

# Quasi-Sliding Mode Control of an Autonomous Underwater Vehicle with Long Sampling Interval

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장주기 샘플링을 갖는 자율무인잠수정의 의사 슬라이딩모드 제어

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**Key Words :** Discrete Control(이산제어), Quasi-Sliding Mode(의사-슬라이딩 모드), Parameter Uncertainty(불확실성), Long Sampling Interval(장주기샘플링), AUV(자율무인잠수정)

## 초 록

본 논문은 AUV의 수직면 운동 제어를 수행하기 위하여 의사 슬라이딩 모드 제어를 이용한 모델링 기법과 제어기 설계법에 관한 것으로서, 샘플링 간격이 길어지는 경우에도 시스템의 강인성이 확보되며 심도 제어가 안정적으로 수행되는 실용성을 실험과 수치 해석을 통하여 검증하였다. 제어기는 참고문헌<sup>9)</sup>에서 제안한 방법을 이용하였으며, 한국기계연구원 선박해양공학연구소(KRISO)에서 개발한 VORAM호를 제어 대상 AUV로 선정하였다. PMM 시험으로 얻어진 운동 계수를 이용하여 수치 해석을 수행하였으며, KRISO의 장수조에서 실험을 수행하였다. 수치 해석과 실험 결과로부터 샘플링이 길어짐에 따라 의사 슬라이딩 모드 제어기는 연속계에 대한 슬라이딩 모드 제어기에서 발생하는 과도한 채터링 및 불안정성을 보이지 않았으며, 시스템의 안정성이 확보되고 불확실성에 대하여 강인한 제어 성능을 보였다. 또한, 본 논문에서는 수치 해석과 실험 결과를 근거로 의사 슬라이딩 모드 제어기의 설계를 위한 제어 변수의 선정 기준을 제시하였다.

## 1. Introduction

The motion of underwater vehicle may be described in terms of the twelve nonlinear equations for the body-fixed coordinate frame and the global reference frame. These equations

include the hydrodynamic added masses and drags acting on the vehicle in the body fixed frame, where the coefficients of the nonlinear equation of motion depend on the operating speed and attitude of the vehicle. The development of the functional form of the

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hydrodynamic forces has been well studied<sup>(1),(2),(3),(4) and 5)</sup>. Those are described by a Taylor series expansion with respect to relative velocities and accelerations.

Generally, the decoupled equations of motion in the vertical plane and in the horizontal plane are used according to mode of vehicle motion. These equations are too complicated to implement the control system. Various simplified models have been developed and used in the studies of motion control of underwater vehicles.

The sliding mode control has been successfully applied to underwater vehicles. Yoerger and Slotine<sup>(6)</sup> proposed a series of SISO continuous-time controllers by using the sliding mode technique on an underwater vehicle. The robustness of their control system in the presence of parameter uncertainties was demonstrated by computer simulation. Cristi et al.<sup>(7)</sup> and Papoulias et al.<sup>(8)</sup> proposed an adaptive sliding mode controller for AUVs based on the dominant linear model and the bounds on the nonlinear perturbations of the dynamics.

In this paper, it is supposed that the vehicle autonomously glides above the sea bottom with the modes of constant altitude keeping and path keeping in vertical plane. The vehicle is remotely operated with the control commands from the supporting ship and with the use of an acoustic telemetry modem(ATM). Information of the position and motion of the vehicle is gathered by an SSBL tracking system and an motion reference unit(MRU).

In discrete-time implementation, as the sampling time becomes large, a continuous-time sliding mode controller can lead the system to chattering along the desired sliding mode and even to instability. Consequently, a discrete-time sliding mode method is needed to guarantee the robustness of control system with parameter uncertainty and long sampling interval.

The dynamic characteristic of an AUV is

introduced and a model is derived for motion control of the AUV in vertical plane. Simulations for the tracking control of VORAM AUV are performed, where the AUV has the parameter uncertainties in system modeling and long sampling interval. The effectiveness and robustness of the proposed discrete-time control systems are illustrated through simulations and experiments on depth keeping control.

## 2. Dynamic Equation of VORAM AUV

The AUV, VORAM, is an unmanned untethered submersible vehicle for research and monitoring in deep sea and is aimed to follow the sea bottom, to keep constant altitude or to keep constant water depth. There are several technique to represent the equation of underwater vehicle's motion. In this paper, it is supposed that the vehicle is operating around a desired forward speed. By adopting a forward speed ratio defined as

$$\eta = \frac{\left(\frac{U}{nD}\right)_{\text{design speed}}}{\left(\frac{u}{nD}\right)_{\text{actual speed}}} \quad (1)$$

where  $U$  is design speed,  $u$  is actual speed,  $n$  is propeller rps, and  $D$  is propeller diameter, we can describe the equation of motion in terms of the forward speed ratio.

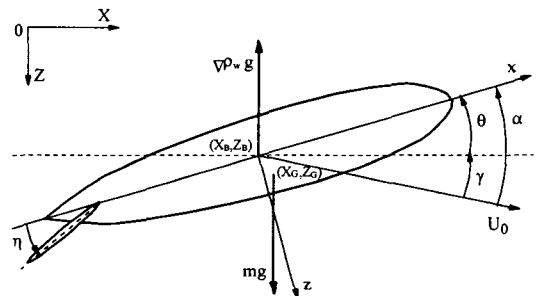


Fig. 1 Coordinate system of AUV

Restricting the motion of the AUV to the vertical plane and ignoring the cross-flow drag fraction, and defining downward positive coordinates as Fig. 1, the equation of motion of the AUV can be described by the following equations:

$$(m - X_{\dot{u}}) \dot{u} + m z_G \dot{q} + (m - Z_{wq}) w q - m x_G q^2 = -(m - \nabla \rho_w) g \sin \theta + F_X \quad (2)$$

$$(m - Z_{\dot{w}}) \dot{w} - (m x_G - Z_{\dot{q}}) \dot{q} - m u q - m z_G q^2 = (m - \nabla \rho_w) g \cos \theta + F_Z \quad (3)$$

$$m z_G \dot{u} - (m x_G + M_{\dot{w}}) \dot{w} + (I_{yy} - M_{\dot{q}}) \dot{q} + m(z_G w + x_G u) q = -(m z_G - \nabla \rho_w z_B) g \sin \theta - (m x_G - \nabla \rho_w x_B) g \cos \theta + M_Y \quad (4)$$

where,

$$F_X = (X_{ww} + X_{w\eta}(\eta - 1.0)) w^2 + (X_{\delta_e \delta_e} + X_{\delta_e \eta}(\eta - 1.0)) \delta_e^2 + X_{qq} q^2 + X_{wq} w q + X_{0\eta}(\eta - 1.0) \quad (5)$$

$$F_Z = Z_0 + Z_{0\eta}(\eta - 1.0) + (Z_w + Z_{w\eta}(\eta - 1.0)) w + (Z_{w|u|} + Z_{w|u|\eta}(\eta - 1.0)) w|u| + Z_{\sigma} q + Z_{q|q|} q|q| + (Z_{\delta_e} + Z_{\delta_e \eta}(\eta - 1.0)) \delta_e + (Z_{\delta_e \delta_e} + Z_{\delta_e \delta_e \eta}(\eta - 1.0)) \delta_e | \delta_e | \quad (6)$$

$$M_Y = M_0 + M_{0\eta}(\eta - 1) + (M_w + M_{w\eta}(\eta - 1)) w + (M_{w|u|} + M_{w|u|\eta}(\eta - 1.0)) w|u| + M_{\sigma} q + M_{q|q|} q|q| + (M_{\delta_e} + M_{\delta_e \eta}(\eta - 1.0)) \delta_e + (M_{\delta_e \delta_e} + M_{\delta_e \delta_e \eta}(\eta - 1.0)) \delta_e | \delta_e | \quad (7)$$

Where  $m$  is the mass of the vehicle;  $I_{yy}$  is the moment of inertia in  $y$  direction;  $u$ ,  $w$  and  $q$  are surge velocity, heave velocity and pitch angular velocity, respectively;  $mg$  and  $\nabla \rho_w g$  are vehicle weight and buoyancy;  $\theta$  is pitch angle, and  $\delta_e$  is the angle of the stern elevator;  $x_G$ ,  $z_G$ ,  $x_B$  and  $z_B$  are gravity center and buoyancy center of the vehicle in  $x$  and  $z$  direction.

$Z_0$  and  $M_0$  are static force and moment in vertical direction and rotational pitch direction,

respectively.  $\eta$  represents the speed dependent variations of forces and moments.  $X_{0\eta}$  includes drag force and propulsion force terms. The hydrodynamic coefficients of the equation of motion are Taylor expansion terms for each component at the forward speed. The coefficients are obtained using planar motion measurement (PMM) test in test basin.

We can show that the effect of vertical velocity is negligible in the steady state according to the variation of elevator angle. In the case of VORAM AUV, let all time derivative terms in (2), (3), and (4) be zero, then we can obtain steady state forward velocity, heave velocity, and pitch angle. From this result, when the vehicle is steadily descending and ascending, we can neglect the heave velocity component for a simplified controller design in vertical plane within 10 % error bounds. On the other hand, the elevator angle must be set less than 30 degrees to avoid forward directional instability, and static offset angle in the elevator is required to keep zero pitch angle.

Generally, underwater vehicle is designed to have symmetric body and neutral buoyancy, it is reasonable to assume that the body fixed coordinate is located at the center of gravity, and the gravity force is equal to the buoyancy force of the vehicle, and  $x_B$  coincides with  $x_G$ . For AUVs shaped like VORAM, pitch motion and depth in vertical plane can be approximately described.

Considering nonlinear components of pitch motion and the coupling terms as system disturbances, the equations of motion are reduced to the following simple forms for the operating condition  $u = U$ .

$$(I_{3 \times 3} - M_q) \dot{q} = M_q q + (m z_G - \nabla \rho_w z_B) g \theta + \Delta f_q(u, w, \eta, \delta_e, M_0) \quad (8)$$

$$\dot{\theta} = q \quad (9)$$

$$\dot{Z} = -\theta + \Delta f_z(\theta, u, w) \quad (10)$$

where,  $\Delta f_q$  and  $\Delta f_z$  represent the uncertainties of the underwater vehicle.

### 3. Design of Discrete-time Sliding Mode Control

The design steps of discrete-time sliding mode controller<sup>9)</sup> are summarized as follows.

1. Construct a mathematical model for control, design a sliding surface for continuous-time system, and determine maximum error bounds.
2. Determine the equivalent control region with the use of discrete-time uncertainty bounds and design the equivalent control law. And select the control parameters  $\rho$ ,  $\phi$  and  $\phi_0$  in the control law<sup>9)</sup>.
3. Design the discrete-time sliding mode controller for inside and outside of the equivalent control region based on Theorem<sup>9)</sup>.

For the case of VORAM with  $U=1.25$  m/sec condition, the non-dimensional state equation is given as

$$\dot{x} = Ax + B\eta + f(x) \quad (11)$$

where,

$$A = \begin{bmatrix} -1.8377 & -1.2007 & 0 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} -1.3212 \\ 0 \\ 0 \end{bmatrix}$$

$$x = \{q \ \theta \ Z\}^T$$

It is desirable to control the vehicle without oscillation in transient state. Since the

eigenvalues of the vehicle are 0 and  $-0.9188 \pm j0.597$ , the sliding surface is designed to locate the poles to 0, -1.10, and -0.75. In this paper, the design of sliding surface is based on the geometric approach technique[10]. This design method requires the availability of the closed loop eigenvector  $W$ . A well known fact related to the linear feedback systems eigenvalue/eigenvector assignment question is that

$$AW - WJ = BL \quad (12)$$

where  $J$  is Jordan block matrix specified by eigenvalues and  $L$  is an arbitrary  $m \times (n-m)$  matrix chosen to provide linear combinations of the column of  $B$ , where  $n=3$  and  $m=1$ . Solving the equation (12), and from the relation,  $G = W^\perp$ , we can get the switching hyperplane matrix as follow:

$$G = [1.0 \ 1.850 \ -0.825] \quad (13)$$

where the subscript " $\perp$ " denotes perpendicular.

Since the dynamic characteristics of AUV depends on operating conditions, attitude, and environmental conditions, the system uncertainties include  $\Delta f_q$ ,  $\Delta f_z$ , and other coupled high order nonlinear terms. In this paper, it is supposed that these are mainly dependent on forward speed and those bounds are proportional to the square of speed variation about normal condition.

Discretizations for the AUV (11) are perform as  $\Phi = \exp^{A\Delta t}$  and  $\Gamma = \int_0^{\Delta t} \exp^{A\tau} B d\tau$ . The control law<sup>9)</sup> is given as

$$u(k) = -(G^T \Gamma)^{-1} G^T (\Phi_0 - I)x(k) + (G^T \Gamma)^{-1} G^T \Delta r(k+1) + \phi^T e(k) - \phi_0 s(k) \quad (14)$$

at the outside of the equivalent control region  $\delta_i$ <sup>9)</sup>. The nonlinear switching gain  $\phi_i$  is

determined with choosing the switching gain matrix  $F$  and the disturbance matrix  $D$ <sup>9)</sup>, and the linear switching gain  $\phi_0$  is selected with the substituted control parameter  $\rho = \phi_0(G^T \Gamma)$  in the range  $0 < \rho < 1$ . The criteria of selection of the switching gain matrices are given in the reference<sup>9)</sup>.

#### 4. Simulations and Experiments

Experiments are performed at KRISO towing tank. Two thin guide wires are connected to VORAM's foreside upper shell in order to prevent the AUV from running into side wall of the tank. The angle of elevator is designed to be saturated to 0.4 radian and the propeller RPM is fixed to 570, that is, the motor RPM is fixed to 2850. The forward speed is 1.25 m/sec at zero angle of attack condition.

Since the length of towing tank is 200 m, running time is limited to around 100 seconds excluding the speed-up region and stop region. Data communications<sup>11)</sup> are carried out every discrete-time interval through RF modem at 1200 bps. Data receiving task is installed after test-run to transmit the control command and AUV's motion data. Each test run is scheduled as follows:

1. Start VORAM at manual control mode.
2. When AUV's speed reaches about 1.0 m/sec, manual mode is changed to DSMC mode with keeping the instantaneous depth.
3. Send depth change command through RF modem at arbitrary instance. The target depth is fixed at 2.5 m.
4. Send return command to initial depth at second arbitrary instance.

Numerical simulations are performed with the full nonlinear equation of motion (2)~(4) and the forces (5)~(7) with the coefficients in the

reference<sup>12)</sup>. The gains and parameters of discrete-time sliding control are selected as the same values both the numerical simulations and the experiments. Fig. 2 and Fig. 3 show the depth keeping results of discrete-time sliding mode control with  $\rho = 0.3$  and 0.8, respectively, where  $\Delta t = 0.5$  second,  $F = [0.4 \ 0.4 \ 0.1]$ , and  $D = [0.1 \ 0.1 \ 0]$ . Dash line represents the desired depth, thin solid lines are experimental data, and thick solid lines show simulation results where initial data are same as the experiments.

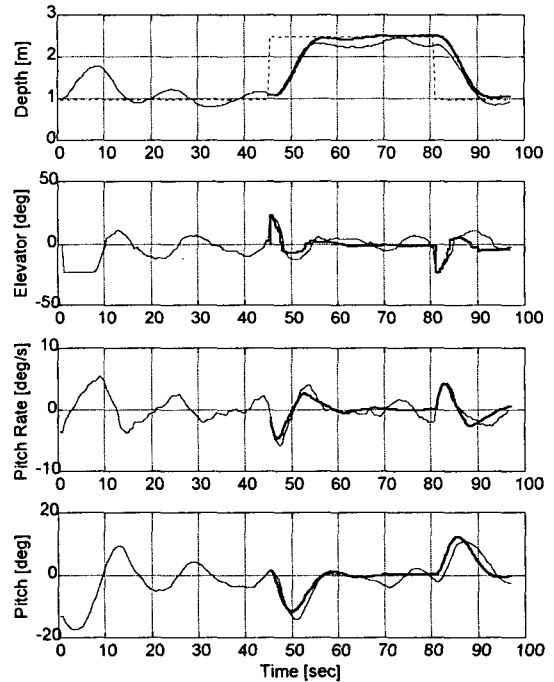


Fig. 2 Experimental and numerical results of depth keeping control with DSMC :  $\Delta t = 0.5$ ,  $\rho = 0.3$ ,  $F = [0.4 \ 0.4 \ 0.1]$ ,  $D = [0.1 \ 0.1 \ 0]$ ; solid line - exp., thick solid - sim., dash line - target depth

The experimental results in general agree with the simulation results. Fig. 2 shows better performance than Fig. 3. Thus, for the case of

small  $\Delta t$ , large  $\rho$  is desirable to keep the depth stably. In these figures, initial large oscillation is caused by transient response which is generated by manual control. This AUV is hard to maneuver with manual control near the free surface because of large inertia and nonlinear hydrodynamic characteristics. In case of depth change, transient response shows good agreement between experiments and calculations. However, steady state response shows static offset error. This must be caused by the vertical tension of the guide wires, since the AUV operates at high speed and generates drag force to the guide wires which react tension forces to the AUV. Free running test without guide wires is required to confirm this fact.

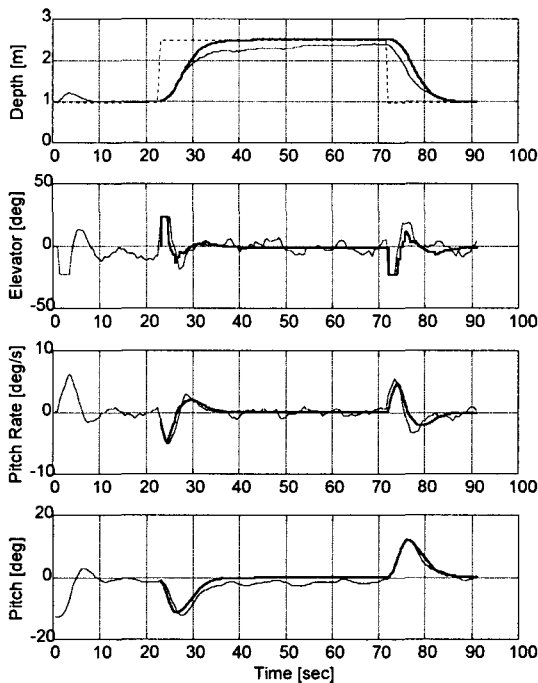


Fig. 3 Experimental and numerical results :  $\Delta t=0.5$ ,  $\rho=0.8$

Fig. 4 and Fig. 5 show the depth keeping results of DSMC with  $\rho=0.3$  and  $0.8$ ,

respectively, where  $\Delta t=1.0$  second,  $F$  and  $D$  matrix are same as previous experiments. In Fig. 4, the experimental result generally agrees with the simulation result. However, in Fig. 5, it shows oscillation and drift phenomenon, since the vehicle has initial deeper submergence and has worse initial conditions than in Fig. 4. While the sliding mode control is intrinsically stable for system uncertainties and initial conditions, this result shows unsatisfactory capability for its position keeping. When  $\Delta t$  becomes large in the range  $0 < \rho < 1$ , large  $\rho$  enlarges the equivalent control region as discussed in the reference<sup>9)</sup>. No switching force is generated inside of the equivalent region, therefore, this controller with large  $\rho$  can make AUV system unstable for large uncertainties or external disturbances.

This phenomenon is clearly exposed in case of  $\Delta t=2.0$  second. Fig. 6 and Fig. 7 show the depth keeping results of DSMC with  $\rho=0.3$  and  $0.8$ , respectively, where  $\Delta t=2.0$  second,  $F$  and  $D$  matrix are same as previous experiments. In Fig. 6 with small  $\rho$ , although the experimental results show overshoot in its depth changing performance, the controller makes the system stable quickly by switching the elevator angle. The experiment generally shows good agreement with the calculation. However, in Fig. 7 with large  $\rho$ , the experiment shows large oscillation in steady state in spite of better initial conditions, while the calculation has better performance than the result in Fig. 6.

For the case of large  $\Delta t$ , therefore, small  $\rho$  is required to make the system stable in the presence of parameter uncertainties, nonlinearities, and external disturbances. For the case of small  $\Delta t$ , large  $\rho$  is desirable to reduce overshoot in depth change and to reduce

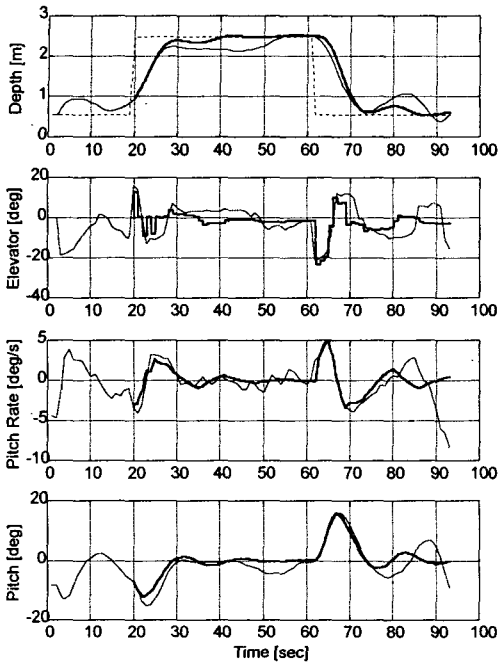


Fig. 4 Experimental and numerical results :  
 $\Delta t=1.0, \rho=0.3$

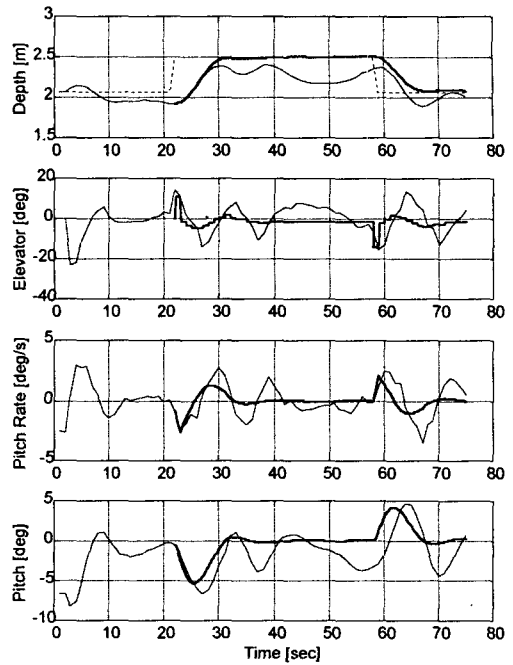


Fig. 5 Experimental and numerical results :  
 $\Delta t=1.0, \rho=0.8$

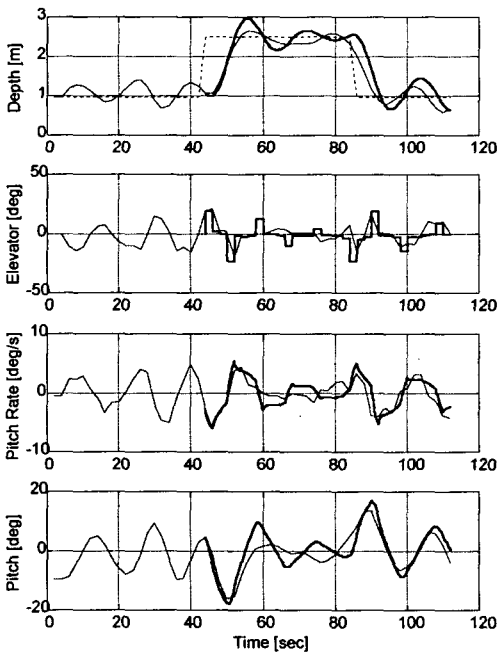


Fig. 6 Experimental and numerical results :  
 $\Delta t=2.0, \rho=0.3$

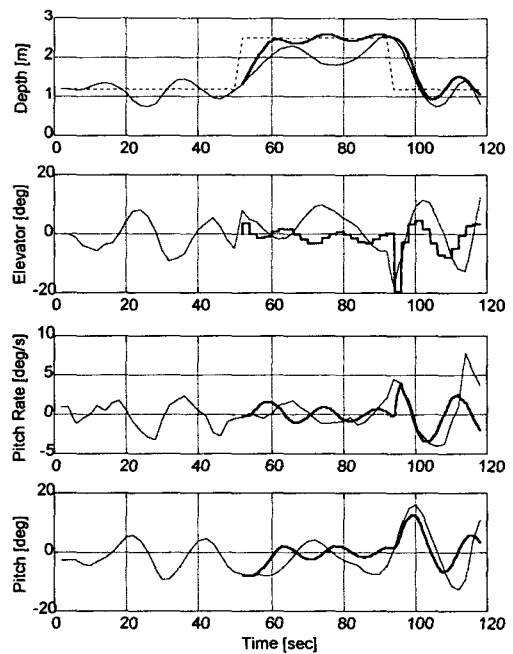


Fig. 7 Experimental and numerical results :  
 $\Delta t=2.0, \rho=0.8$

chattering phenomenon.

Through the experiment of VORAM AUV's depth keeping control, the effectiveness of the proposed discrete-time sliding mode controller is verified and the selection of design parameter is guided on the basis of experimental and numerical simulation results.

## 5. Conclusion

Numerical simulations and experiments on depth keeping controls of an AUV are carried out using the proposed discrete-time sliding mode control law. Experiments are performed in KRISO towing tank. The AUV, named VORAM developed by KRISO, is used as a model for the verification of the proposed control algorithm. As sampling interval becomes large, the proposed control law is very effective and make the system stable in the presence of system uncertainties, nonlinearities, and even external disturbances. When the AUV has large sampling interval, in the range  $0 < \rho < 1$ , it is required to have small  $\rho$  in the control law to make the system stable. When the AUV has smaller sampling interval, large  $\rho$  is desirable to reduce overshoot in depth change and to reduce chattering phenomenon.

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