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Modeling Phased Array Ultrasonic Testing of a Flat-Bottom Hole in a Single Medium

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Abstract The expanded multi-Gaussian beam model has recently been developed that can calculate the radiation beam field from a single, rectangular transducer with great computational efficiency. In this study, this model is adopted to calculate the radiation beam field for a phased array transducer with various time delays to achieve steering and/or focusing. The calculation beam fields are compared to those obtained by well known Rayleigh-Sommerfeld integral that provides the exact solution in order to explore the validity of the expanded multi-Gaussian beam model. And then, this study proposes a complete ultrasonic measurement model including the expanded beam model, far-field scattering model and system efficiency. Using the proposed model, phased array ultrasonic testing signals for a flat-bottomed hole with/without focusing were performed.

Keywords: radiation beam, phased array ultrasonic transducer, modeling, expanded multi-Gaussian beam

1. Introduction

The ultrasonic array transducers currently used in most field applications consist of many small piezoelectric elements with rectangular cross-sections. Therefore, for the prediction of radiation beam fields from an ultrasonic array transducer it is necessary to have the beam models that have a capability of describing rectangular cross-sections carefully. Obviously, the Rayleigh-Sommerfeld integral (RSI) model (Schmerr, 1998) is able to do such a task. In addition, the RSI model prediction is considered as an "exact" solution so that it has been widely adopted in the efforts of modeling various ultrasonic inspection systems up to now (Song et al., 2002). However, it is computationally expensive since it involves a two-dimensional integral over the transducer surface.

Recently, Ding et al. have proposed a beam model that can predict the radiation beam field from a rectangular transducer by the superposition of two-dimensional Gaussian beams (Ding et al., 2003). In fact, this beam model is an extension of the multi-Gaussian beam (MGB) model that can be constructed by adopting the coefficients proposed by Wen and Breazeale (Wen et al., 1988) to describe the beam field from transducers with circular cross-sections with an outstanding computational efficiency. Therefore, this is often called the expanded multi-Gaussian beam (EMGB) model.

To develop a complete ultrasonic measurement model for a phased array ultrasonic testing, four main components which are the ultrasonic beam model for a phased array transducer, time delays for focusing and/or steering, a far-field scattering model for a flaw, and system efficiency factor for

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the characteristics of the system.

For the beam model, we adopt the expanded multi-Gaussian beam model which allows the rapid evaluation of the wave field incident on the flaw. For the far-field scattering model, the Kirchhoff approximation under the small flaw assumption is improved. To calculate the proper time delay for focusing and/or steering of ultrasonic beams, we use the geometrical ray path method. And, the system efficiency factor is determined by deconvolution of an experimental front surface reflection signal by a reference reflector model. In this study, we have adopted the artificial signal instead of experimental signal for the calculation of system efficiency factor. Combining these four components, we develop a complete ultrasonic measurement model for a phased ultrasonic testing that can predict flaw signals from a flat-bottomed hole in single medium with computational efficiency.

In this paper, we described the developed measurement model including four major components. And then capability of the EMGB model to predict the signal of phased array ultrasonic testing from a linear array transducer in single medium was shown. Specifically, the radiation beam fields from a linear array transducer into a single medium are calculated with time delays for focusing and/or steering based on the EMGB model. And for the prediction of a phased array ultrasonic testing signal we have calculated beam forming by the superposition of flaw signals produced by individual elements with proper time delays. Predicted phased array ultrasonic flaw signals scattered from a flat-bottomed hole in single medium using the developed measurement model were also presented in this paper.

2. Measurement Model for a Phased Array Ultrasonic Testing

The contribution of the components of the ultrasonic measurement model (such as the beam radiation, scattering from a flaw, and system efficiency factor) can be considered as a simple multiplication in the frequency domain under the linear time invariant (LTI) system (Schmerr, 1998). Taking this LTI approach is, in fact, equivalent to combining the paraxial beam approximation and the small flaw assumption.

Fig. 1 shows the procedure of a measurement model for a phased array ultrasonic testing interrogating a flaw part immersed in water. Thus, for the calculation of the prediction of signal produced by an array transducer, it is necessary to calculate the radiation beam field, time delay for radiation and reception process, scattering model, and system efficiency factor. In this section, the four components are described by steps involved in this calculation.

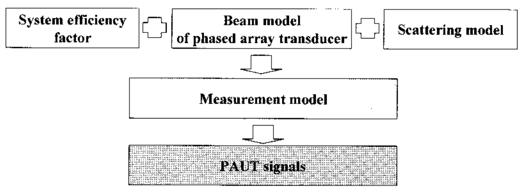


Fig. 1 Schematic diagram for modeling of phased array ultrasonic testing

2.1. Beam Model

Radiation beam field from a phased array ultrasonic transducer can be predicted by the superposition of radiation beams produced by individual elements with proper time delays applied for focusing and/or steering. Therefore, for the calculation of the beam field produced by an array transducer it is necessary to calculate the radiation beam field from a single rectangular element and the time delay to be applied to the element. Then, they are combined together to produce the radiation beam from the array transducer.

In a single fluid medium, the radiation beam field from the n-th rectangular piston source (in a linear array transducer with N elements) can be calculated by use of the EMGB model given by

Eq. (1). The radiated sound field, $p_n(\mathbf{x},\omega)$, produced by the same n-th element can be calculated by the superposition of two-dimensional Gaussian beams (Ding et al., 2003).

$$p_n(\mathbf{x},\omega)=v_0(\omega)\left[\sum_{m=1}^{M}A_m G_1(x,z,\omega)\right]\cdot\left[\sum_{m=1}^{M}A_m G_1(y,z,\omega)\right] \quad (1)$$

where the one-dimensional Gaussian beam, G_1, is defined by

$$G_l(x,z,\omega)=\frac{1}{\sqrt{1+i\frac{2z}{ka^2}B_m}}\exp\left(-\frac{B_m\frac{x^2}{a^2}}{1+i\frac{2z}{ka^2}B_m}\right)$$
$$G_l(y,z,\omega)=\frac{1}{\sqrt{1+i\frac{2z}{ka^2}B_m}}\exp\left(-\frac{B_m\frac{y^2}{b^2}}{1+i\frac{2z}{ka^2}B_m}\right) \quad (2)$$

where a and b are the half-major and half-minor axes of the transducer, λ is the wavelength and A_m and B_m are complex constants found to optimize the circular piston source (Wen et al., 1988).

Once we have a time delay for each transducer element, we can calculate the radiation beam fields at the pint $\mathbf{x}$ in the fluid medium by using Eq. (3) with or without focusing and/or steering, as generated by the linear array transducer (with N elements).

$$p(\mathbf{x},\omega)=\sum_{n=0}^{N-1}p_n(\mathbf{x},\omega)\,\exp(i\omega t_n) \quad (3)$$

where time delays, t_n, can be written by Eq. (4)

2.2. Time Delay for Focusing and Steering

One of great advantage of phased ultrasonic testing systems is focusing and/or steering the radiated beam by controlling the applied time delay of each transducer element. However, to achieve desired focusing and/or steering, methods for calculation of time delay are needed. To get proper time delays for focusing and/or steering, geometrical based ray acoustic method (Wooh et al., 1999) is used widely. Thus, in this study, we adopted the ray acoustic method.

Fig. 2 schematically shows the time delay configuration for the phased array transducer. In the ray acoustic method, time delay to be applied to the n-th element in a linear array transducer (with N elements) can be determined by Eq. (4).

$$t_n=\frac{F}{c}\left\{\left[1-\left(\frac{\overline{N}d}{F}\right)^2+\frac{2\overline{N}d}{F}\sin\theta_s\right]^{1/2}-\left[1-\left(\frac{(n-\overline{N})d}{F}\right)^2+\frac{2(n-\overline{N})d}{F}\sin\theta_s\right]^{1/2}\right\} \quad (4)$$

where F is the focal length from the center of the array, d is the center-to-center spacing between elements, θ_S is the steering angle from the center of the array transducer, $\overline{N}=(N-1)/2$, and is the wave speed.

Substituting Eq. (4) into Eq. (3), we can predict ultrasonic beam field radiated from a linear phased array transducer with/without steering and/or focusing.

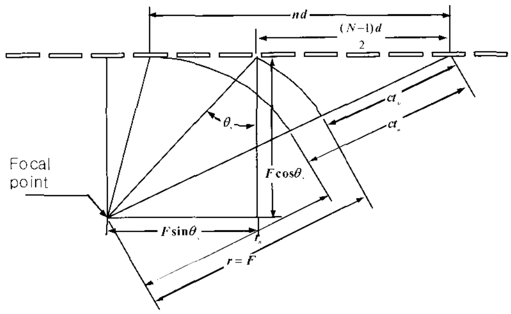


Fig. 2 The Geometry used for deriving the focusing formula of a linear phased array transducer

2.3. Far Field Scattering Amplitude

When ultrasound beam hits a flaw, scattered waves from the flaw are generated as shown in Fig. 3. The distribution of scattered waves, of course, depends strongly on the geometric and material properties of the flaw. If some simplifying assumptions are adopted, the scattering solution can be obtained easily. One of the assumptions typically selected for this purpose is that the wave field incident on a flaw is considered as a plane wave locally around the flaw. This assumption is valid for paraxial beams as in the EMGB model. Under this assumption, solving a scattering problem is simplified to determine the far-field scattering amplitude of the flaw for an incident plane wave of unit amplitude. Thus, in this study, we adopt two simplifying assumption. First, the flat-bottom hole is considered as a traction-free penny-shaped crack embedded in an isotropic, elastic solid. Second, the high frequency assumption is adopted in the calculation of the far-field scattering amplitude. Under the Kirchhoff approximation, the phased array ultrasonic pitch-catch signals from small, isolated flaws such as a planar circular crack can also be calculated by using Eq. (5).

$$A_n^{P;P}(e_i^P;e_s^P) = -\frac{ik_1}{4\pi}C^{P;P}\int_S \exp[i(k_\beta e_i^P - k_1 e_S^P)\cdot x_s]dS(x_s) \quad (5)$$

where e_i^s and e_s^s are the unit vectors along the propagation directions of the incident and scattered beams to/from the point, x_S, on the flat bottom hole surface.

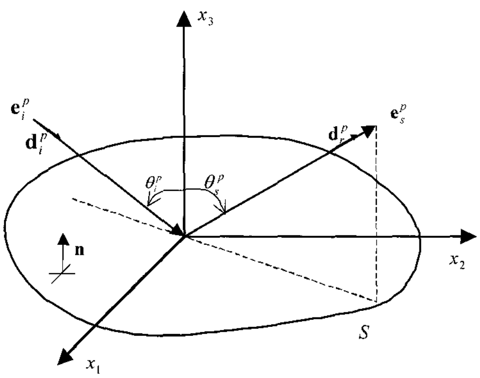


Fig. 3 The Geometry of crack scattering using Kirchoff approximation

2.4. System Efficiency factor

Fig. 4 shows a schematic diagram of the reference experimental setup proposed in the present work in order to determine the system efficiency factor. In this setup, the radiated beam from a phased array ultrasonic transducer propagates into the water as the longitudinal wave, and reflects at surface of steel specimen. Reaching the surface of steel, the beam is reflected back to the phased array ultrasonic transducer. Since the ultrasonic beam radiated from multi elements is represented by the supposition of radiated beam of one element, the beam radiated from one element is considered in this case.

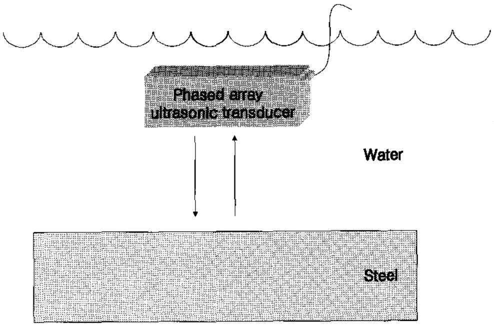
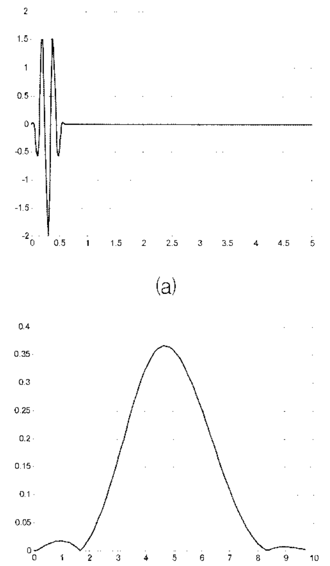


Fig. 4 An ultrasonic immersion testing setup for the determination of the system efficiency factor of a phased array transducer

The system efficiency factor is determined by the deconvolution of an experimental reflection signal captured from the surface of the steel specimen by the reference reflector model as:

$$\beta(\omega) = \frac{V_0(\omega)}{V_R(\omega)}W(\omega) \quad (6)$$

where $V_0(\omega)$ is the measured voltage by the experiments, and $W(\omega)$ is the Wiener filter adopted for the desensitization of the deconvolution to noise. In this study, we have adopted the artificial signal by Eq. (7) for $V_0(\omega)$ and this signal at (a) time domain and (b) frequency domain is shown in Fig. 5.



(a)

(b)

Fig. 5 (a) An artificial reference reflection signal (5 MHz center frequency), and (b) the system efficiency factor determined from the signal

$$V(t)=\begin{cases}\left[1-\cos\left(\frac{2\pi f}{3}t\right)\right] & for \quad 0\le t\ge\frac{3.0}{f} \\ 0 & ohterwise\end{cases} \qquad (7)$$

where $V_R(\omega)$ by the theoretical reference reflector signal can be written by Eq. (8)

$$V_R(\omega)=\frac{1}{S}\int_S v_R(\omega,\mathbf{x})dS \qquad (8)$$

where $v_R(\omega,x)$ can be found by Eq. (9)

$$v_R(\omega,\mathbf{x})=\sum_{n=1}^{N}\mathbf{d}^p R_{12}^{p;p} A_m \frac{\sqrt{\mathbf{G}_1^p(0)_x}}{\sqrt{\mathbf{G}_1^p(z_1)_x}}\exp\left[\frac{ik_1 x_1^2/(q(z_1)_x)}{2}\right] \bullet A_m \frac{\sqrt{\mathbf{G}_1^p(0)_y}}{\sqrt{\mathbf{G}_1^p(z_1)_y}}\exp\left[\frac{ik_1 y_1^2/(q(z_1)_y)}{2}\right]\exp(2ik_1 z_1) \qquad (9)$$

where z_1 is the propagating distances in the water, $\mathbf{d}^p$ is the polarization vector for the transmitted wave of type p and $R_{12}^{p;p}$ is the reflection coefficient of plane wave from the surface of steel specimen. And G-terms can be defined by Eq. (10)

$$\begin{aligned}\mathbf{G}_1^p(0)_x &= \frac{ik_1 w_x^2}{2B_n}\\ \mathbf{G}_1^p(z_1)_x &= z_1-\frac{ik_1 w_x^2}{2B_n}\\ \mathbf{G}_1^p(0)_y &= \frac{ik_1 w_y^2}{2B_n}\\ \mathbf{G}_1^p(z_1)_y &= z_1-\frac{ik_1 w_y^2}{2B_n}\end{aligned} \qquad (10)$$

Fig. 5 (a) shows a front surface reflection signal (captured with normal incidence) from which the system efficiency factor was determined by Eq. (6) as shown in Fig. 5 (b). This system efficiency factor was applied throughout the model predictions of waveforms made in the present study.

3. Ultrasonic Measurement model for a phased array testing system

The average velocity received by the n^{th} element, which is due to the wave generated from the mth element can be written as

$$V_{nm}(\omega)=\beta(\omega)\exp(ik_1 z_1^{nf})\exp(ik_1 z_2^{fm})C_{nf}(\omega)C_{fm}(\omega)A_{nm}(\omega)\left[\frac{2\pi}{-ik_2 S}\right] \qquad (11)$$

where $C_{nf}(\omega)$ and $C_{fm}(\omega)$ are the diffraction coefficient for nth and m^{th} element, respectively, that can be obtained by Eq. (1) and S is a face of single element of linear array transducer. The far-field scattering amplitude, $A_{nm}(\omega)$, is given by Eq. (5) based on the Kirchhoff approximation. Substituting Eq. (11) into Eq. (3) and modifying pressure term to velocity term, we have a complete measurement model for a linear phased array ultrasonic testing as:

$$\begin{aligned}V(\omega) &= \sum_{m=0}^{N-1} V_{Nm}(\omega)\exp(-i\omega t_m)\\ &= \sum_{m=0}^{N-1}\sum_{n=0}^{N-1} V_{nm}(\omega)\exp(-i\omega t_n)\exp(-i\omega t_m)\end{aligned} \qquad (12)$$

where time delay, t_m, is given by Eq. (4).

4. Ultrasonic Beam Fields in a Single Medium

The configuration of an array transducer used in this simulation study is shown in Fig. 6. In this study, we have chosen a linear phased array transducer that has the center frequency of 5 MHz, the number of elements of 32, the element width of 0.8 mm, the element height of 10 mm, and the inter-element spacing of 0.2 mm.

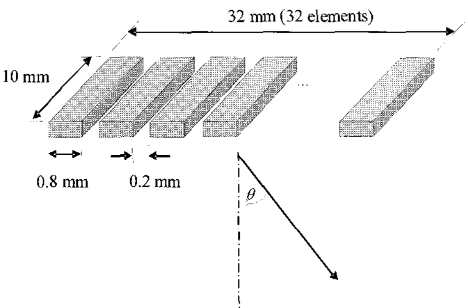
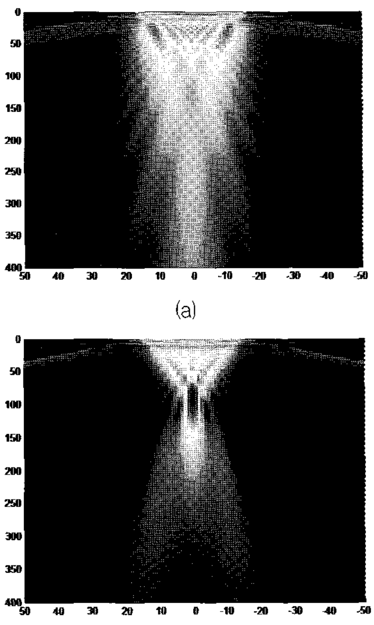


Fig. 6 Geometry of a linear phased array transducer



(a)

(b)

Fig. 7 Calculated radiation beam fields in the steel specimen (a) without focusing and (b) with focusing at 107.5 mm by using the expanded multi-Gaussian beam model

Fig. 7 (a) and (b) show the results of calculated radiation beam fields produced in the steel specimen by use of the EMGB model. Fig. 7 (a) shows radiation beam field without focusing in steel specimen. Fig. 7 (b) shows results when the beam was focused at 107.5 mm (with the steering angle of 0°).

5. Prediction of Phased Array Ultrasonic Signals

Fig. 8 represents a phased array ultrasonic testing setup in a single medium for the ultrasonic testing simulation. Fig. 9 shows predicted time domain waveforms scattered from a flat bottom hole with the radius of 1 mm in water by using Eq. (12) where receiving element is the 8th. Fig. 9 (a), (b) and (c) show signals received by the 16th element radiated from 2nd, 8th, and 12th, respectively.

$$\begin{aligned} V(\omega) &= \sum_{m=0}^{N-1} V_{Nm}(\omega)\exp(-i\omega t_m) \\ &= \sum_{m=0}^{N-1}\sum_{n=0}^{N-1} V_{nm}(\omega)\exp(-i\omega t_n)\exp(-i\omega t_m) \end{aligned} \tag{13}$$

Fig. 10 shows the predicted ultrasonic flaw signals for a flat-bottomed hole with/without focusing. Fig. 10 (a) shows an ultrasonic phased array signal from a the flat-bottomed hole without focusing. And Fig. 10 (b) shows an flaw signal with focusing at 107.5 mm where lies the flat-bottomed hole surface. As shown in Fig. 8, model predicted signal with focusing is a greater than the signal without focusing as expected.

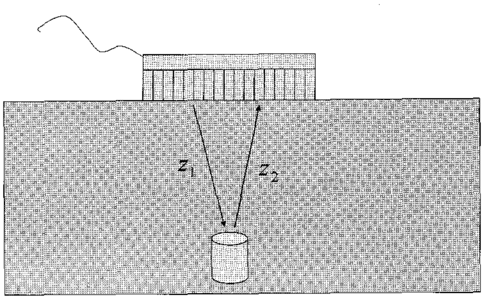


Fig. 8 A phased array ultrasonic testing setup in a single medium for the ultrasonic testing simulation

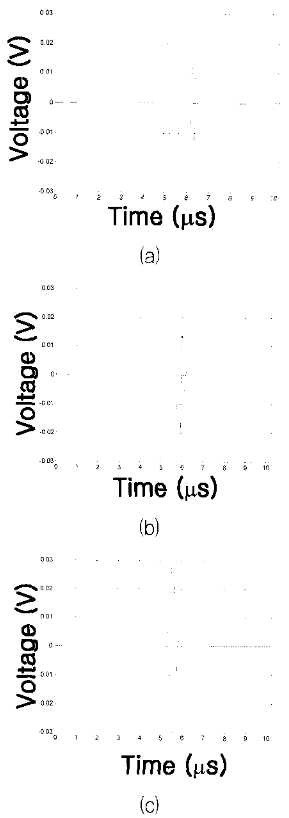


Fig. 9 Signals received by the 16th element in case to the individual element. Firing elements are (a) 2nd, (b) 8th, and (d) 12th element

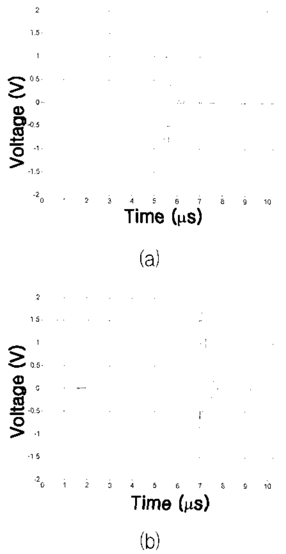


Fig. 10 Received signal by the ultrasonic phased array transducer (a) without focusing, (b) with focusing

6. Summary

In this study, we have proposed a complete measurement model for a phased array ultrasonic testing system to simulate phased array ultrasonic signals from the flat-bottomed hole. To develop the measurement model, we have used the EMGB model for a rectangular transducer, the Kirchhoff approximation for far-field scattering amplitude, the ray acoustic method for time delay and reference model for a phased array transducer to get system efficiency factor. Using the EMGB model, the radiated beam fields from a linear phased array transducer in a single medium were calculated with focusing and/or steering. And, we have performed the prediction of ultrasonic phased array signals scattered from a flat-bottomed hole using the proposed measurement model. The proposed model can be a valuable tool for the prediction of ultrasonic beam fields and flaw signal for a phased array ultrasonic testing system.

Acknowledgements

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