determine u satisfying $m_d=Bu$ for a given m_d. The inputs are constrained by $0 \leqq u_i \leqq 1$. Generally, the dimension of u is greater than the one of m_d. One kind of the logic is a right pseudo inverse solution as $u=B^T(BB^T)^{-1}m_d=B^+m_d$. If this solution satisfies the inequality constraint, it is an optimal solution because the 2-norm of u is minimal. However, if the solution violates the inequality constraints, the solution has to be modified[2].

In this paper, the distribution logic is focused on when there are faults at thrusters. The distribution logic has to be changed if faults occur at thrusters. This is the reconfiguration problem and the reconfiguration is to change the distribution logic to accommodate thrusters' faults. An algorithm which treats multiple faults and uses minimal fuel is proposed.

Faults Model

Faults that may occur at thrusters are a locked state, a leakage, thruster efficiency reduction[4, 5]. A temporary variation of thruster efficiency is not considered here. If there is a leakage, a fuel line has to be closed to save fuel. So a leakage is treated as a locked state. If the line is independent of others, only a thruster related to the leakage will be locked. If not, a group of thrusters will be locked [6(pp.382-385)].

These faults can be modeled by changing the columns of B corresponding to faulty thrusters: a locked state is represented by dropping or making the corresponding column null and thruster efficiency reduction by scaling down the corresponding column. Then, the faulty system is represented as

$$m_d=B_f u\ ,\ B_f=BE \ \text{ where } E=\text{diag}(\alpha_1,\cdots,\alpha_m) \ \text{ and } 0 \leqq \alpha_i \leqq 1\ . \tag{4}$$

The parameters or fault modes are assumed to be identified by fault diagnosis [5] even though a diagnosis algorithm is not presented here. And the remaining control power is assumed to be sufficient to cover the number and severeness of faults.

Reconfiguration Algorithm Based on Allocation

Optimal Method To Minimize Fuel Consumption

It is desirable to minimize the fuel consumption during a maneuver. If thrusters share a common propellant tank, the total fuel consumption is proportional to the summation of all the thrusters' on-times. Hence this summation becomes a performance measure. A desired change of the angular momentum (m_d) must be obtained with the thrusters actuation (Bu), and the thrusters' on-times must be between 0 and 1. This thrust allocation problem is expressed as the following constrained optimization problem[3].

$$\text{minimize} \quad P_1 = \sum_i u_i = c^T u \ , \tag{5}$$

$$\text{subject to} \quad 0 \leqq u_i \leqq 1 \quad \text{and} \quad m_d = Bu \ ,$$

where $c^T = [1,\cdots,1]$. This is a linear programming (LP) problem. The solution u is the optimal thrusters' actuation for a given maneuver command.

The simplex method is a well-known method to solve a linear programming problem [7(pp.150-194), 8(pp.255-362)]. It consists of two phase; to find a basic feasible solution and then to find an optimal solution. Since this LP problem and the solving methods are well known, the solving methods are skipped. And we have used 'linprog', a function of MATLAB, to obtain a solution.

When faults occur, the reconfiguration is done by changing the control input matrix and the performance index as

$$\text{minimize} \quad P_2 = \sum_i u_i = c_f^T u \ , \tag{6}$$

$$\text{subject to} \quad 0 \leqq u_i \leqq 1 \quad \text{and} \quad m_d = B_f u \ .$$

The elements of c_f corresponding to locked thrusters are 0 and the others are 1. B_f is given by Eq. (4). The attainable space of desired momentum changes will be reduced by faults. It means that the system becomes slower due to faults.

This reconfiguration algorithm is simple: the only thing to do is to change several parameters related with the control input matrix and the performance index. Since the algorithm of a linear programming problem is the same, there is no need to modify the algorithm.

Pseudo Inverse Method and Grouping Method

For a faulty system, a right pseudo inverse solution is $u = B_f^+ m_d$. But this provides infeasible solutions whose elements do not satisfy the inequality constraints of $0 \leqq u_i \leqq 1$. Such elements will be set to 0 or 1.

A grouping method that considers several thrusters as one actuator is a general method and it is used in practice. But the distribution logic for one of groups is also based on the pseudo inverse method. This logic also uses B_f and finally sets some elements 0 or 1 to satisfy the inequality constraints.

Since the algorithm for these methods are largely dependent on the control input matrix, the algorithm will be presented with example systems at the following section.

Numerical Examples

The three methods are compared: the grouping method (GM), the pseudo inverse method

(PIM) and the LP method (LPM). The first example system is a geostationary and three-axis stabilized satellite having 12 thrusters [3]. The numerical data of the satellite are given as

$$B=\begin{bmatrix} 0.4 & 0.4 & -0.4 & -0.4 & 0.3 & -0.3 & -0.3 & 0.3 & -0.3 & -0.3 & 0.3 & 0.3 \\ 0.4 & -0.4 & -0.4 & 0.4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.5 & -0.5 & 0.5 & 0.5 & -0.5 & 0.5 & 0.5 & -0.5 \end{bmatrix},\ m_d=\begin{bmatrix} m_y \\ m_r \\ m_p \end{bmatrix} \tag{7}$$

For a grouping method, possible sets are prepared as shown in Table 1. For the first set, the thrusters of 5 ~ 8 are used for pitch control and the thrusters of 1 ~ 4 are used for yaw and roll control. If the thruster 5 gets out of order, other sets not including the thruster 5 will be selected. However, this way can not consider all possible sets and not accommodate multiple faults easily.

Table 1. Possible sets for grouping thrusters

Set No.	Thruster No. (P: Pitch, Y:Yaw, R:Roll)	
1	P: 5, 6, 7, 8	Y/R: 1, 2, 3, 4
2	P: 9,10,11,12	Y/R: 1, 2, 3, 4
3	P: 5, 7, 9,11	Y/R: 1, 2, 3, 4
4	P: 6, 8,10,12	Y/R: 1, 2, 3, 4
5	R: 1, 2, 3, 4	Y/P: 5, 6, 7, 8
6	R: 1, 2, 3, 4	Y/P: 9,10,11,12

The first set has been selected for comparison so the first eight columns of Eq. (7) are used. A right pseudo inverse solution, $u=B^T(BB^T)^{-1}m_d=B^+m_d$, has to be modified to satisfy the inequality constrains as

$$\begin{aligned} &\text{if } u_i-u_{i+2}>0\ ,\ u_i=u_i-u_{i+2} \text{ and } u_{i+2}=0 \\ &\text{else} \quad u_{i+2}=u_{i+2}-u_i \text{ and } u_i=0\ .\quad i=1,2,5,6. \\ &0\leq u_i\leq 1\ ,\quad i=1,\cdots,8. \end{aligned} \tag{8}$$

For a grouping method, the first four thrusters are allocated to yaw and roll control, and last four thrusters to pitch control as

$$\begin{aligned} &\begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix}=\frac{1}{1.6}\begin{bmatrix} 1 & 1 \\ 1 & -1 \\ -1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} m_y \\ m_r \end{bmatrix}\ ,\quad \begin{bmatrix} u_5 \\ u_6 \\ u_7 \\ u_8 \end{bmatrix}=\begin{bmatrix} -1 \\ -1 \\ 1 \\ 1 \end{bmatrix}m_p\ , \\ &\text{if } u_i-u_{i+2}>0\ ,\ u_i=u_i-u_{i+2} \text{ and } u_{i+2}=0 \\ &\text{else} \quad u_{i+2}=u_{i+2}-u_i \text{ and } u_i=0\ .\quad i=1,2. \\ &\text{if } m_{d,p}>0\ ,\quad u_5=u_6=0 \\ &\text{else} \quad u_7=u_8=0\ , \\ &0\leq u_i\leq 1\ ,\quad i=1,\cdots,8. \end{aligned} \tag{9}$$

A test set of desired momentum changes and a fault scenario are given as

$$\begin{aligned} &-1\leq m_y\leq 1\ ,\ -1\leq m_r\leq 1\ ,\ -1\leq m_p\leq 1\ , \\ &\text{Thruster 1 = no operation, Thruster 5 = reduced efficiency by 50 \%.} \end{aligned} \tag{10}$$

Fig. 3 shows the distribution of solutions by each method. The area P is provided by the PIM, G by the GM, and L by the LPM. Some solutions which are not inside P or G but L may require more fuel consumption for rapid maneuver.

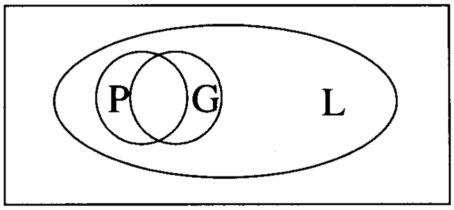

Fig. 3. Diagram of solution distribution

Table 2 shows the results. Additional fuel consumption means that the percentage of average differences between the PIM (or GM) and the LPM over the area P (or G). A positive value means that a method uses more fuel than the LPM.

Table 2. Test results of the pseudo inverse method, grouping method and LP method

		PIM	GM	LPM
Normal case	$m_d = Bu$ satisfied	33 %	33 %	51 %
	Additional fuel consumption compared with optimal	4 %	8.5 %	-
Faulty case	$m_d = Bu$ satisfied	14 %	13 %	25 %
	Additional fuel consumption compared with optimal	7.4 %	14 %	-

The fuel consumption of the GM is the largest. The LP method has found solutions for more m_d. It means that some desired momentum changes cannot be followed by the PIM or the GM but can be followed by the LPM. For a faulty condition, The percentages of times that solutions were found decreased for all the methods.

The reconfiguration algorithm of the GM is as

$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} = \frac{1}{1.6} \begin{bmatrix} 0 & 0 \\ 1 & -1 \\ -1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} m_y \\ m_r \end{bmatrix} , \quad \begin{bmatrix} u_5 \\ u_6 \\ u_7 \\ u_8 \end{bmatrix} = \begin{bmatrix} -2 \\ -1 \\ 1 \\ 1 \end{bmatrix} m_p . \tag{11}$$

The fifth on-time command has doubled to compensate the fifth thruster. For the PIM, B_f has to be used. However, the matrix B_f^+ is too complicated to devise a reconfiguration algorithm. If this matrix and Eq. (8) is used, there are very few m_d that this reconfiguration can provide solutions. Hence the reconfiguration algorithm has been modified as

$$\begin{aligned} & u = B^T(BB^T)^{-1} m_d = B^+ m_d \\ & \text{if } u_i - u_{i+2} > 0 , \ u_i = u_i - u_{i+2} \ \text{ and } \ u_{i+2} = 0 \\ & \text{else} \qquad u_{i+2} = u_{i+2} - u_i \ \text{ and } \ u_i = 0 . \quad i = 1,2,5,6. \\ & u_5 = 2u_5 , \\ & u_1 = 0 , \ 0 \leq u_i \leq 1 , \ i = 2,\cdots,8. \end{aligned} \tag{12}$$

Also, the PIM showed inefficiency to find a solution of desired momentum changes whose directions were similar to the columns of B. For example, if $m_d^T = [0.4, 0.4, 0.1]$ which is similar to the fifth column, solutions by the LPM and the PIM are given as Eq. (13). The sum of u_1 is 1.2 and the sum of u_2 is 1.48. The difference is 23 %.

$$u_1^T = [0.9532 \quad 0 \quad 0 \quad 0.0468 \quad 0 \quad 0 \quad 0.0376 \quad 0.1624] , \qquad (13)$$
$$u_2^T = [0.82 \quad 0 \quad 0 \quad 0.18 \quad 0.14 \quad 0 \quad 0 \quad 0.34] .$$

The second example system is a launch vehicle's upper stage having 8 thrusters [2]. The numerical data are given as

$$B=\begin{bmatrix} 2 & 2 & -2 & -2 & 0 & 0 & 0 & 0 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\ 0 & 0 & 0 & 0 & -2 & -2 & 2 & 2 \end{bmatrix} , \ m_d = \begin{bmatrix} m_y \\ m_r \\ m_p \end{bmatrix} . \qquad (14)$$

The test set of desired momentum changes is given as

$$-4 \leqq m_y \leqq 4 \ , \ -2 \leqq m_r \leqq 2 \ , \ -4 \leqq m_p \leqq 4 \ . \qquad (15)$$

A fault scenario is the same as Eq. (10). Table 3 shows the test results. These results also show a similar trend to Table 2.

The PIM and the GM are not easy to devise a reconfiguration algorithm for multiple faults. And the combinations of possible faults are too diverse to consider all of them. But the LPM is very simple. There is no need to devise a reconfiguration algorithm for each scenario. This advantage will greatly reduce the burden of developing control and distribution logic.

Table 3. Test results of the pseudo inverse method, grouping method and LP method

		PIM	GM	LPM
Normal case	$m_d = Bu$ satisfied	58 %	49 %	81 %
	Additional fuel consumption compared with optimal	2.8 %	12 %	-
Faulty case	$m_d = Bu$ satisfied	25 %	19 %	41 %
	Additional fuel consumption compared with optimal	5.6 %	18 %	-

Conclusion

A reconfiguration algorithm for managing redundant thrusters was presented. The proposed algorithm was based on the linear programming (LP) method. A reconfiguration problem was defined as an optimization problem whose performance index was a linear cost function related with total fuel consumption by thrusters. The pseudo inverse method, the grouping method, and the proposed LP method were compared with the percentage of times that solutions were found and the amount of consumed fuel. Even though it took the longest time for the LP method to find a solution, the LP method showed the best performance. And also the reconfiguration algorithm could easily treat multiple faults.

The LP method may not be used right now due to the limited computing power but this limitation will be overcome by rapidly developing technologies in the near future. So this will be used as a useful distribution or management logic for redundant thrusters.

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