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On the Implementation of Fuzzy Arithmetic for Prediction Model Equation of Corrosion Initiation

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ABSTRACT

For critical structures and application, where a given reliability must be met, it is necessary to account for uncertainties and variability in material properties, structural parameters affecting the corrosion process, in addition to the statistical and decision uncertainties. This paper presents an approach to the fuzzy arithmetic based modeling of the chloride-induced corrosion of reinforcement in concrete structures that takes into account the uncertainties in the physical models of chloride penetration into concrete and corrosion of steel reinforcement, as well as the uncertainties in the governing parameters, including concrete diffusivity, concrete cover depth, surface chloride concentration and critical chloride level for corrosion initiation. The parameters of the models are regarded as fuzzy numbers with proper membership function adapted to statistical data of the governing parameters and the fuzziness of the corrosion time is determined by the fuzzy arithmetic of interval arithmetic and extension principle

Keywords **:** ***corrosion, service life, prediction model, fuzzy number, stochastic model***

1. Introduction

Durability of reinforced concrete structures is the ability of a structure to withstand various forms of attack from the environment, which are susceptible to a variety of deterioration mechanisms including chloride ingress, carbonation, alkali-silica reaction, freeze-thaw action, and thaumasite-sulphate reaction. The last three are main forms of attack on the concrete itself and the first two are main forms of attack on the reinforcement. Deterioration of reinforced concrete structures can, that is, be classified into concrete deterioration and reinforcement deterioration.

In the case of attack on the reinforcement embedded in concrete, on the contrary to the case of concrete degradation, the corrosion of steel reinforcement in concrete structures leads posi steel and concrete, and reduction in strength and ductility. As a result, the safety, serviceability and durability structures are reduced, while their life cycle costs are increased. Normally, concrete protects steel reinforcement from corrosion by forming a passive film around the steel due to the high alkalinity of the concrete pore solution. When chloride ions from the origin such as deicing salts or seawater penetrate into the concrete and reach the steel surface, they disrupt the passive film and initiate corrosion. The corrosion of steel reinforcement will start immediately after the chloride content of concrete near the embedded steel reaches a critical level, which defines the resistance of steel to corrosion. Consequently, the onset of corrosion is governed by the surface chloride concentration, concrete diffusivity, concrete cover depth of the steel, corrosion critical level, as well as moisture level in terms of the pore solution, and the availability of oxygen.[2)]

Herein, the prediction of the onset of corrosion is very important in order to reduce life cycle costs and enlarge the

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service life of RC structure as earlier stated. Thus, a reliable prediction model of chloride penetration into reinforced concrete structures is critical for predicting the time to onset of corrosion of steel reinforcement.[5] Mathematical models of chloride ingress currently being developed are primarily based on chloride diffusion, which can be used as starting points in the development of service life prediction tools and performance-based specifications.

Even if chloride ingress into concrete is complex, models are constructed around Fick's second law of diffusion and the error function solution by Crank. Fick's second law of diffusion concerns the rate of change of concentration with respect to time as follows:
tively to concrete fracture, loss of bond between

$$\frac{\delta C}{\delta t} = D\frac{\partial^2 C}{\partial x^2} \tag{1}$$

with boundary condition of $C_x = 0$ at $t = 0$ and $0 < x < \infty$, $C_x = C_s$ at $t = 0$ and $0 < x < \infty$

Crank's solution of Fick's second law of diffusion can be stated as follows, using an apparent diffusion coefficient:

$$\frac{C_x}{C_s} = 1 - erf\left[\frac{x}{2\sqrt{D_{ca}t}}\right] \tag{2}$$

where C_x = chloride concentration at depth x at time t
C_s = surface chloride concentration, kg/m^3
D_{ca} = apparent diffusion coefficient, cm^3/year
t = time of exposure, year
erf = error function

It can be noted from Eq. (2) that the determination of time to corrosion initiation requires the values of D_{ca}, C_s, C_x and x. These variables of the deterministic prediction model assume the uncertainty associated with governing parameters such as exposure condition, type and quality of concrete, and quality of construction. In recent years, there has been much study about processing the uncertainty of the variables by using Monte Carlo (MC) simulation, by which the parameters of the models are modeled as random variables and the distribution of the corrosion time and probability of corrosion are determined. MC simulation proves to be well applicable but it is very difficult to be manually calculated because it generates the relevant random values of a number of variables.[1]

In this paper, the uncertainties of the deterministic model associated with various conditions are treated by fuzzy arithmetic which is a successful tool to solve engineering problems with uncertain parameters. The shape of variable derived from measured data is modeled as the standard form of triangular fuzzy numbers(TFN) which are just a rough approximation of the really existing uncertainty.[3]

The prediction capability of fuzzy variable treated-prediction model is illustrated in a case study of a reinforced concrete building structure in coastal environment.

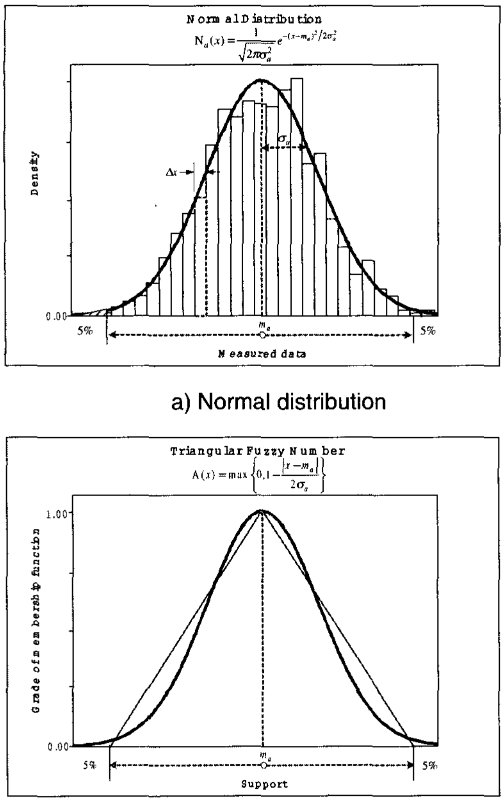
2. Uncertainty in prediction model and its process

To achieve reliable results for the numerical solution of the deterministic prediction problem, exact values for the parameters for the problem equations should be available. In practice, however, exact values can not be provided. The model parameters exhibit variability, e.g. due to both irregularities in manufacturing when considering the physical properties of a material and uncertainties in measuring when considering the environmental condition.

Theses uncertainties can be grouped into aleatoric uncertainty and epistemic uncertainty. The aleatoric uncertainty arises from the physical or inherent uncertainty identified with the random nature of the basic parameters that govern the chloride penetration and corrosion mechanisms. This uncertainty is associated with variability of the concrete cover depth, uncertainty of the chloride concentration at the surface, and uncertainty of the chloride diffusion coefficient.

The epistemic uncertainty arises from the uncertainty in the models for chloride transport and corrosion initiation. The model uncertainty results from the use of a simplified physical model of the actual phenomenon, such as assumption of chloride transport mechanism governed by diffusion use of simplified models of the diffusion coefficient and driving chloride concentration and use of simplified chloride critical level to define the corrosion resistance of steel reinforcement. The epistemic uncertainty also arises from statistical uncertainty due to estimating statistical representative value of an average from a limited sample size. Thus, it is clear that a deterministic prediction model can be quite improper in predicting the actual structural response to environmental condition.[1]

To solve this limitation, the application of fuzzy set theory proves to be a practical approach. More specifically, the uncertainties in the model parameters can be taken into account by representing the effects of scatter by fuzzy numbers with their shape derived from statistical data. The elementary mathematical operations like addition, multiplication, etc. must then be carried out using generalized versions of the operations that ensure the handling of fuzzy numbers. By this technique, one can demonstrate how initially assumed uncertainties are processed through the cal-



a) Normal distribution

b) Adaptation of normal distribution to TFN

Fig.1 Normal distribution of any parameter and its adaptation to membership function of triangular fuzzy number(TFN)

culation procedure leading finally to fuzzy results that reflect the reliability of the problem solution. Additionally, the fuzzy results allow the computation of a crisp value as the most likely result for the problem which in general differs from the result achieved by an initially non-fuzzy approach using only crisp parameters.

3. Implementation of fuzzy numbers

3.1 Definition of fuzzy number and fuzzy arithmetic

To qualify as a fuzzy number, a fuzzy set A on real numbers must be normal and convex. The fuzzy set must be normal, since the concept of a set of "real numbers close to a given real number R" is fully satisfied by R itself; hence the membership grade of R in any fuzzy set that attempts to capture a fuzzy number must be 1. The bounded support of a fuzzy number and all its α-cuts for $\alpha \neq 0$ must be closed intervals to allow definition of arithmetic operations on fuzzy numbers in terms of standard arithmetic operations on closed intervals. Since α-cuts of any fuzzy number are required to be closed intervals for all $\alpha \in$ [0, 1], every fuzzy number is a convex fuzzy set.

A fuzzy number is represented as an ordered set of confidence intervals, each of them providing the related numerical value at a given presumption level $\alpha \in$ [0, 1]. These confidence intervals should comply with the relation

$$\alpha_1 > \alpha_2 \Rightarrow {}^{\alpha_1}A \subset {}^{\alpha_2}A$$

where $\alpha_1 > \alpha_2 \in$ [0, 1] and ${}^{\alpha_1}A$, ${}^{\alpha_2}A$ are the confidence intervals at presumption levels α_1 and α_2 respectively.

The four basic arithmetic operations on fuzzy numbers (addition, subtraction, multiplication, and division) can be described as sequences of operations among confidence intervals. In particular, let A and B be fuzzy numbers and let $\otimes$ be a generic arithmetic operator.

The fuzzy number $A \otimes B$ is obtained by computing the operation ${}^{\alpha}A \otimes {}^{\alpha}B$ for each $\alpha \in$ [0, 1], where ${}^{\alpha}A$ and ${}^{\alpha}B$ are the confidence interval of A and B at presumption level α. It was proved that this approach complies with the extension principle of Zadeh as

$$C(z) = \sup_{z=x\otimes y} \min[A(x), B(y)] \tag{3}$$

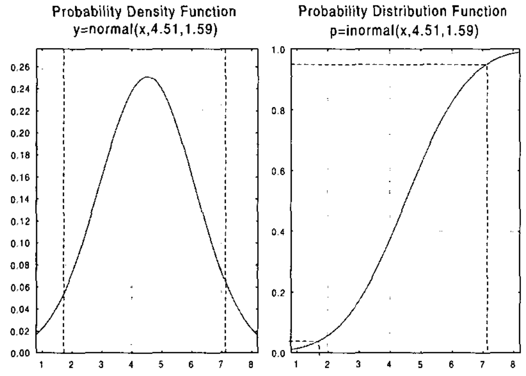
3.2 Triangular fuzzy number(TFN)

To include uncertainties into the solution procedures of deterministic prediction model, the fuzzy numbers that are used to represent the uncertain model parameters was implemented in an standard form of TFN due to those simplicity in both calculation and just three components.

Considering a definite uncertain parameter a, measured data for the parameter are assumed to be available from which a normalized distribution function $N_a(x)$ can be derived that expresses the frequency of occurrence of a certain measured value x for the parameter a within the interval Δx. In most cases, these data approximately show Gaussian distribution, i.e. normal distribution. The uncertainty in the parameter a can then be approximately modeled by a fuzzy number $\tilde{a}$ with the membership function A(x) of equation (4), which has the support of $2\times 2\sigma_a$ set up for around 95% confidence interval of a normalized distribution function $N_a(x)$.

$$A(x) = \begin{cases} \dfrac{x-a}{m-a}, & if\ a < x \le m \\ \dfrac{b-x}{b-m}, & if\ m < x \le a \\ 0, & otherwise \end{cases} \tag{4}$$

Table 1 Statistic properties of all parameters in corrosion prediction model

<table>
<tr><td rowspan="2">Parameter</td><td rowspan="2">Cover thickness, cm</td><td>Mean value</td><td>4.51</td><td rowspan="2">Surface Concentration, kg/m^3</td><td>Mean value</td><td>3.09</td></tr>
<tr><td>Standard variation</td><td>1.59</td><td>Standard variation</td><td>0.44</td></tr>
<tr><td>Probability density function and probability distribution function</td><td colspan="3">


Lower bound:1.89, Mode: 4.51, Upper bound:7.13
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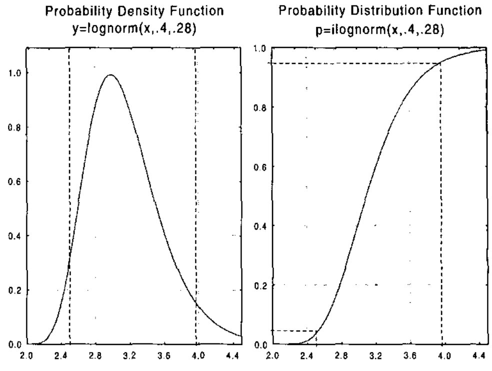
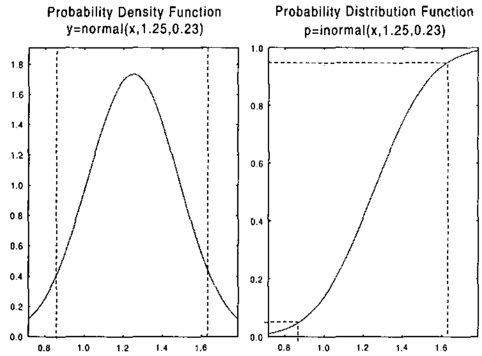


Lower bound:2.54, Mode: 2.98, Upper bound:3.96
</td></tr>
<tr><td rowspan="2">Parameter</td><td rowspan="2">Critical concentration, kg/m^3</td><td>Mean value</td><td>1.25</td><td rowspan="2">Diffusion coefficient, cm^3/year</td><td>Mean value</td><td>1.26</td></tr>
<tr><td>Standard variation</td><td>0.23</td><td>Standard variation</td><td>0.37</td></tr>
<tr><td>Probability density function and probability distribution function</td><td colspan="3">


Lower bound:0.87, Mode: 1.25, Upper bound:1.63
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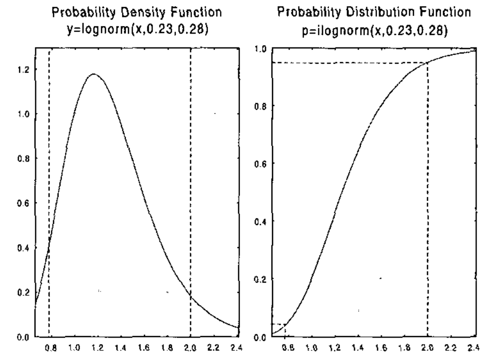

Lower bound:0.79, Mode: 1.16, Upper bound:1.99
</td></tr>
</table>

where m_a and σ_a are the mean value and the standard deviation of the Gaussian distribution in Fig. 1.

Considering an uncertain parameter b showing log-normal distribution, similarly to an uncertain parameter a of normal distribution, a triangular form of membership function is identified as shown in Fig. 2.

4. Calculating the prediction model by means of fuzzy arithmetic(case study)

4.1 Explanation of overall calculation procedure of time to corrosion initiation, T_{corr}

As stated earlier, the prediction model by Fick's 2nd law like Eq. (1) has been widely used due to its simplicity. To calculate the deterministic prediction model by using fuzzy arithmetic in this study, the approximation model of Crank's solution of Fick's second law presented by RILEM like Eq.(5) is used.[4)]

$$C_{cr} = (C_0 - C_{init}) \cdot \left\{1 - \frac{x}{2\sqrt{3D \cdot T_{corr}}}\right\}^2 + C_{init} \tag{5}$$

Eq. (5) is rewritten as follows:

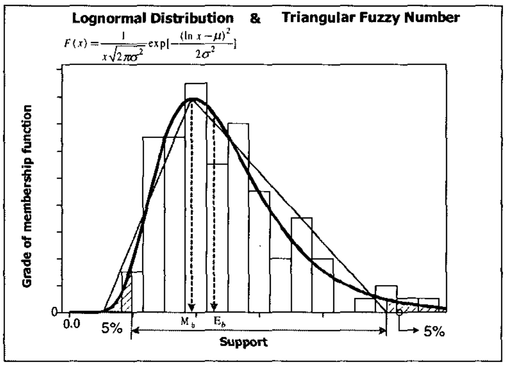

Fig. 2 Lognormal distribution of any parameter and its adaptation to membership function of triangular fuzzy number

$$T_{corr} = \frac{1}{12D} \cdot \left(\frac{x}{1 - \sqrt{\frac{C_{cr}}{C_o}}} \right)^2 \quad (6)$$

$$T_{corr} = \left(\frac{x^2}{12D} \right) \cdot \left(\frac{1}{1 - \sqrt{\frac{C_{cr}}{C_o}}} \right)^2 \quad (7)$$

Thus fuzzy arithmetic is applied to calculate the time T_{corr} to initiation of reinforcement corrosion separated into two parts like Eq.(7). Normally, in stochastic model by Monte Carlo Simulation(MC simulation), all parameters of Eq. (6) are taken into consideration by modeling them as random variables which have probabilistic density functions (PDF) that are obtained from field measurements or from the survey analysis but in this study, they are treated as fuzzy variables with proper core and support by conforming to the procedure illustrated in Figs. 1 and 2.

The mean value and standard deviations of all parameters which are used to apply fuzzy arithmetic to solving corrosion prediction problems with uncertain parameters are listed in Table 1.

Overall procedure of calculating the time T_{corr} to initiation of reinforcement corrosion by Eq.(6) based on fuzzy arithmetic is represented in Fig.3. Crisp value of time to corrosion initiation is acquired by multiplying the defuzzified value of membership function of $\tilde{x}^2/12\tilde{D}$ by the crisp value of $\left[1/\left(1-\sqrt{\tilde{C}_{cr}/\tilde{C}_o}\right)\right]^2$ calculated by inserting the defuzzified value of $\sqrt{\tilde{C}_{cr}/\tilde{C}_o}$, where each membership function is defuzzified by fuzzy centroid method, i.e. Center of Area(CoA), by way of the following Eq. (8).

$$\bar{y} = \frac{\int A(y) \cdot y \, dy}{\int A(y) \, dy} \quad (8)$$

4.2 Detailed fuzzy arithmetic of governing equation based on extension principle and interval arithmetic.

In this study, arithmetic operation of addition, subtraction, division, and multiplication on fuzzy numbers is carried out by using fuzzy interval arithmetic. As mapping fuzzy numbers via functions, the extension principle is applied to those transformations.

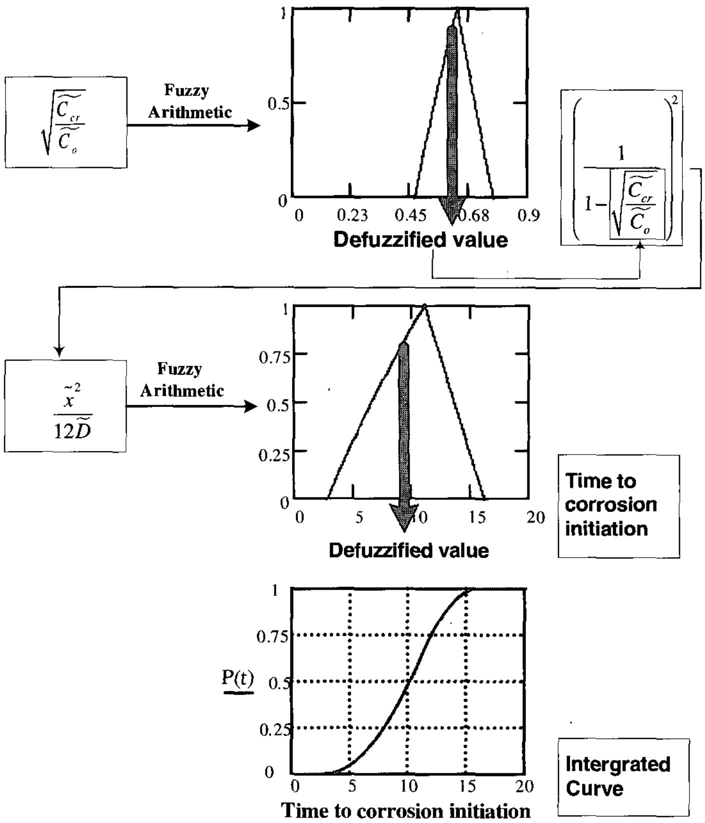

Fig. 3 Overall calculation procedure of time to corrosion initiation

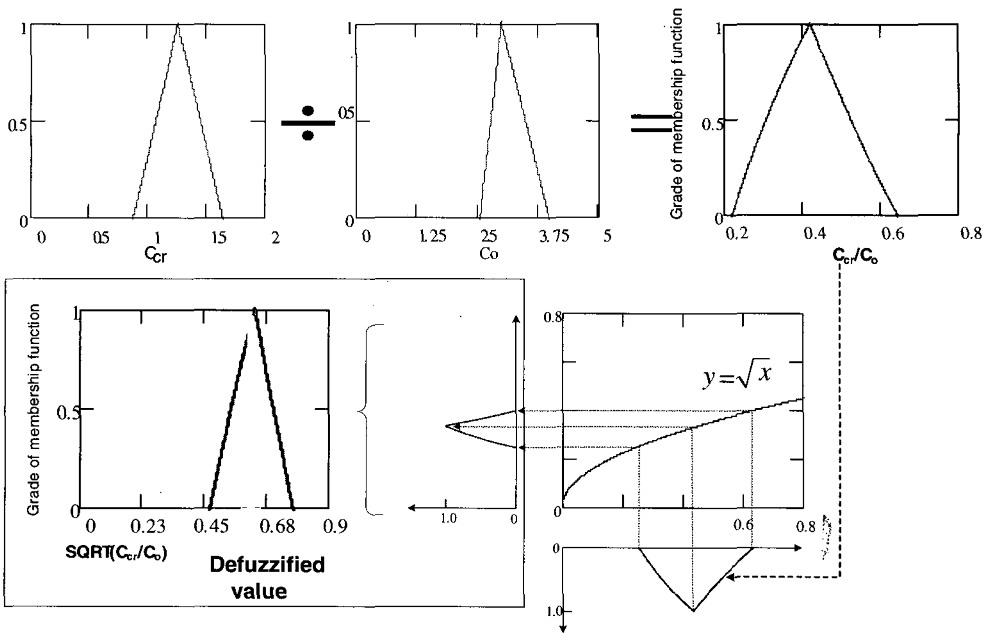

Fig.4 Membership function of fuzzy number critical chloride concentration(Ccr) divided by fuzzy number surface chloride concentration (Co) and its square root.

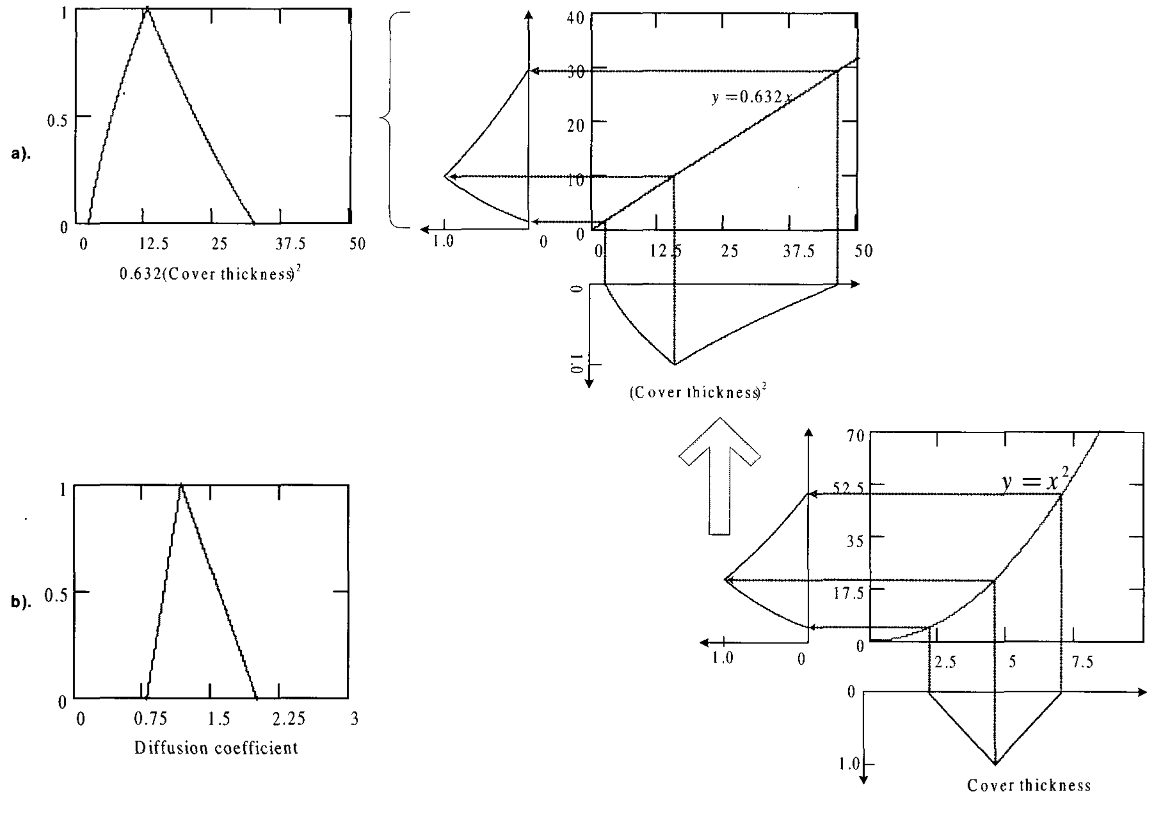

Fig. 5 Membership function of a) cover thickness(x) and transformation to 0.632(cover thickness)2 based on extension principle and b) diffusion coefficient(C),

Fig. 4 represents the shape of membership function of fuzzy number $\widetilde{C_{cr}}$ and $\widetilde{C_o}$ as well as the fuzzy arithmetic procedure of $\sqrt{\widetilde{C_{cr}}/\widetilde{C_o}}$ based on fuzzy interval arithmetic and extension principle. Fig. 5 represents the membership function of fuzzy number $\underset{\sim}{x}$ and $\widetilde{D}$, and also those transformations via each function. They show that mathematical operation of division increases the variability that each parameter possesses.

As shown in Fig. 4, the defuzzification of $\sqrt{\widetilde{C_{cr}}/\widetilde{C_o}}$ with the Center of Area (CoA) leads to the crisp value

$$\sqrt{\widetilde{C_{cr}}/\widetilde{C_o}} = 0.637 \tag{9}$$

which can be considered as the expected value for $\sqrt{\tilde{C}_{cr}/\tilde{C}_o}$

By inserting the above expected value of $\sqrt{\tilde{C}_{cr}/\tilde{C}_o}$ into $\left[1/\left(1-\sqrt{\tilde{C}_{cr}/\tilde{C}_o}\right)\right]^2$, the crisp value is as follow:

$$\left[1/\left(1-\sqrt{\tilde{C}_{cr}/\tilde{C}_o}\right)\right]^2 = 7.589 \tag{10}$$

Fig. 6 is to represent the membership function of $7.589\times\left(\tilde{x}^2/12\tilde{D}\right)$ and its integrated distribution function, and $7.589\times\left(\tilde{x}^2/12\tilde{D}\right)$ defuzzified by CoA is as follow:

$$7.589\times\frac{\tilde{x}^2}{12\tilde{D}} \longrightarrow \frac{0.632\cdot\tilde{x}^2}{\tilde{D}} = 9.912 \tag{11}$$

Finally, the time T_{corr} to initiation of reinforcement corrosion determined is as follows:

$$T_{corr} = \left(\frac{\tilde{x}^2}{12\tilde{D}}\right)\times\left(\frac{1}{1-\sqrt{\frac{\tilde{C}_{cr}}{\tilde{C}_o}}}\right)^2 = 9.912 \text{ years} \tag{12}$$

And also the integrated distribution of the time to corrosion initiation is generated and is shown in Fig. 6, from the integrated distribution function, the possibility that reinforcement corrosion is initiated in 9.912 years is estimated about to 50%. From this, it is understood that corrosion after 16 years will undoubtedly occur.

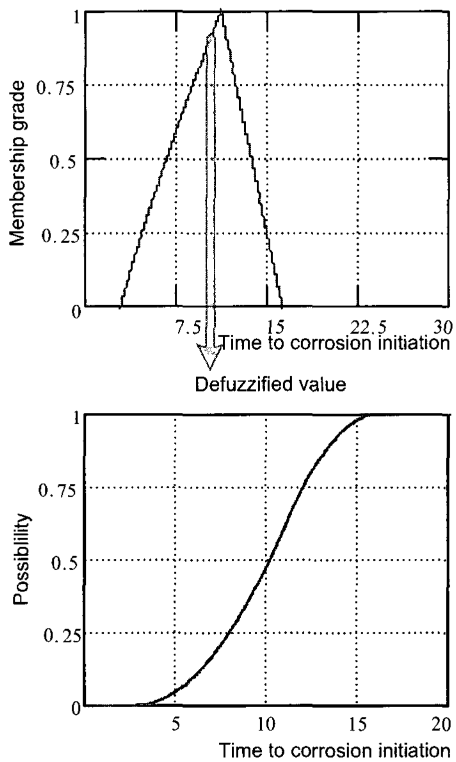

Fig. 6 Membership function of $0.632\cdot\tilde{x}^2/\tilde{D}$ and its integrated distribution

5. Conclusion

This paper presented the application of a fuzzy arithmetic approach for the modeling and prediction of reinforcement corrosion in building structures that are subjected to chloride-corrosive environment. The approach took into account the uncertainties in the physical modeling, and variability of the material and structural parameters affecting the corrosion process, in addition to the statistical and decision uncertainties. The proposed fuzzy arithmetic-based prediction model is thought to overcome the shortcomings of existing deterministic prediction models. The implementation of this tool will, if more studied in the future, provide more extensive predictions and will enable decision-makers to select cost-effective repair strategies that will extend the life of structures and will reduce the life cycle.

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