International Journal of Precision Engineering and Manufacturing Vol. 6, No.1, January 2005.

A Study on the Influence of Nonlinearity Coefficients in Air-Bearing Spindle Parametric Vibration

Y.A. Chernopyatov[1], C.M. Lee[2#], W.J. Chung[2] and K.S. Dolotov[3]

[1] Researcher, Machine Tool Research Center (RRC), Changwon National University, Changwon, Kyongnam 641-773, Korea
[2] Professor, School of Mechatronics, Changwon National University,Changwon, Kyongnam 641-773, Korea
[3] Ph.D., TTM Dept., MSUT STANKIN, Moscow, Russia

ABSTRACT

The development of the high-efficiency machine-tools equipment and new cutting tool materials with high hardness, heat- and wear-resistance has opened the way to application of high-speed cutting process. The basic argument of using of high-speed cutting processes is the reduction of time and the respective increase of machining productivity. In this sense, the spindle units may be regarded as one of the most important units, directly affecting many parameters of high-speed machining efficiency. One of the possible types of spindle units for high-speed cutting is the air-bearing type. In this paper, we propose the mathematical model of the dynamic behavior of the air-bearing spindle. To provide the high-level of speed capacity and spindle rotation accuracy we need the adequate model of "spindle-bearings" system. This model should consider characteristics of the interactions between system components and environment. To find the working characteristics of spindle unit we should derive the equations of spindle axis movement under the affecting factors, and solve these equations together with equations which describe the behavior of lubricant layer in bearing (bearing stiffness equations). In this paper, the three influence coefficients are introduced, which describe the center of spindle mass displacement, angle of shaft rotation around the axes under the unit force application and that under the unit torque application. These coefficients are operated in the system of differential equations, which describes the spindle axis spatial movement. This system is solved by Runge-Kutta method. Obtained trajectories and amplitude-frequency characteristics were then compared to experimental ones. The analysis shows good agreement between theoretical and experimental results, which confirms that the proposed model of air-bearing spindle is correct

Key Words : CAE, air-bearing spindle, vibration, trajectory.

1. Introduction

The experience of the world machine-tool industry and leading specialists indicates that current maximum cutting speeds lie in the range of 300 m/s, and soon should reach 500m/s. The usual machining precision is in the range of 1μm, but in some cases it is necessary to provide 0.1μm precision. The spindle unit is one of the main possible sources of out-of-shape defects, that is why industry raises exact requirements to its speed capacity, load capacity and precision. These requirements force manufacturers to use different types of bearings – rolling, magnetic and air-bearings, in high-speed high-precision machining centers.

The rolling bearings provide high stiffness, and thus high load capacity, however their speed capacity is limited to 30,000-50,000 rpm. This limitation is stipulated by durability and bearing manufacturing

Manuscript received: August 29, 2003 ;
Accepted: August 4, 2004
Corresponding Author:
Email: cmlee@sarim.changwon.ac.kr
Tel: +82-55-279-7572, Fax: +82-55-263-7574

accuracy. More reasons for this limitation lie in the difficullty of manufacturing rigid shaft which will work in ranges above the first critical speed. As a rule, the rotation accuracy of precision spindle with rolling bearings is not better than 0.5μm.

The magnetic bearings have several advantages. The main advantage is the possibility to control the spindle position during machining. Moreover, it is possible to adjust the stiffness and damping. However, there are some crucial drawbacks in the usage of the magnetic bearings. They are, namely: the complexity of construction and control, usage of diamagnetic materials, spindle heating, unhandiness and high sluggishness of induction system, etc. All these disadvantages degrade the wide appliance of this bearing type.

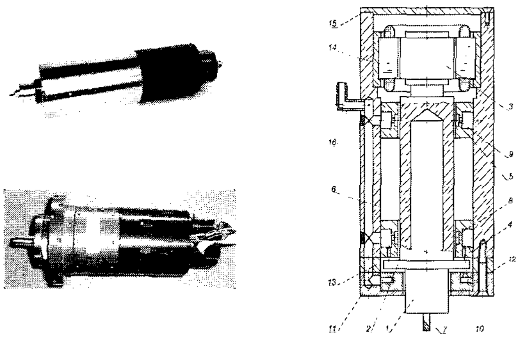

Fig. 1 Various examples of air-bearing spindles (a) and spindle structure (b)

At the same time, the spindle units on air bearings are the most apparent from the different points of view. For example, they provide higher speed and load capacity while being precise enough. Currently, the rotation speed of air-bearing spindles reaches 150,000 rpm, the spindle rotation accuracy is 0.1 to 0.5μm (some samples may have accuracy of 0.05μm). However, there is a popular but incorrect bias that air-bearings could not provide enough stiffness. The static rigidity of the air-bearing is not more than 10-12 N/μm, however the real stiffness of spindle unit is defined in general by many other parameters. There is distinction between air-bearings and rolling bearings; an air-bearing spindle works as a spindle with elastic bearings working at higher speeds than the first critical speed. At these conditions, spindle behaves like an absolute rigid body, which, at high angular speed, possesses the self-centering property. Air-bearing spindle passes the first and second critical speeds, having the cylindrical and conical precessions. Due to the high gyroscopic torque and high capacity of kinetic energy the real stiffness of this spindle unit with air-static bearings is much higher than static stiffness.

2. Nonlinear components of spindle-bearing model

Without doubt, the prognosis of tool axis mechanical trajectories at the cutting speed of 50 m/min and higher, for an air-bearing spindle, is a very important and complicated problem. The air-bearing spindles are widely used for drilling and milling process of printed-circuit boards (PCB), for grinding and boring small diameter holes, with ceramics or cermet cutting, tails when both high speeds and high precision are required. As a rule, the tool stiffness is quite low, or considerably less than bearing stiffness.

The modern scheme of this equipment is based on the following principle: an asynchronous electric motor rotates the spindle shaft with a tool. This shaft rotates on aerostatics bearings, which excludes any mechanical contact of rotation surfaces. To control the speed of shaft rotation the operator changes the frequency of current supply. The current tendency turns to wide usage of built-in motors. These constructions employ the spindle shaft combined together with the motor shaft, i.e. the spindle shaft becomes a part of motor rotor, while the stator winding is located inside the spindle body. Tools are replaced automatically with built-in pneumatic cylinder.

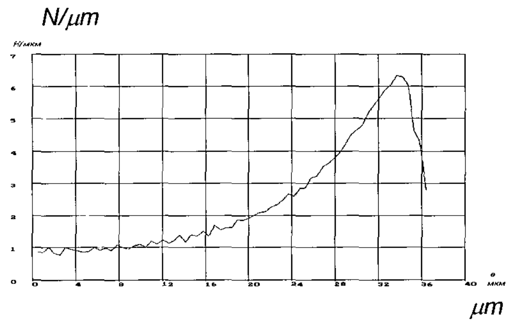

Fig. 2 Stiffness coefficient characteristics

To provide the high-level of speed capicty and spindle rotation accuracy we need mathematical model of the "spindle-bearings" to be adequate enough. This model should consider the characteristics of interactions between system components and environment. The experience shows that system must consider almost ten nonlinear effects which are most influencing the accuracy and precision of spindle unit. Some of these coefficients will be decribed later.

To find the working characteristics of spindle unit we should comprise the equations of spindle axis movement under the affecting factors, and solve these equations along with equations which describe the behavior of lubricant layer in bearing (bearing stiffnes equations).

In general, the processes taking place in a cutting zone are difficult to study. However, we can confidently maintain that only a spindle axis is oscillating and therefore transmits the significant part of errors through the tool axis to the contact point of tool and machining surface. Thus, the surface obtains the out-of-shape error (considering the allowance for mandrel and tool set-up error). The tool axis is continuation of the spindle axis, that is, the spindle axis oscillation seriously affects the accuracy of machining. When the spindle axis is rotating,

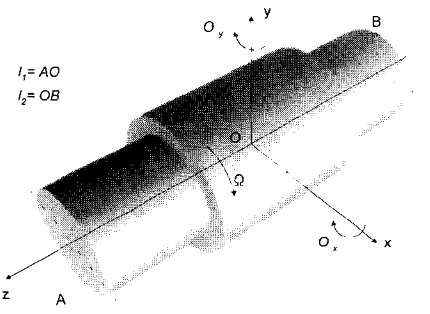

Fig. 3 Spindle calculation scheme

a complicated movement in the form of cylindrical or cone precession takes place. The determination of the forms of this movement would allow us to define the quality of machining. Spindle is the object, which has both static and dynamic out-of-balance with complicated constraints (non-linear parametric oscillation of the solid). The tool and tool equipment brings about in some additional out-of-balance. Also, the influence of cutting forces are being transferred through the tool. Hence, it is possible to define the laws of cutting edge movement and consequently the accuracy of machining on the base of the equations, which are describing the spindle movement in terms of the given tool and cutting force parameters[1].

The radial stiffness of the bearings is considerably less than the bending stiffness of a spindle itself, thus it will oscillate as a rigid solid body. The spindle rotation axis Z (its symmetry axis at the same time) will describe the certain spatial conical surface. It means that the spindle will perform some angular oscillations about the X and Y axes (Fig. 3). Due to these angular oscillations the angular inertial torques appear, spindle's what?.

Let C_A and C_B be the stiffness coefficients of *A* and *B* spindle bearings, respectively. The stiffness coefficients have the non-linear characteristic, as clearly shown in Fig. 2 The form of the characteristics depends on various structure parameters (bearing diameters, center shaft diameters, bearing width, number of air-nozzles) and control parameter (air pressure). Spindle bearings are also influenced by inertial forces $m\ddot{x}$ and $m\ddot{y}$ of the spindle, where m is the mass of the spindle, x and y are the projections of displacements of the spindle center of mass to the axes *X* and *Y*, respectively. Stiffness coefficients are also depend on center shaft displacement in bearing inner diameter $C_i=f(x, y)$.

Let us set up the equations of the spindle movement, using influence coefficients δ_{ij}. To make this let l_1 be the distance of first bearing (*A*) from shaft center of mass (*O*); l_2 be the distance of second bearing (*B*) from center of mass; *Po* – main force vector, and *Mo* – main torque vector, applied in the shaft center of masses.

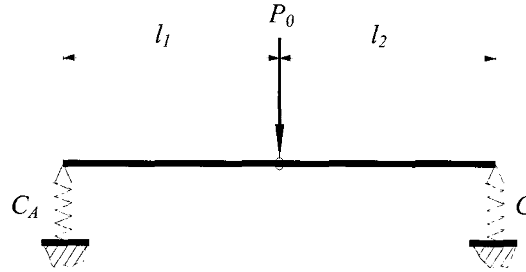

Fig. 4 Spindle calculation scheme

From the equilibrium equation, we can derive:

$$P_A + P_B = P_O$$

$$P_O l_1 = P_B (l_1 + l_2)$$

$$P_B = P_O \frac{l_1}{l_1 + l_2}$$

$$P_A = P_O - P_O \frac{l_1}{l_1 + l_2} = P_O \left(1 - \frac{l_1}{l_1 + l_2} \right) = P_O \frac{l_2}{l_1 + l_2}$$

Thus, the shaft displacement under the *Po* force will be:

$$x_A = \frac{P_O}{C_A} \cdot \frac{l_2}{l_1 + l_2}, \; x_B = \frac{P_O}{C_B} \cdot \frac{l_1}{l_1 + l_2}$$

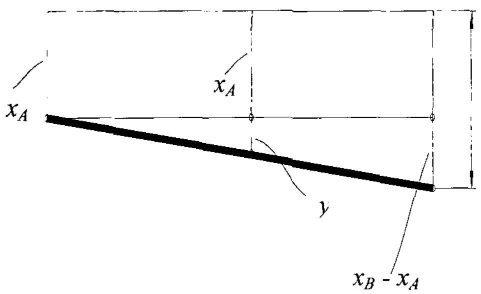

Fig. 5 Scheme for axis displacements calculation

From triangles similarity (fig. 5) we derive:

$$\frac{y}{x_B - x_A} = \frac{l_1}{l_1 + l_2}$$

$$y = \frac{l_1}{l_1 + l_2} (x_B - x_A)$$

$$y + x_A = x_B \frac{l_1}{l_1 + l_2} + x_A \left(1 - \frac{l_1}{l_1 + l_2} \right) =$$

$$= x_B \frac{l_1}{l_1 + l_2} + x_A \frac{l_1}{l_1 + l_2} =$$

$$= P_0 \left(\frac{l_1^2}{C_B (l_1 + l_2)^2} + \frac{l_2^2}{C_A (l_1 + l_2)^2} \right)$$

$$y + x_A = \delta_{11} P_O$$

The coefficient δ_{11} is the displacement of the spindle center of mass under the unit force application:

$$\delta_{11} = \frac{l_1^2}{C_B (l_1 + l_2)^2} + \frac{l_2^2}{C_A (l_1 + l_2)^2} \qquad (1)$$

The coefficient δ_{12} is the angle of shaft rotation around the axes *X* or *Y* under the unit force application:

$$\delta_{12} = \frac{l_1}{C_B (l_1 + l_2)^2} - \frac{l_2}{C_A (l_1 + l_2)^2} \qquad (2)$$

The coefficient δ_{22} is the angle of shaft rotation around the axes *X* or *Y* under the unit torque of force application.

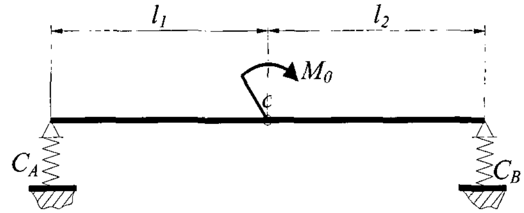

Fig. 6 Spindle calculation scheme with applied vector of main torque

From the equilibrium equation it follows that:

$$R_A + R_B = 0$$

$$R_A = -R_B$$

$$R_A l_1 - R_B l_2 + M_O = 0$$

$$M_o = R_B (l_1 + l_2)$$

$$R_B = \frac{M_O}{l_1 + l_2}, \; R_A = -\frac{M_O}{l_1 + l_2}$$

$$x_B = \frac{M_O}{C_B (l_1 + l_2)}, \; x_A = -\frac{M_O}{C_A (l_1 + l_2)}$$

$$M_O \delta_{22} = \frac{x_B - x_A}{l_1 + l_2} =$$

$$= M_O \left(\frac{1}{C_B (l_1 + l_2)^2} + \frac{1}{C_A (l_1 + l_2)^2} \right) =$$

$$= M_O \left(\frac{C_A + C_B}{C_A C_B (l_1 + l_2)^2} \right)$$

Thus the δ_{22} will be:

$$\delta_{22} = \frac{C_A + C_B}{C_A C_B (l_1 + l_2)^2} \tag{3}$$

Using these influence coefficients one can express the displacements and turning angles through the application forces:

$$\begin{cases} x = -m\ddot{x}\delta_{11} - (J\ddot{\Theta}_y - J_z\Omega\dot{\Theta}_x)\delta_{12} \\ \Theta_y = -m\ddot{x}\delta_{12} - (J\ddot{\Theta}_y - J_z\Omega\dot{\Theta}_x)\delta_{22} \\ y = -m\ddot{y}\delta_{11} + (J\ddot{\Theta}_x + J_z\Omega\dot{\Theta}_y)\delta_{12} \\ \Theta_x = m\ddot{y}\delta_{12} - (J\ddot{\Theta}_x + J_z\Omega\dot{\Theta}_y)\delta_{22} \end{cases} \tag{4}$$

where θ_x and θ_y – spindle rotation angles about X an Y axes,

J_x=J_y=J – mass moment of inertia about X, Y axes correspondingly,

J_z – mass moment of inertia,

δ_{11} – displacement of the spindle center of mass under unit force application,

δ_{12} – is the angle of shaft rotation around the axes X or Y under unit force application,

δ_{22} – the angle of shaft rotation around the axes X or Y under the unit force torque application,

Ω - the spindle rotation frequency.

A system of differential equatims (4) describes the spatial spindle oscillations as a solid body.

Taking into account that the influence coefficients δ_{11}, δ_{12} and δ_{22} are complex functions of the turning angle of the spindle around Z axis, and the radial spindle displacements in air-bearing[2], it is advisable to use numerical methods when integrating the system of equations (4). Brief analysis of the equation system shows that it can be solved using one-step methods of Cauchi problem, in which for next solution point on curve $y = f(x)$ we must have the information about only one previous step. One of the such one-step methods is the Runge-Kutta method, the further improvement of the Euler's method[3]:

$$y_{n+1} = y_n + \frac{h}{6}(k_{n1} + 2k_{n2} + 2k_{n3} + k_{n4}) \tag{5}$$

where

$$k_{n1} = f(t_n, y_n)$$
$$k_{n2} = f(t_n + h/2, y_n + hk_{n1}/2)$$
$$k_{n3} = f(t_n + h/2, y_n + hk_{n2}/2)$$
$$k_{n4} = f(t_n + h/2, y_n + hk_{n3})$$

The Runge-Kutta's method is easy to implement as the software algorithm. Moreover, the method has a local truncation error proportional to h^5, and thus a total error proportional to $(t_f - t_0)h_4$. Actually, numeric experiments show, that the error in calculation using Runge-Kutta method drops by a factor of 10,000 each time the step size is reduced by a factor of 10. The high accuracy and relative easiness of evaluation makes it a very popular method.

3. Experimental results and discussions

Examples of trajectory calculation of the spindle center under different conditions are shown in Figs. 7, 8, where m – shaft weight, p_s – feed pressure. Amplitude-frequency characteristic shown on Fig. 9 Moreover, Fig. 10 shows the comparison of experimental and calculated trajectories under the same conditions.

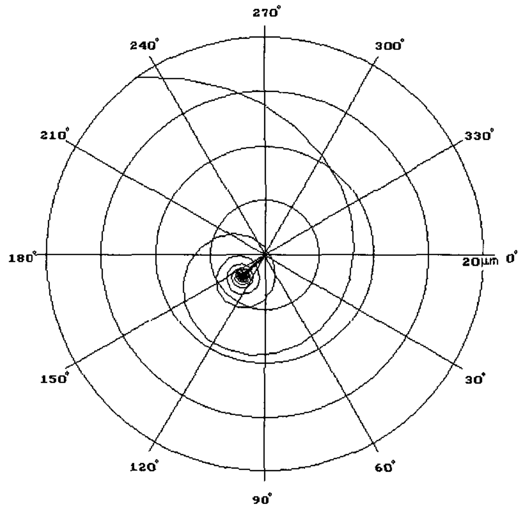

Fig. 7 Calculated trajectory (m=0.407 kg, J_x=J_y=13.5·10^{-5} kg·м^2, J_z=0.956·10^{-5} kg·м^2 , p_s=0 MPa, Ω=3000 rpm).

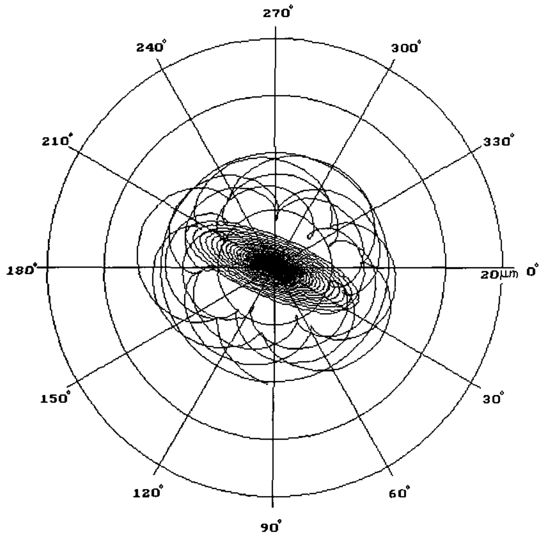

Fig. 8 Calculated trajectory (m=0.407 kg, J_x=J_y=13.5·10^{-5} kg·м^2, J_z=0.956·10^{-5} kg·м^2 , p_s=0.6 MPa, Ω=129000 rpm).

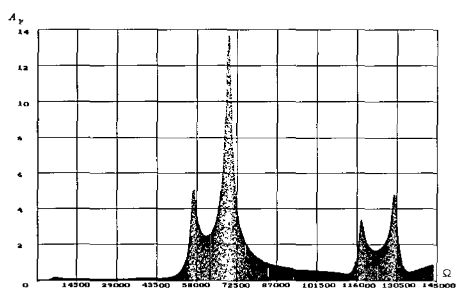

Fig. 9 Sample amplitude-frequency characteristic (experimental, m=0.407 kg, J_x=J_y=13.5·10^{-5} kg·м^2, J_z=0.956·10-5 kg·м^2, p_s=0.6MPa).

Fig. 7 represents the numerical experiment – acceleration of the spindle at zero feed pressure (i.e. atmospheric pressure) for researching the behavior of the air layer. Fig. 8 represents another numerical experiment – researching of the spindle behavior after the acceleration. The spindle has finished the acceleration (curly part of trajectory) at 129000 rpm, its rotation become stable and its center of masses tends to the stability. Fig. 8 shows that during acceleration the amplitude reaches its highest value at 72000 rpm. At that moment there is the swap of precession axes, and then amplitude drops. If spindle passes the critical zone during acceleration, it can work at 87000..116000 rpm, however the rotation error will be appx. 1..2 m. In the range of 0..43500 rpm, error will be 0.1 to 0.2 m.

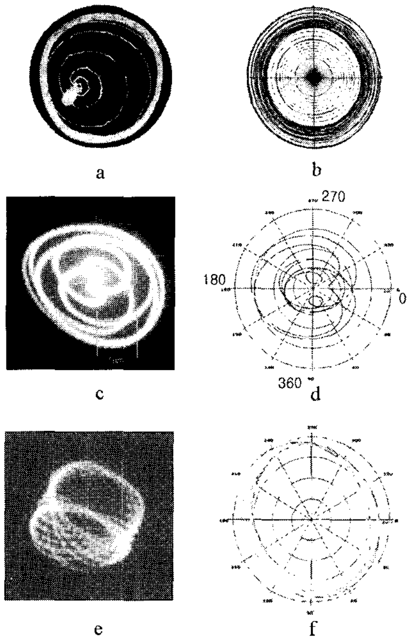

Fig. 10 Comparing various experimental spindle trajectories (a, c, e) with calculated trajectories (b, d, f)

Figs. 10a to 10f represent various comparisons between real and numeric experiments. Figs. 10a and 10b represent examples of acceleration of the spindle from 0 to 5000 rpm. Figs. 10c and 10d show the example of phenomena known as "half-speed whirl". It was described, for example, in[6]. The figs. 10e and 10f show another example of spindle behavior modeled using proposed methods.

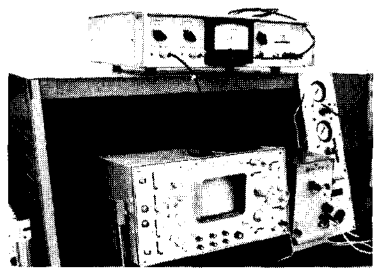

Fig. 11 The test stand for checking and measuring the spindle unit micro-displacements.

Experiments have been conducted using the special test stand. The general view of test stand is shown in fig. 8 The stand is intended for checking and measuring the spindle unit micro-displacements in the range of 0.2..60 m in static and dynamic modes, respectively. The stand sensors are capacitive, thus the direct contact between sensors and shaft are not needed. Capacitive sensors provide measurement sensitivity not less than 0.2 m. The stand power supply is DC 18V.

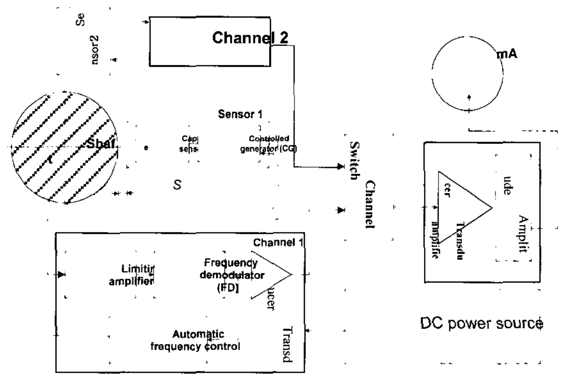

Fig. 12 The functional block diagram of the test stand for checking and measuring the spindle unit micro-displacements

The functional block diagram of the test stand is shown in fig. 12 The capacitive sensor (CS) represents the capacitor, generated by conducting surface of the target object (shaft) and sensor's working surface. The capacitor is a part of contour capacity of controlled generator (CG), therefore the distance variation between shaft surface and sensor's working surface leads to corresponding alteration of CG frequency. Then the signal goes to the frequency demodulator (FD) through the limiting amplifier (LA). The voltage on the FD output repeats the frequency law form of CG. On the next step it is amplified and through the channel switch goes to the micro-displacement measurer. The voltage from the FD output also goes to the automatic frequency control device (AFCD). AFCD makes the degenerative feedback and changes the frequency of CG, thus extending measuring range. The second measuring channel has the same scheme.

A main advantage of the capacitive measuring method is the absence of influences between the target object and the measurer, high and stable sensitivity of measuring channel, wide static and dynamic working ranges.

4. Conclusion

As conclusion, we may summarize the following results. Having the purpose of accuracy prognostication of high-speed precise machining with axial tools, we have investigated and analyzed the factors that affect the air-bearing spindle rotation accuracy. The study was based on the spindle's movement trajectory calculation. Various factors were analyzed, including parameters of tool, tooling, and the cutting forces influences transferred through the tool.

The thorough study allowed to derive several "influence coefficients", which represent displacement of the spindle center of mass under a unit force application, the angle of shaft rotation around the axes X or Y under unit force application, and the angle of shaft rotation around the axes X or Y under the unit force tor torque. Then these coefficients were used in the differential equations very awkward! which describe the coordinates and angles at the spindle rotation. The differential equation system was solved using Runge-Kutta numeric method of 4^{th} order.

The obtained results (i.e. images of spindle trajectory) were compared with experimental data results, and their coincidence allowed us to make a conclusion about the correctness of used model.

Acknowledgement

This work was supported by the Korea Science and Engineering Foundation (KOSEF) through the Machine Tool Research Center at Changwon National University.

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