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Using Mean Residual Life Functions for Unique Insights into Strengths of Materials Data

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Abstract. We show how comparative mean residual life functions (MRL) can be used to give unique insights into strengths of materials data. Recall that Weibull's original reliability function was developed studying and fitting strengths for various materials. This creative comparing of MRL functions approach can be used for regular life data or any time to response data. We apply graphical MRL's to real data from tests of tensile strength of high quality engineered wood.

Key Words : *mean residual life, strength of materials, internal bond, tensile strength, mean residual life plots, medium density fiberboard.*

1. INTRODUCTION

Mean residual life (MRL) functions have many theoretical insights and practical applications. Recall for a random life variable X, the MRL is defined as the conditional

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expectation $E(X-t \mid X>t)$. Note, for example, the MRL function can exist, while the failure rate function might not exist, while, vice a versa, the failure rate function can exist without the MRL function existing. As the reliability function can be represented as a function of the failure rate function; the reliability function can be represented as a function of the MRL, when each respectively exists.

MRL history is ancient, going back to at least the second century A.D. in life tables for annuities. Compare the helpful review and references on MRL in Guess and Proschan (1988). For more on specific classes of distributions connected with MRL widely studied and tested in a variety of situations see the important classical Hollander and Proschan (1984) and more specific Hollander and Proschan (1975), among others references.

Also, observe these useful papers on a wide variety of MRL topics ranging from testing in regards to MRL such as trend changes, to basic modeling, to comparing MRL to other reliability measures, etc., by Anis, Basu, and Mitra (2004), Bradley and Gupta (2003), Asadi and Ebrahimi (2000). Na and Kim, (2000), Lim and Park (1998), Guess, Nam, and Park (1998), Guess, Walker, and Gallant (1992), Abouammoh (1988), Berger, Boos, and Guess (1988), Guess and Park (1988), Guess, Hollander and Proschan (1986). This is a short list among many other helpful papers, which we deeply regret not listing here. We plan soon to do a more comprehensive review of over 200 papers on MRL, but we have other goals in this note than such an extensive literature review.

Other functions are useful, for example see the insightful paper of Klefsjö (1991) on total time on test (TTT) plots. Contrast, also, Iskandar, Klefsjö, and Sandoh (2000) on TTT plot applications. We recommend TTT plots and other function tools in addition to MRL, but investigate in this paper some helpful comparative usages of MRL.

In Section 2, we show how comparative mean residual life functions (MRL) can be used to give unique insights into strength of materials data. Recall that Weibull's original reliability function was developed studying and fitting strengths for various materials, see Weibull (1939, 1951). The approach of comparing MRL functions can be used for regular life data or any time to response data. We apply graphical MRL's to real data from tests of tensile strength of high quality engineered wood, called medium density fiberboard (MDF).

Compare Young and Guess (2002) for how such data is stored and used in a real time data base with regression modeling to predict strength. This provides quick feedback to the manufacturer in order to minimize process inputs and maximize product quality within specified limits. One key metric used by manufacturers of the quality or reliability of MDF is internal bond (IB). Samples from a cross section of the MDF are tested by being pulled apart. The IB at failure is then measured in pounds per square inch (p.s.i.) or corresponding metric units (kilograms per cubic meter).

Product "life" for MDF can be measured in terms of the strength to failure, as opposed to the time to failure. The strength or pounds per square inches to failure is a crucial reliability parameter of the product. It naturally allows the producer to make assurances to customers about the quality, safety, and useful "life" range of the product. In Section 3 we provide concluding remarks and recommendations on MRL usages for comparative insights.

2. COMPARING MEAN RESIDUAL LIFE FUNCTIONS FOR UNIQUE INSIGHTS

We graph the empirical MRL's without outliers in Figure 1, then with outliers included in Figure 2 for pressure to failure data of a particular MDF product having a density of 46 pounds per cubic foot (lbs/ft^3), thickness of 0.625 inches, and width of 61 inches. The empirical MRL is plotted at each unique failure then linearized between points as seen in both graphs. Also, note that the MRL functions in both cases were linear and decreasing on the interval from 0 to 100. We stress the unique behavior and difference in the upper interval after 100 psi.

Figure 1 illustrates a decreasing empirical MRL suggesting the classical DMRL class, tested in Hollander and Proschan (1975) and several others DMRL tests. Figure 2 illustrates a subset of outliers with unusually stronger MRL near the end. Also, the "hooking upward" pattern of the MRL is an unexpected outcome for this material.

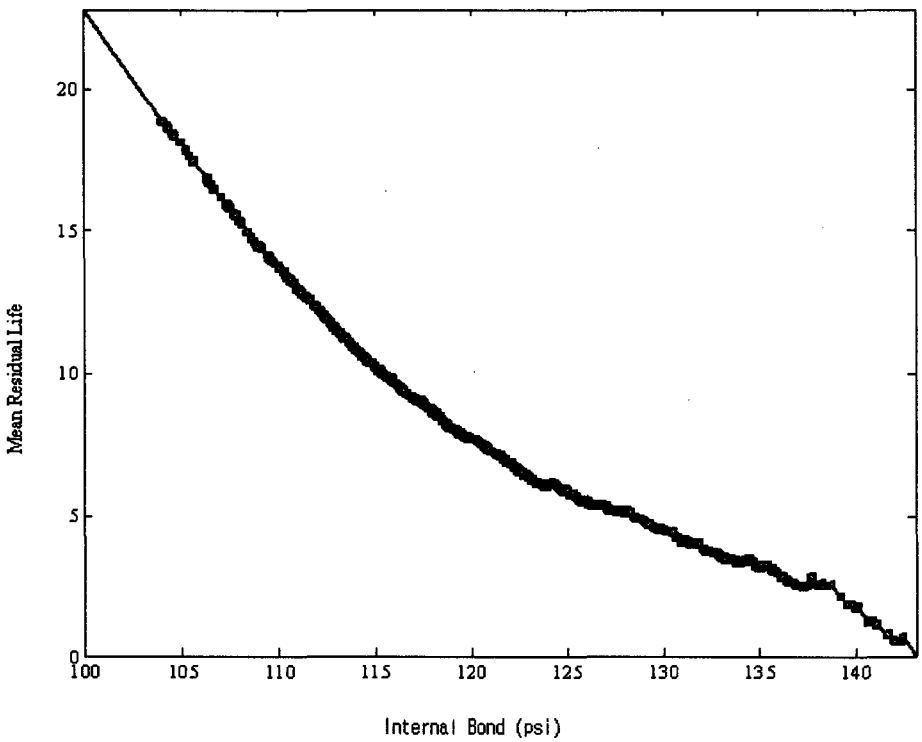

Figure 1. Mean Residual Life function without outliers (i.e., outliers omitted).

The increase in MRL above 135 p.s.i. was a surprise. This may be the result of setup change by the manufacturer to a higher targeted strength product, i.e., the manufacturer produces a higher strength product with more resin and a slower line speed, and is unable

to instantaneously meet target specifications during setup change from the nominal strength to higher strength product. It is obvious to practitioners from the MRL graph in Figure 2 that a hybrid product, or medium-strength product, is being produced. The MRL graph in Figure 2 reveals an opportunity cost for the manufacturer, e.g., improve setup change time to minimize resin usage, optimize line speed targets during product change, etc. The MRL graph in Figure 2 may have, also, identified a new product opportunity for the manufacturer. These empirical MRL and behaviors can be powerful diagnostic tools to facilitate training, process improvements and cost savings.

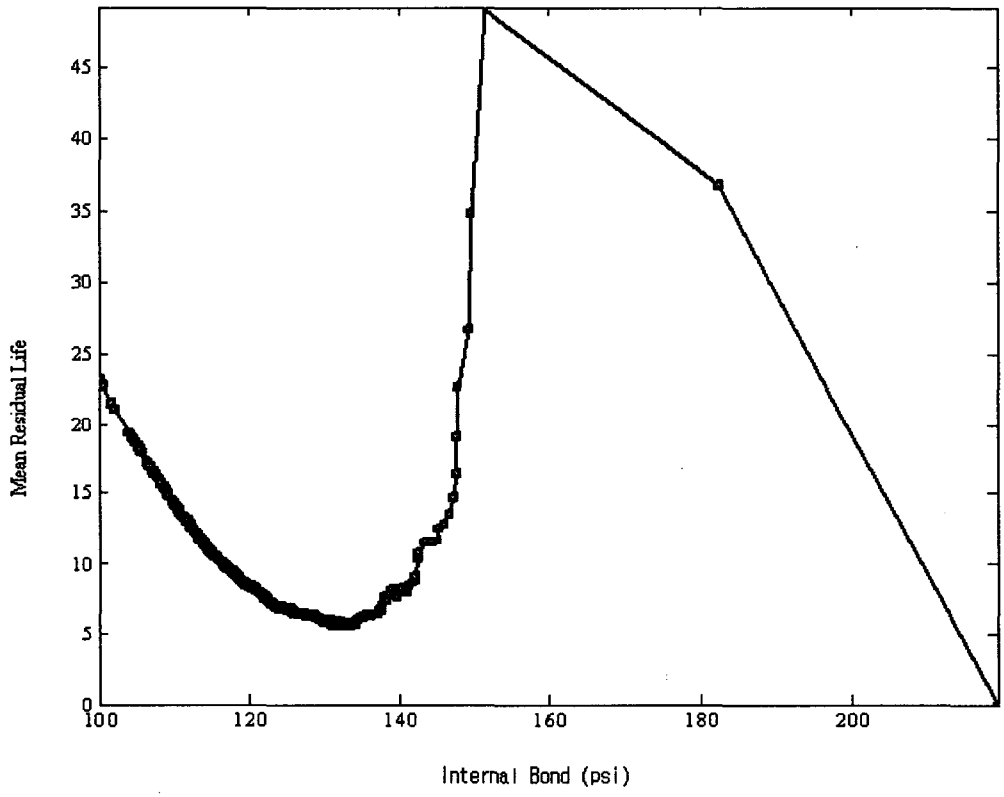

Figure 2. Mean Residual Life function with outliers (all data included).

We also illustrate the graphs of histograms, boxplots, and normal quantile plots in Figure 3 with no outliers and in Figure 4 with all data included. These graphs suggest that more detailed study is required, but the previously illustrated MRL plots were more revealing in identifying the behavior of a subgroup with superior strength.

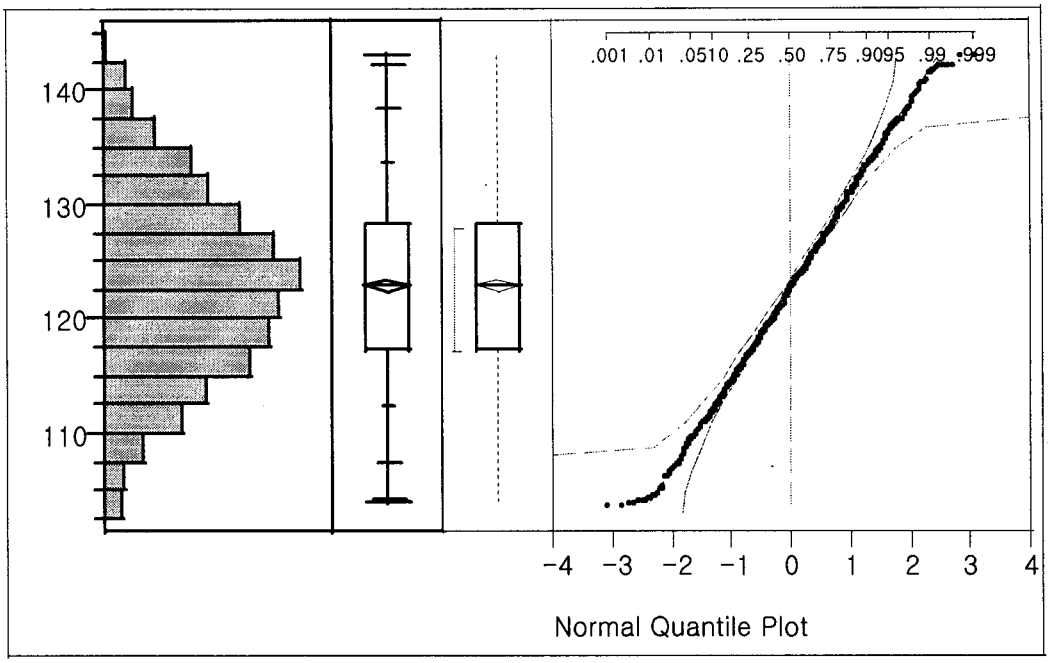

Figure 3. Histogram, boxplots, and normal quantile plot without outliers (i.e., outliers omitted).

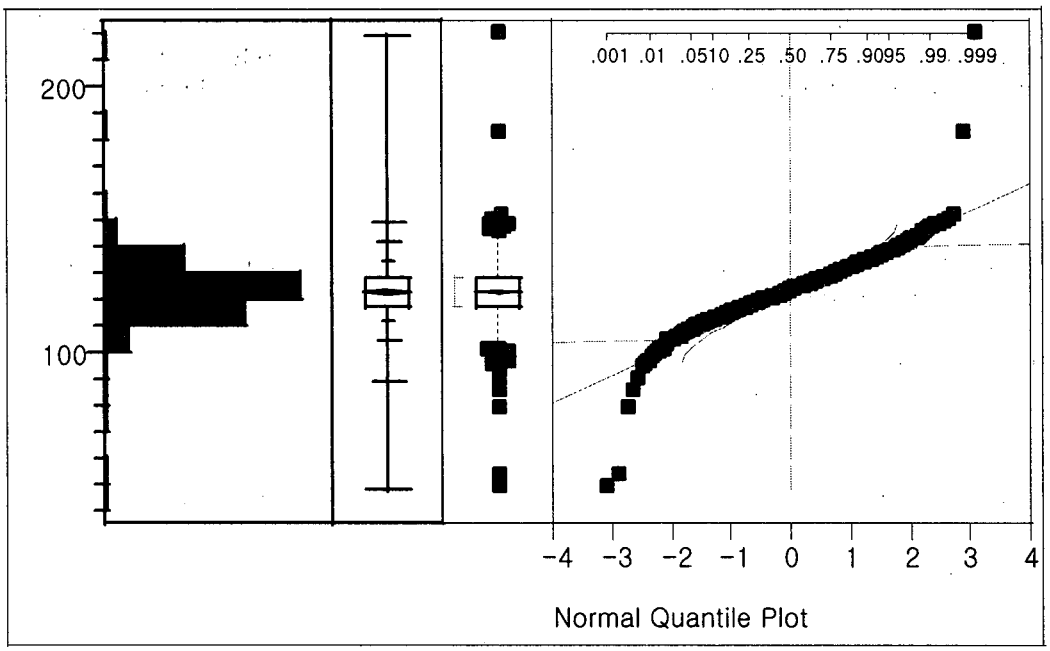

Figure 4. Histogram, boxplots, and normal quantile plot with outliers (all data included).

The mean with no outliers was estimated at 122.73 p.s.i., while the mean with all the data (outliers included) was only slight different at 122.86 p.s.i. The means by themselves mask the unexpected difference going on in a subgroup of superior strength which was quickly identified by the MRL plots. Obviously all of these techniques, including the MRL plots, are needed for process improvement and training.

3. SUMMARY AND CLOSING REMARKS

We recommend viewing data from many different perspectives ranging from histograms, boxplots, quantile plots, TTT plots, etc., but have also illustrated how unique, striking behaviors can be identified by comparative MRL plots. It is always highly recommended to explore data with many different techniques. MRL plots can be readily constructed with MATLAB, Maple, R, S+, or other software. Comparative empirical MRL without and with outliers can be a powerful tool for facilitating training, process improvement and cost savings.

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