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# Alternative Cone Tip Resistance Analysis Method using Rescaled Range Analysis

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**Abstract**

In this study, R/S analysis which was proposed by Mandelbrot & Wallis (1969) was applied to evaluate the presence of the fractal property in the cone tip resistance of in-situ CPT data. Hurst exponents (*H*) were evaluated in the range of 0.660~0.990 and the average was 0.875. It was confirmed that a cone tip resistance data had the characteristic of fractals and it was expected that cone tip resistance data sets are well approximated by a fBm process with an Hurst exponent near 0.875. It was also observed that the boundary between layers were obviously identified as a result of R/S analysis and it will be usage in practices.

*Keywords : Rescaled range (R/S) analysis, Cone penetrometer test, Cone tip resistance, Fractal, Hurst coefficient, Fractal dimension*

## I. Introduction

Many observations of nature consist of records in time or space. And a physical phenomenon or the data representing it are considered random when a future time or space record cannot be predicted within resonable experimental error from an experiment. Such random processes with long range power law correlation have been observed in various of fields including economics, geosciences, physics and biology.

There are roughly two types of tools used in assessing the presence of such correlation in continuous series data; spectral domain methods represented by power spectrum analysis, and random walk methods in the space domain represented by the rescaled (R/S) range analysis (Mandelbrot & Wallis, 1968, 1969a, 1969b; Mandelbrot, 1983).

Some scientists and engineers claimed that a fractal-based function is a natural candidate for representing random processes, especially on hierarchical soil structures (Molz et al., 1998).

It has been well known that in-situ geotechnical investigation technologies, such as CPT, can support the continuous data with the depth of

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layer and is not appeared smooth but the extreme random pattern. However, many contemporary interpolation procedures, such as averaging, Kriging and the polynomial interpolation act to smooth the basic data.

In this study, R/S analysis which maintains the randomness of data over a range of measurement scale as large as desired was applied to evaluate the presence of the fractal property in the cone tip resistance of in-situ CPT data.

## II. Fractal, Fractional Brownian Motion (fBm) and Rescaled Range (R/S) Analysis

Fractals are a means of describing the complicated irregular features, the complexity, of variation at all spatial scales and attempt to model multiscale variation in terms of self-similarity or self-affinity. These complexity can be described quantitatively by the fractal dimension *D*. For example, true fractals which is self-similar yield linear graphs on double log plots of these relations; the slopes of the graphs give experimental estimates of fractal dimension *D*, of the phenomenon being studied.

Brownian motion was introduced by Robert Brown in 1828 and Wiener, in 1923, introduced the random function X (t) for Brownian motion for any two times $t$ and $t_0$ as fellows.

$$X(t) - X(t_0) \sim r \mid t - t_0 \mid^{H} \quad (t \geq t_0) \cdots\cdots\cdots (1)$$

where, $r$ is sclae ratio and $H$ (Hurst exponent) = 1/2. This equation means that very long range correlations were found in the time-series data, which may contradict the intuitive notion that natural time series have short term memory, so that events separated by several years or decades may be considered independent in a statistical sense.

The concept of R/S analysis was originally evolved by Hurst in 1951 and long range correlations was characterized by the hurst exponent *H*. Mandelbrot (1983) has introduced the concept of fBm (fractional brownian motion) as a generalization of the random function X (t) by changing the exponent from $H$ = 1/2 to any real number in the range $0 < H < 1$ in equation (1). When $H > 1/2$ the process is said to have positive long range correlation or persistence, while $H < 1/2$ means the process has long range anti-correlation or anti-persistence. When $H = 1/2$, we say that the process has short range correlation (Feder, 1988; Mandelbrot & Van Ness, 1968 & Mandelbrot, 1983).

The appropriate methodology for calculating $H$ from a data set displaying the properties of fBm is called rescaled range (R/S) analysis. soils are also a representative heterogeneous media which showed the random process in most of their physical or mechanical behaviors, so that geotechnical investigation results appeared the extremely random pattern even though these were collected in the same place. Therefore, it is possible that such random patterns can be predicted by fractal theory when it is analyzed by R/S in the appropriate range of Hurst exponent. Molz et al, (1993, 1998) applied R/S analysis to the spatial variation of continuous data of porosity (n) and hydraulic conductivity (*K*) of porous media and found the results that averaged $H$ was about 0.820 and 0.875.

These $H$ could be applied to model sparse data of geological information using stochastic modeling techniques, such as Monte-Carlo simulation.

In R/S analysis, $R$ is the range which was normalized (common mean removed and difference divided by the common standard deviation) and S is the standard deviation.

Computationally, on the CPT data set, the variance and R/S values were obtained for each lag size, $n \cdot \Delta z$, using the formulas given below(Molz & Boman, 1993).
Where, $Gq_c$ is the normalized cone tip resistance, $z$ is the depth at which cone tip resistance was measured and u is the point considered in the range.

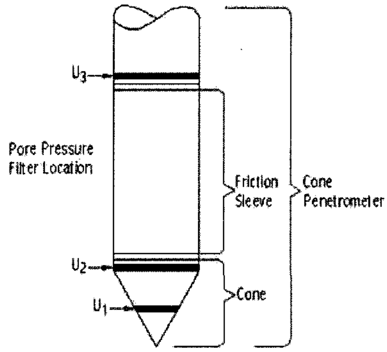



Fig. 1 Cone penetrometer

$$R(z, n \cdot \Delta z) = \max\left(Gq_c(z+u) - \left[Gq_c(z) + \frac{u}{n \cdot \Delta z}(Gq_c(z+n \cdot \Delta z) - Gq_c(z))\right]\right) - \min\left(Gq_c(z+u) - \left[Gq_c(z) + \frac{u}{n \cdot \Delta z}(Gq_c(z+n \cdot \Delta z) - Gq_c(z))\right]\right) \quad (2)$$

$$S(z, n \cdot \Delta z) = \left(\frac{\Sigma Gq_c^2(z+u)}{n \cdot \Delta z - 1} - \frac{(\Sigma Gq_c(z+u))^2}{(n \cdot \Delta z)^2 - n \cdot \Delta z}\right)^{1/2} \quad (3)$$

## Ⅲ. Cone Petrometer Test and Site Description

The CPT is performed with a cylindrical penetrometer with a conical tip (cone) penetrating into the ground at a constant rate of penetration. A cone penetrometer with a 10 $cm^2$ base area cone with an apex angle of 60 degrees is accepted as the reference and has been specified in the International Reference Test Procedure (IRTP, 1999).

During the penetration, the forces on the cone and the friction sleeve are measured. The measurements are carried out using electronic transfer and data logging, with a measurement frequency that can secure detailed information about the soil conditions (See Fig. 1). The results from a cone penetration test can in principle be used to evaluate stratification, soil type, soil density and in situ stress conditions and shear strength parameters.

A data sets used in this study were obtained from the exist geotechnical investigation reports which was carried out in Busan city area, Korea (Chung-suk, 2002). This area was developed to the new port zone and CPT were collected to assess the subsurface condition for the construction of the road in project procedures. In this study, eight representative profiles among 38 CPTs were selected with the characteristics of the cone tip resistance profile. The layers were in the order of filled layer (5~15 m), sedimentary (clayey) layer (16~32 m), weathered soil layer and weathered rock layer. The weak sedimentary clayey layer were shown irregular N-value (from 1 to 50/2).

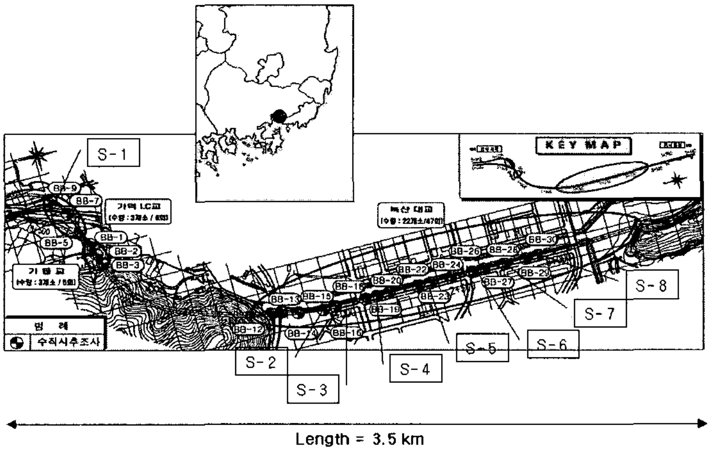



Fig. 2 Location of site and CPTs

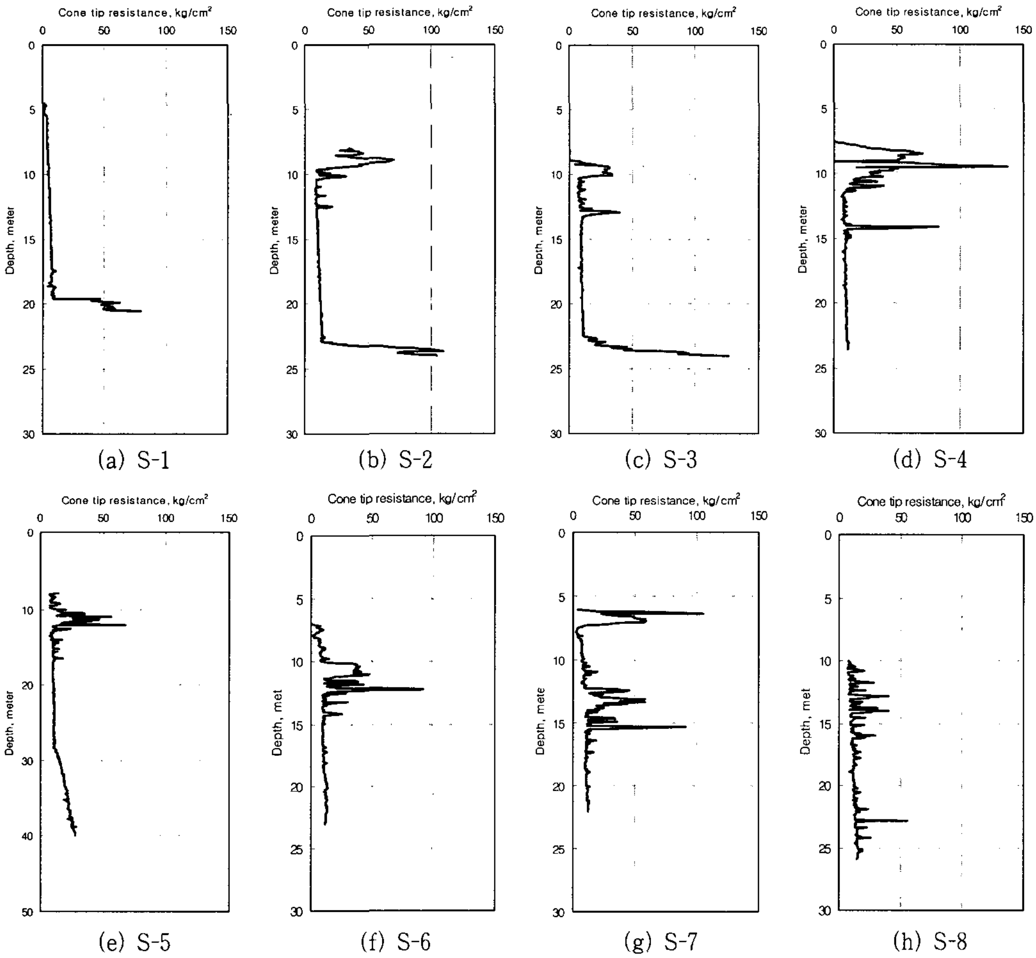



Fig. 3 Cone tip resistances profile of in-situ CPTs

Test location and used profiles of cone tip resistance of CPTs were shown in Fig. 2 and Fig. 3. Each figures in Fig. 3 was presented the cone tip resistance only and those figures showed the different and random pattern with the characteristics of layers composed. Fig. 3 (a) was the profile of a layer which consist of weak clay layer down to about 19m and showed the very stationary pattern. Fig. 3 (b) ~Fig. 3 (f) were the profile of layers which upper layer were filled by coarse (sand & gravel) materials, so that upper layer was shown the high cone tip resistance value. On the other hand, Fig. 3 (g) and Fig. 3 (h) were shown the very irregular pattern of cone tip resistance in almost of all the depth, and it could be resulted from the mixing of natural subsurface soils and fill materials.

## Ⅳ. Results and Discussion

Normalization was performed and their results were shown in Fig. 4. In Fig. 4, the result of each step of normalization procedure was plotted with the decreasing lag sizes ($\Delta z$=16, 8, 4, 2, 1, 0.5m). The profile of cone tip resistance in each test points were plotted to compare cone tip resistance profile to the each step of normalization.

With the lag sizes, it was obvious that similar type of layers could be distinguished with others and could classify the same type of soil (or layer) from the X (t) range because same type of soil must be in the same deviation range. It was very interesting results that the boundary between layers (or the soil type) were obviously identified in the procedure of normalization.

Because, in the conventional analysis, the *friction ratio* [Rf = (fs/qc)×100%] has been mainly used to identify the soil type and there are charts available from several researchers (Lunne et al., 1997). However such methods were a little tedious and implicated procedure in practice.

Fig. 5 showed the results of R/S analysis and summarized in Table 1. It was appeared that Hurst exponents (*H*) evaluated were in the range of 0.660~0.990 and the average was about 0.875. It was recognized that analyses were reasonable because these values were in the range of what Hurst or Madelbort & wallis claimed. Even Hurst exponent at S-8 which showed the very complex profile was 0.660, it was greater than 0.5 and it was meant that, in the fractal viewpoint, cone tip resistance values have positive long range correlation or persistence and Hurst coefficient could be applied stochastic modeling techniques to model sparse data of geological information.

When we refer the other values (Feder, 1988), it was also remarkable that the average value *H*=0.875 was similar with the result of Molz & Boman (1993) in which spatial porosity (*H*=0.82) and hydraulic conductivity (*H*=0.855) variations were considered (According to Feder (1988), natural phenomena have a their own Hurst exponent value, river discharges (*H*=0.72), sunspot numbers (*H*=0.75), temperature (*H*=0.68) etc). But future researches with the various data sets might be needed to confirm the appropriate range of Hurst exponent in the case of geotechnical investigation results.

As a result of the R/S analysis, it was confirmed that cone tip resistance data sets are well approximated by a fBm process with an Hurst

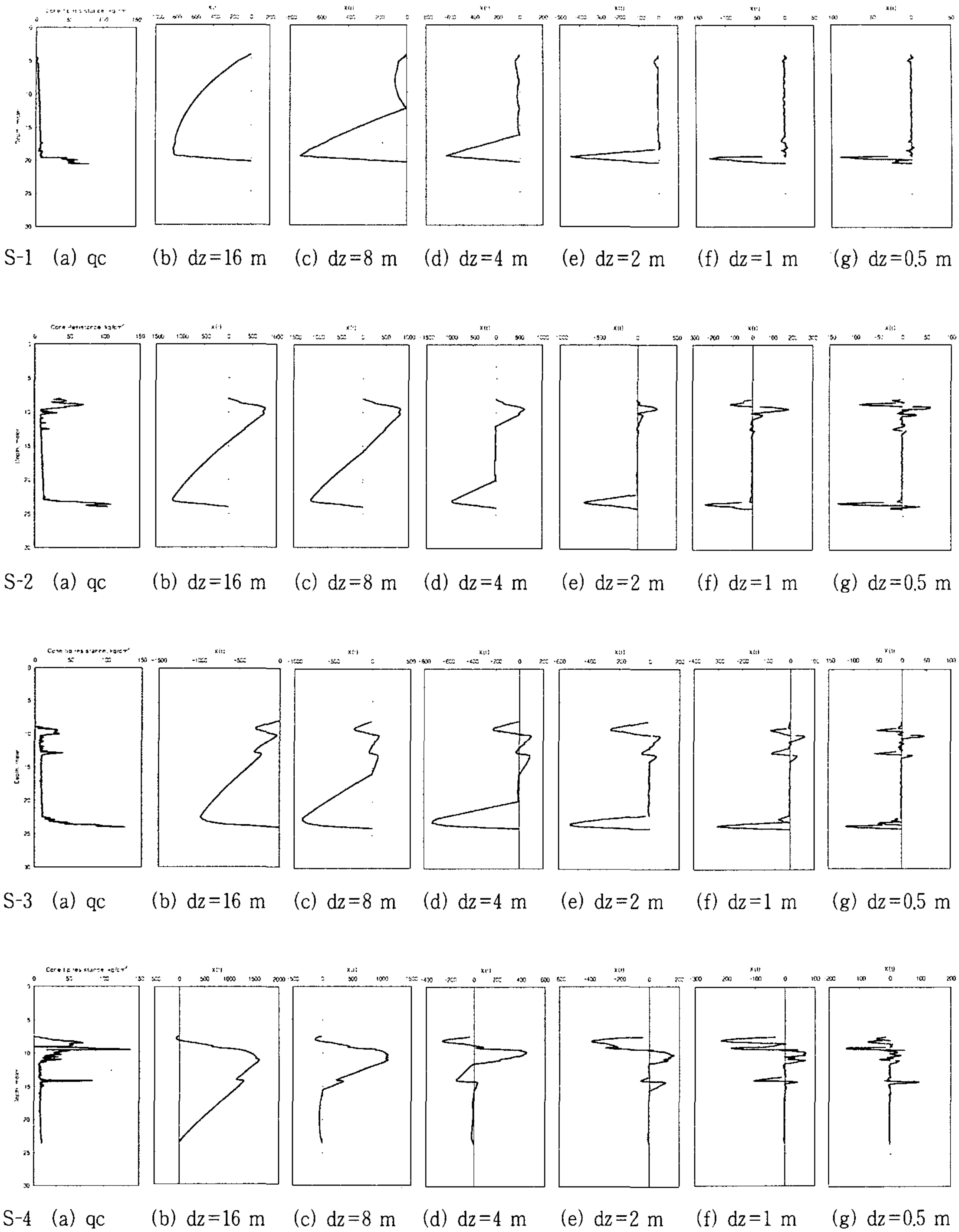
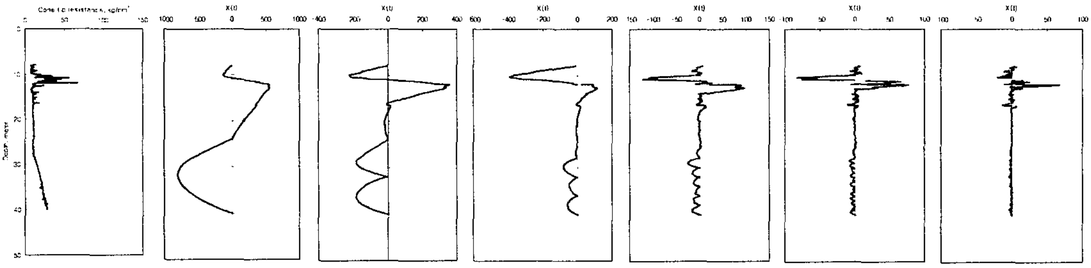
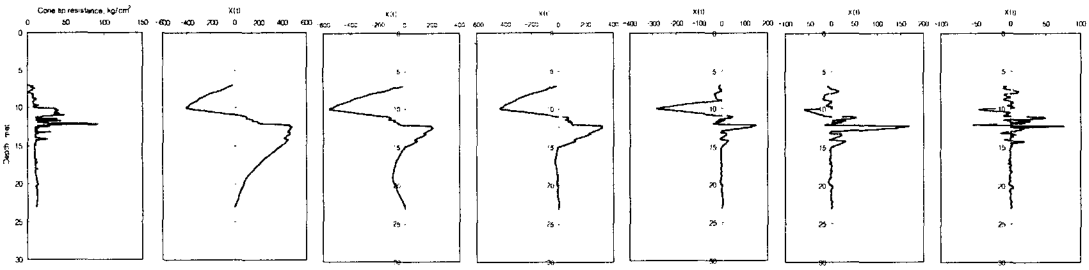


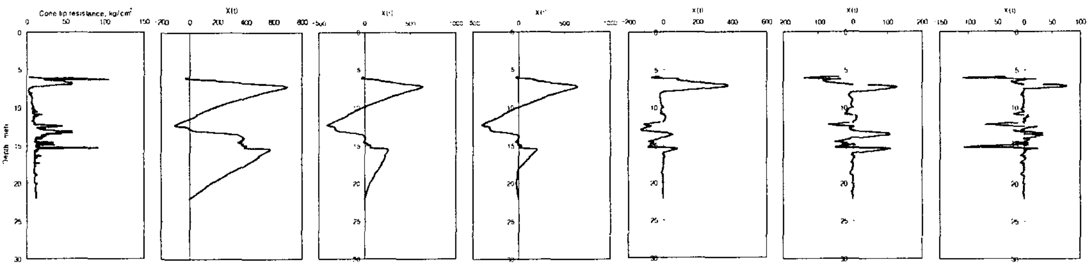

**Fig. 4 Normalization analysis results with △z = 16, 8, 4, 2, 1, 0.5 m**



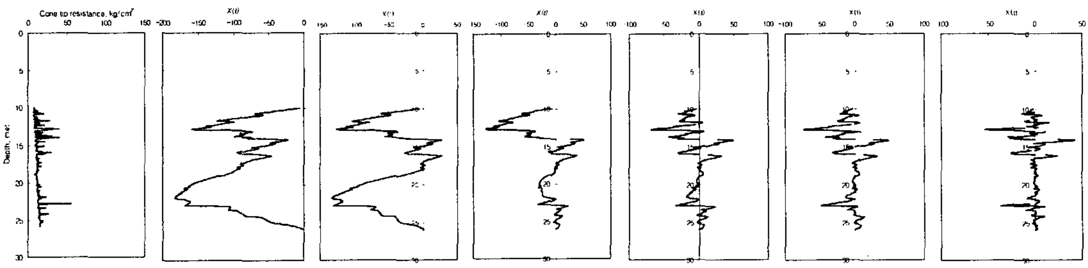
S-5 (a) qc (b) dz=16 m (c) dz=8 m (d) dz=4 m (e) dz=2 m (f) dz=1 m (g) dz=0.5 m



S-6 (a) qc (b) dz=16 m (c) dz=8 m (d) dz=4 m (e) dz=2 m (f) dz=1 m (g) dz=0.5 m



S-7 (a) qc (b) dz=16 m (c) dz=8 m (d) dz=4 m (e) dz=2 m (f) dz=1 m (g) dz=0.5 m



S-8 (a) qc (b) dz=16 m (c) dz=8 m (d) dz=4 m (e) dz=2 m (f) dz=1 m (g) dz=0.5 m

**Fig. 4 Normalization analysis results with $\triangle z$ = 16, 8, 4, 2, 1, 0.5 m (continued)**

exponent near 0.875.

For relationship between the Hurst exponent and the fractal dimension, according to Feder (1988), the relation $D = 2-H$ hold in the high-resolution or local limit in the analysis of structure of the record of a fractal function. Fractal dimensions, therefore, could be also calculated and summarized in Table 1.

But, in the other viewpoint, according to Rangarajan (2000), it must be noted that short or unappropriate range data set (=transient range) might be resulted in the inappropriate $H$ value and, hence, more researches are needed to identified the upper and lower limit of lag size to apply R/S analysis.

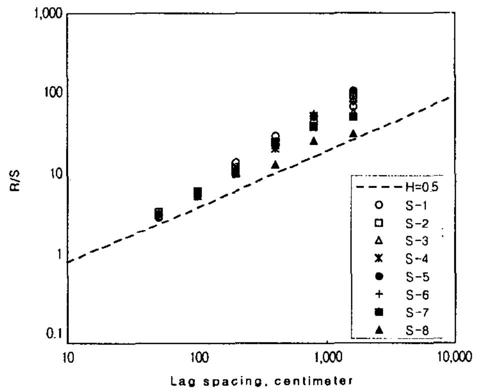



**Fig. 5 Rescaled range analysis results of CPTs**

**Table 1 Summary of R/S analysis**

| Data | Thickness of layer (m) | Hurst's exponents | Local fractal dimension |
|---|---|---|---|
| S-1 | 16 | 0.900 | 1.100 |
| S-2 | 16 | 0.935 | 1.065 |
| S-3 | 16 | 0.819 | 1.181 |
| S-4 | 16 | 0.949 | 1.051 |
| S-5 | 32 | 0.990 | 1.010 |
| S-6 | 16 | 0.932 | 1.068 |
| S-7 | 16 | 0.818 | 1.182 |
| S-8 | 16 | 0.660 | 1.340 |
| Average | | 0.875 | - |

After the evaluating work of Hurst exponents, next step in the stochastic interpolation process is to generate values between investigation points based on the properties of fBm. There are several methodologies for doing this even though a clear alternatives have not been supported in a field of geotechnical engineering yet rather a subject for future research (see ref. [1] for more information).

## V. Conclusion

In this study, R/S (rescaled range) analysis was applied to the cone tip resistance data to evaluate the Hurst exponent ($H$) and the conclusions were as below;

1. Hurst exponents ($H$) were evaluated in the range of 0.660~0.990 and the average was 0.875. These results were shown reliable when compared to the results of Feder (1988) and Molz & Boman (1993).

2. It was confirmed that a cone tip resistance data had the characteristic of fractals and it was expected that cone tip resistance data sets are well approximated by a fBm process with an Hurst exponent near 0.875.

3. It was also observed that the boundary between layers (or the soil type) were obviously identified as a result of R/S analysis and it will be usage in practices to identify the layers or soil type.

But future researches were needed to select the appropriate data set and to perform the stochastic interpolation process in order to use R/S analysis in practice.

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