International Journal of Precision Engineering and Manufacturing Vol. 6, No. 1, January 2005.

Analysis of Elasto-Plastic Dynamic Behavior of Plates Subjected to Low Velocity Impact

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ABSTRACT

In this study, a computer program is developed for analyzing the elasto-plastic dynamic behaviors of the plates subjected to line-loading by a low-velocity impactor. The equilibrium equation associated with the Hertzian contact law is formulated to evaluate the transient dynamic behavior of the impacted plates. Compared with an elastic analysis, the effects of material plasticity are considered. Consequently, in the case of elasto-plastic analysis, impulse decreases, displacements increase and contact duration time is longer than that of the elastic case for the same plate structures. And the time variation of the impacting load is not significant due to the plasticity except at the beginning of impact duration, and the induced stresses of plate are more realistic.

Key Words : Elasto-plastic dynamic behaviors, Line-loading, Low-velocity impact, Hertzian contact law, Finite element analysis

1. Introduction

Interest in impact problems is growing in many technical applications mainly due to safety requirements. Previous studies on a plate impacted by a low velocity impactor have been limited to elastic analysis[1], which is unrealistic in nature since plastic deformations are found in most cases.

In this study, elasto-plastic dynamic behaviors of a plate subjected to line-loading by a low-velocity impactor are analyzed to focus on the plasticity effects in the low velocity impact problems[2]. Assuming that the behaviors of the plate are uniform across its width, the plane strain approach is adopted for analyzing the plate. Also, assuming that the impactor is much stiffer than the plate, the impactor is treated as a rigid body.

Manuscript received: May 3, 2004;
Accepted: July 14, 2004
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The equilibrium equation associated with the Hertzian contact law is formulated to evaluate the transient dynamic behaviour of the impacted plate[1,3,4]. A plain strain finite element formulation and a central difference method are employed for the numerical calculation of space and time domains, respectively. In order to consider the effect of plasticity, the plastic flow rule associated with von Mises yield criterion and an isotropic hardening property are applied.

Compared with elastic analysis, the effects of material plasticity are discussed. In the case of elasto-plastic analysis, it is concluded that the impulse decreases, displacements increase and contact duration time is longer than that of the elastic case.

The time variation of the impacting load is not significant due to the plasticity except for the beginning of the impact, and more realistic resultant stresses are induced.

2. Problem Description

A rectangular plate with length L, width W, and thickness h impacted by a line-nose impactor of mass m, velocity v, and nose radius r is presented in Fig. 1. Two parallel edges of the plate are clamped, and the other two edges are free. Assuming that the load distribution across the width is uniform, and there is no free edge effect. The impactor is much stiffer than the plate, and its motion can be described as that of a rigid body. Also, it is assumed that the contact force between the impactor and the plate is a point force.

The two-dimensional schematic presentation is shown in Fig. 2.

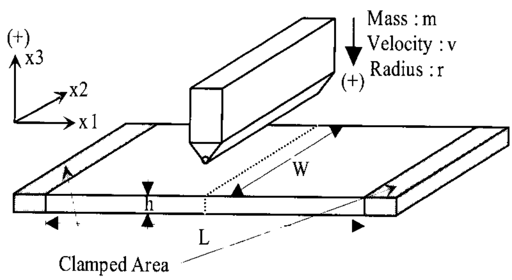


Fig. 1 Description of the problem

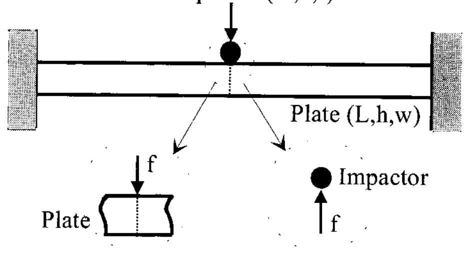
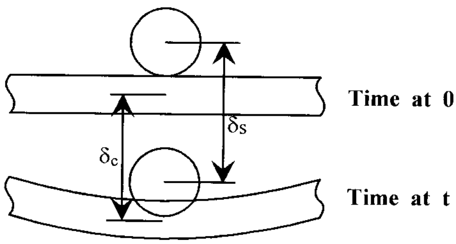


Fig. 2 2-D modeling of the impact problem



$$\alpha = \delta_S - \delta_C$$

Fig. 3 Definition of the indentation depth

3. Theory and Analysis

3.1 Governing equations

For the dynamic analysis of the plate shown in Fig. 2, the following three equations are considered. Two of them are the equations of motion of the impactor and the plate, and the other is the contact law which is used to determine the contact force between the impactor and the plate.

Since the impactor moves along a perpendicular line to the plate, the equations of motion of the impactor are written as follows:

$$m_s \ddot{w}_s + f = 0 \tag{1}$$

where f is the contact force, and m_s and $\ddot{w}_s$ are the mass and the acceleration of the impactor, respectively. In the time domain, the semi-discretized equations of motion of the plate are[5]

$$[M]\{\ddot{U}\} + [K_T]\{U\} = \{F\} \tag{2}$$

where $[M]$ and $[K_T]$ are the mass and tangent stiffness matrices, and $\{F\}$, $\{U\}$ and $\{\ddot{U}\}$ are the force, displacements, and acceleration vectors, respectively.

With negligible body forces, the contact force vector is given by:

$$\{F\} = f\{I\} \tag{3}$$

where $\{I\}$ is a vector whose components are zeros except for the components related to the contact forces. Each component denoting the contact point has the value of -1.

In order to solve Eqs. (1) and (2), the contact force between the impactor and the plate must be known. In this study, the Hertzian contact law is adopted to determine the contact force. By assuming Hertzian pressure distribution, the contact force f is related to the indentation depth α as follows[6]:

$$\alpha = f(\chi_s + \chi_p)\{1 - \ln[fr(\chi_s + \chi_p)]\} \tag{4}$$

where $\chi_s = (1-\nu_s^2)/\pi E_s$ and $\chi_p = 1/\pi E$. The values of ν_s, E_s, and E are Poisson's ratio, Young's modulus of impactor and Young's modulus of plate respectively.

The indentation depth α is the difference in displacement between the center of the impactor and the mid-plane of the plate. The first contact between the impactor and the plate occurs at time 0, and let δ_S and δ_C be the displacements of the center of the impactor and the plate at time t, respectively as shown in Fig. 3. If δ_C is larger than δ_S, the plate is not in contact with the impactor, so the contact force must be zero. Otherwise the contact force is calculated by Eq. (4). Thus,

$$\alpha = \delta_S - \delta_C \quad (5)$$

$$\alpha = f(\chi_s + \chi_p)\{1 - \ln[fr(\chi_s + \chi_p)]\} \quad (\delta_S \geq \delta_C)$$
$$0 = f \quad (\delta_S < \delta_C) \quad (6)$$

Since the contact force is coupled with displacements of the impactor and the plate, Eqs. (1), (2), and (6) must be solved simultaneously.

3.2 Explicit time integration

At time t, the semi-discretized equations of motion of the plate are

$$[M]\{\ddot{U}\}^t + [K_T]^t\{U\}^t = \{F\}^t \quad (7)$$

To solve this system of differential equations, a finite difference approximation is used in the time domain. In the above plate problem, the stiffness matrix and the force vector are the functions of the displacements, and implicit time integration methods are efficient. In this study, the central difference method is adopted for the explicit time integration. The velocity and acceleration vectors at time t are

$$\{\dot{U}\}^t = \frac{1}{2\Delta t}(\{U\}^{t+\Delta t} - \{U\}^{t-\Delta t}) \quad (8)$$

$$\{\ddot{U}\}^t = \frac{1}{\Delta t^2}(\{U\}^{t+\Delta t} - 2\{U\}^t + \{U\}^{t-\Delta t}) \quad (9)$$

By substituting Eq. (9) into Eq. (7), we have

$$\left(\frac{1}{\Delta t^2}[M]\right)\{U\}^{t+\Delta t} = \{\mathcal{F}\}^t \quad (10)$$

where $\{\mathcal{F}\}^t$ is the effective force vector defined as:

$$\{\mathcal{F}\}^t = \{F\}^t - [K_T]^t\{U\}^t + \frac{1}{\Delta t^2}[M](2\{U\}^t - \{U\}^{t-\Delta t}) \quad (11)$$

Using a lumped mass matrix which is a diagonal and nonsingular matrix, Eq. (10) can be rewritten by the following components form:

$$U_i^{t+\Delta t} = \mathcal{F}_i^t\left(\frac{\Delta t^2}{M_{ii}}\right) \quad (12)$$

In Eq. (11), the internal force term $[K_T]^t\{U\}^t$ is calculated by assembling the element nodal point forces $\{F^e\}^t$ that are equivalent to the element internal stresses as follows:

$$[K_T]^t\{U\}^t = \sum_{e=1}^{N_{el}}\{F^e\}^t \quad (13)$$

where N_{el} is the number of elements.

The element nodal point forces are expressed as follows:

$$\{F^e\}^t = \int_{V^m}[B]^T\{\sigma^e\}^t dV \quad (14)$$

where $[B]$ is the strain-displacement matrix and $\{\sigma^e\}^t$ is the element stress vector.

With the central difference method there is no iteration in solving non-linear equations and there is no assembly of the stiffness matrix. Also, by using the lumped mass matrix there is no need to solve the system of equations.

The explicit integration method is conditionally stable, and the stability condition to the time step Δt is given by:

$$\Delta t \leq \frac{2}{\omega_{\max}} \quad (15)$$

where $\omega_{\max}$ is the maximum circular frequency of the element.

3.3 Incremental constitutive relations

Incremental constitutive relations for isotropic hardening materials can be written as:[7]

Elastic Region : $\{d\sigma\} = [D]\{d\varepsilon\}$

Plastic Region :

loading stage $\{d\sigma\} = [D_{ep}]\{d\varepsilon\}$ (16)

unloading stage $\{d\sigma\} = [D]\{d\varepsilon\}$

3.4 Calculation of the contact force

At time $t+\Delta t$, the contact force is governed by:

$$\delta_S \geq \delta_C :$$
$$\alpha^{t+\Delta t} = f^{t+\Delta t}(\chi_s+\chi_p)\{1-\ln[f^{t+\Delta t} r(\chi_s+\chi_p)]\}$$
$$\delta_S < \delta_C : \qquad 0 = f^{t+\Delta t} \tag{17}$$

In Eq. (17) χ_s, χ_p, and r are time-invariant terms, but α varies with time. At time $t+\Delta t$, α is expressed as Eq. (5) by $\alpha^{t+\Delta t} = \delta_S^{t+\Delta t} - \delta_C^{t+\Delta t}$.

The displacement $\delta_C^{t+\Delta t}$ of the center of the plate at time $t+\Delta t$ is calculated from the displacement vector $\{U\}^{t+\Delta t}$, which is known by solving the equations of motion of the plate at time t as follows:

$$\delta_C^{t+\Delta t} = -U^{t+\Delta t}\Big|_{at\ \frac{L}{2},\frac{h}{2}} \tag{18}$$

$\delta_S^{t+\Delta t}$ is the displacement of the impactor. Assuming linear variation of the contact force from time t and $t+\Delta t$, the velocity and the displacement of the impactor at time $t+\Delta t$ are[1]

$$v^{t+\Delta t} = v^t - \frac{(f^t + f^{t+\Delta t})}{2m}\Delta t \tag{19}$$

$$\delta_S^{t+\Delta t} = \frac{1}{m}\int_0^{t+\Delta t}\int_0^{t+\Delta t} f\,dtdt$$
$$= \delta_S^t + v^t\Delta t - \frac{1}{6m}(2f^t + f^{t+\Delta t})\,\Delta t^2 \tag{20}$$

where m and v^t are the mass and the velocity at time t of the impactor.

By combining Eqs. (17) - (20), the contact force is expressed as:

$$\delta_S \geq \delta_C :$$
$$0 = f^{t+\Delta t}(\chi_s+\chi_p)\{1-\ln[f^{t+\Delta t} r(\chi_s+\chi_p)]\}$$
$$-\delta_S^t - v^t\Delta t + \frac{1}{6m}(2f^t + f^{t+\Delta t})\,\Delta t^2 + \delta_C^{t+\Delta t}$$
$$\delta_S < \delta_C ; \qquad 0 = f^{t+\Delta t} \tag{21}$$

3.5 Solution procedure

Step 1

Initialize the vectors which denote the displacement and the stress of the plate, the velocity and the displacement of the impactor, and the contact force.

Step 2

Solve the equtions of the motion of the plate at time t, and calculate the displacement of the plate at time $t+\Delta t$.

Step 3

Compute the contact force at time $t+\Delta t$ from Eq. (21). Compute the velocity, the displacement, and the stresses of the impactor at time $t+\Delta t$ from Eqs. (19) and (20).

Step 4

Print the results at time $t+\Delta t$. For each time step, repeat from Step 2 to Step 4.

4. Results and Discussion

The finite element model for the impacted plate is shown in Fig. 4. Due to the symmetry of the boundary and loading conditions only half of the plate is to be analyzed.

Before these numerical simulations, a simple verification is carried out to confirm the validity of the present approach. The same plate which is loaded by a rectangular pulse of 86.5 kN is analyzed, and the resultant displacements and stresses at the center point are compared in Fig. 5 and 6. These comparisons show good agreement with the results of a commercial FE program ANSYS.

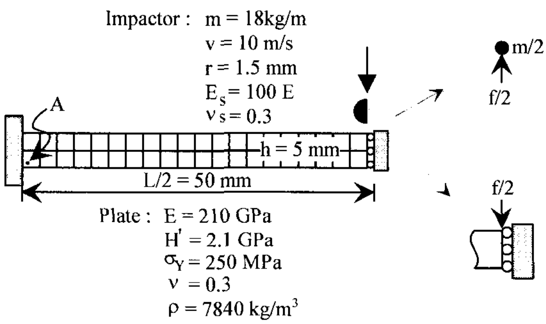


Fig. 4 Numerical model for impacted plate

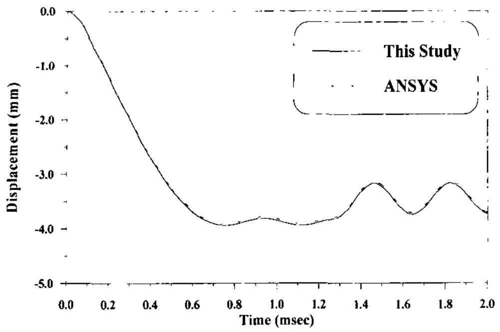


Fig. 5 Comparison of displacements with ANSYS

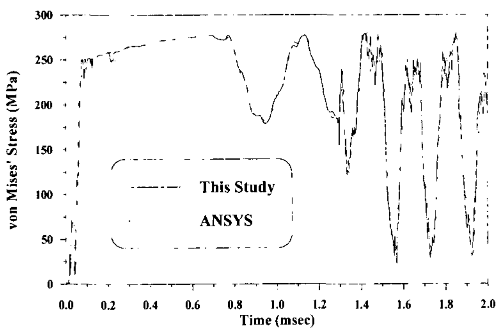


Fig. 6 Comparison of Mises stresses with ANSYS

Fig. 7 shows the time history of the contact force for elastic and elasto-plastic cases. It is noted that there exist more fluctuations, peak values for elastic contact force. In the elasto-plastic analysis, the frequent fluctuations vanish, maximum value decreased, and contact duration time increased, which represent more realistic results.

Fig. 8 shows the time history of the displacement of the center of the plate. In the elastic analysis, it oscillates, but in the elasto-plastic behavior of the plate, the permanent strain appears.

Fig. 9 shows the time history of the equivalent stress at the point A. In the elastic analysis, the maximum value is almost 10 times as large as that of the yield stress. It is evident that the material is in pseudo-elastic state, and it is unreal. The elasto-plastic analysis shows the strain hardening behavior of the material after yielding, and it is more realistic.

Numerical values of the plasticity effects on this problem are summarized in Table 1. To investigate the plasticity effect in the small deformation range, another numerical calculations are executed by using the impactor which has the mass of 10 *kg* and various velocities. The velocity of the impactor is limited so that the maximum displacement does not exceed the thickness of the plate. In Fig. 10, the impulses for the elasto-plastic analysis and the elastic case are compared. Plasticity effect is found in most range of small deformation.

The impactor is treated as a rigid-body, and must satisfy the impulse-momentum balance. Table 2 shows impulse-momentum balance of the impactor for various velocities.

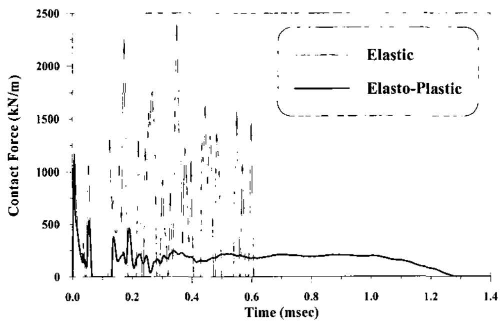


Fig. 7 Time history of contact force

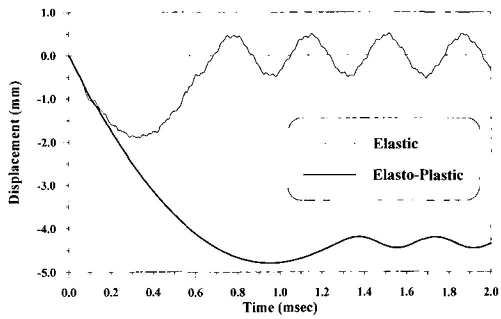


Fig. 8 Displacements of the center of the plate

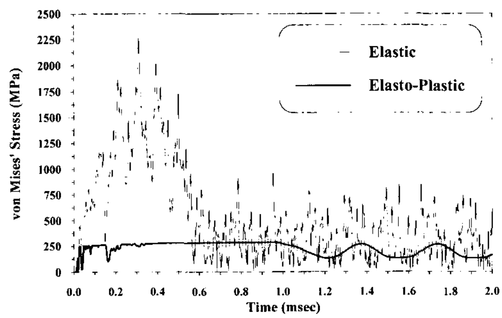


Fig. 9 Von Mises' stresses at the point A

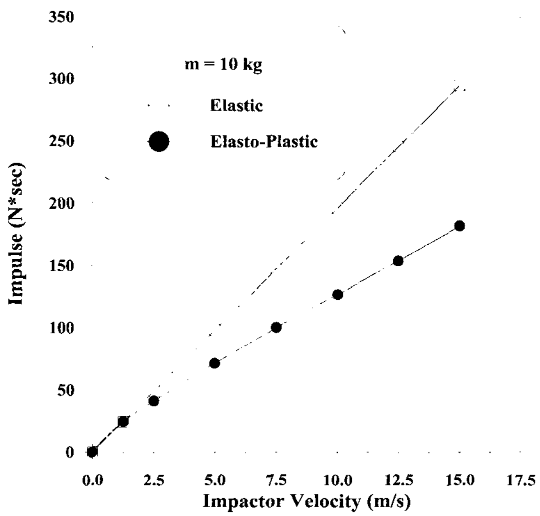


Fig. 10 Impulse comparison

Table 1 Plasticity effects on the impact problem

	Elastic	Elasto -Plastic	Ratio (EP/E)
Impulse (N*sec)	351.8	221.2	0.629
Max. Contact Load (kN)	2394	1106	0.462
Max. Displ. of the Plate (mm)	-1.903	-4.802	2.523
Contact Duration (msec)	0.605	1.279	2.114
Final Velocity of the Impactor (m/s)	-9.54	-2.29	0.240
Max. Effective Stress at the Point A (MPa)	2260	285.4	0.126

Table 2 Momentum and impulse for various impactor velocities (m = 10 *kg*)

Initial Velocity (m/s)	Initial Mom. (kg*m/s)	Elastic		Elasto-Plastic	
		Final Mom. (kg*m/s)	Impulse (N*sec)	Final Mom. (kg*m/s)	Impulse (N*sec)
1.25	12.5	-11.96	24.46	-11.96	24.46
2.5	25	-23.95	48.95	-16.15	41.15
5	50	-47.97	97.97	-21.23	71.23
7.5	75	-72.02	147.0	-24.72	99.72
10	100	-96.08	196.1	-26.65	126.7
12.5	125	-120.2	245.2	-28.45	153.4
15	150	-144.2	294.2	-31.45	181.5

5. Conclusions

In this study, elasto-plastic dynamic behaviors of a plate subjected to line-loading by a low-velocity impactor are analyzed. Compared with elastic analysis, the impacted plate with plastic behavior revealed as follows:

- Impulse decreases.
- Displacement increases.
- Contact duration time increases.
- Stress is more realistic.

From numerical results for various impactor velocities, it is concluded that material plasticity must be considered in low-velocity impact problems. The impact analysis approach in this study may be extended and applicable to other types of plate impact problems.

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