

The Optimization of Truss Structures with Genetic Algorithms

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Abstract

This paper investigated the optimum design of truss structures based on Genetic Algorithms (GA's). With GA's characteristic of running side by side, the overall optimization and feasible operation, the optimum design model of truss structures was established. Elite models were used to assure that the best units of the previous generation had access to the evolution of current generation. Using of non-uniformity mutation brought the obvious mutation at earlier stage and stable mutation in the later stage; this benefited the convergence of units to the best result. In addition, to avoid GA's drawback of converging to local optimization easily, by the limit value of each variable was changed respectively and the genetic operation was performed two times, so the program could work more efficiently and obtained more precise results. Finally, by simulating evolution process of nature biology of a kind self-organize, self-adapt artificial intelligence, this paper established continuous structural optimization model for ten bars cantilever truss, and obtained satisfactory result of optimum design. This paper further explained that structural optimization is practicable with GA's, and provided the theoretic basis for the GA's optimum design of structural engineering.

Keywords : genetic algorithm; punishment function; elite models; truss structures; optimum design

Genetic Algorithms(GA's) is based on the bionics of biologic evolution, simulating Darwin's natural evolution of "natural selection and the survival of the fittest" and Mendel's law, so GA's is one search algorithm which is intelligent, adaptive self and probabilistic global optimizer¹⁾. It is mostly used to deal with optimization problem, especially the complicated and nonlinear questions which the traditional optimized method can't solved. GA's is an efficient concealed and collateral optimum search method, and pursues global optimization. From 1960s', professor HOLLAND of MICHIGAN University in America began to research the self-adaptive action of nature and artificial systems. In 1975, HOLLAND published the first

monograph of «Adaptation in Nature and Artificial Systems», systemically discussed the genetic algorithms and artificial self-adaptive system. At the same time, JONG did a lot of numerical experiments about the optimization of pure numeric function combining the mode theorem in his doctor thesis, established the work frame of genetic algorithms, found some important and instructive conclusions²⁾. In 1989, GOLDBERG published the monograph of «Genetic Algorithms in Search, Optimization and Machine Learning». In 1991, DAVIS published the «Handbook of Genetic Algorithms». In 1985, from the first International Conference of Genetic Algorithm in America and the International GA's institute coming into existence, people will convoke a meeting every two years; also establish the international node of communicating information about genetic algorithms³⁾. Genetic algorithms are more in-depth studied, widely applied in many subjects,

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and the applying area becomes more and more extensive.

This paper uses GA's collateral character, global optimization and operable character, researches the optimum design of truss structures. With GA's three genetic operators: selection, crossover and mutation, using ameliorative measure such as the elite models and non-uniformity mutation, seeking the best matching of the design parameters under the actual criterion, so the program of structural design can satisfy the aim of safety, credibility and low cost. Finally, a numerical example of the ten bars truss was presented to research the optimum design, provides the theoretic basis for the GA's optimum design of structural engineering.

1 GA's concrete process in the optimum design of structures

GA's process includes many basic operations such as: coding, making the initiatory colony, calculating of the fitness function, selection, crossover and mutation.

(1) Coding. When the design variables have many valid numerals or too many variables need solution, the binary coding has a long character bunch; it takes the computer's long time during the operations such as selection, crossover and mutation. So this paper uses the decimal coding, needs no transforming number system, also it's easy in understanding, and can save the time in genetic operations.

(2) Making the initiatory colony. Also this paper uses random creating method to establish the initiatory colony. Every decimal number in the colony uses the following formula to calculate:

$$X = (X_{\max} - X_{\min}) \cdot \text{Rand} + X_{\min} \quad (1)$$

(3) Calculation of the fitness function. The calculation of fitness and the choice of the punishment function are correlative; the punishment function has many kinds.

This paper introduces three schemes as follows4):

Method 1: JOINES and HOUCK Method

To seek the minimum problem of nonlinear programming, the evolution function can be constructed as additive format; the punishment part is formed by the variable punishment gene and the punishment of disobeying constraint.

Method 2: YOKOTA, GEN, IDA and TAGUCHI Method

The punishment part is formed according the opposite punishment coefficient of the disobeying constraint. This method is used in the maximum questions of nonlinear programming; the construction of the evolution function uses the multiplicative format.

Method 3: GEN and CHENG Method

This method can adjust the punishment proportion automatically in every generation, keep the equilibrium between the information reservation and the unfeasible punishment pressure, as to avoid the excessive punishment. This paper uses the methods.

(4) Selection. In the ready colonies, the excellent colonies are selected to add the new colony, join the next generation's evolution.

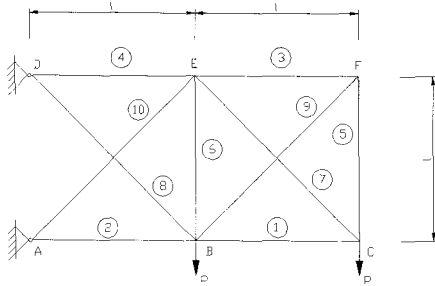
(5) Crossover. In the continuous genetic algorithm, every decimal number is seen as a genetic whole, join the crossover.

(6) Mutation. In the continuous genetic algorithm, the mutation of every decimal number (gene) is changing during the allowable area. There are two methods to do the mutation operation: the first method uses the same thought as the initiatory colony to produce the mutation individual; the second method is named non-uniformity mutation, along with the increase of the iteration times, the number will gradually reduce. In this paper non-uniformity mutation operation is used to make the obvious mutation at earlier stage and stable mutation in the later stage.

2. Numerical example and discussion

2.1 Design conditions

The design conditions of the ten bars truss are presented as in Fig.1. Where, $p = 444.92kN$, $l = 9.144m$; $\rho = 7850kg/m^3$.



(Fig.1) Ten bars truss

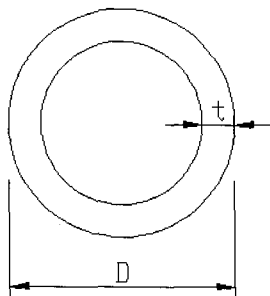
2.2. Design variables and objective function

The design variables are supposed as follows: the section area of upside and downside chord bars is x_1 ; the section area of erect bars is x_2 ; the section area of diagonal bars is x_3 .

The weight of the ten bars truss is selected as objective function, so

$$w = \rho l(4x_1 + 2x_2 + 4\sqrt{2}x_3) = 2\rho l(2x_1 + x_2 + 2\sqrt{2}x_3) \quad (2)$$

Where, $x_i = \pi(D_i - t_i)t_i$, ($i = 1, 2, 3$), D_i is the steel tube's diameter, t_i is the thickness of the steel tube's wall, such as Fig.2.



(Fig. 2) Steel tubular section of truss bar

According to the research and calculation, we get internal force of the ten bars truss as follows:

$$N_1 = -\left(\frac{x_2}{x_1^2} + \frac{\sqrt{2}x_2}{x_1x_3} + \frac{5}{2x_1} + \frac{1}{2x_2} + \frac{3\sqrt{2}}{x_3}\right) \cdot \frac{p}{\Delta}$$

$$N_2 = -\left(\frac{4x_2}{x_1^2} + \frac{4x_2}{x_3^2} + \frac{6\sqrt{2}x_2}{x_1x_3} + \frac{13}{2x_1} + \frac{1}{x_2} + \frac{5\sqrt{2}}{x_3}\right) \cdot \frac{p}{\Delta}$$

$$N_3 = \left(\frac{x_2}{x_1^2} + \frac{4x_2}{x_3^2} + \frac{3\sqrt{2}x_2}{x_1x_3} + \frac{1}{2x_1}\right) \cdot \frac{p}{\Delta}$$

$$N_4 = \left(\frac{4x_2}{x_1^2} + \frac{12x_2}{x_3^2} + \frac{10\sqrt{2}x_2}{x_1x_3} + \frac{11}{2x_1} + \frac{1}{x_2} + \frac{7\sqrt{2}}{x_3}\right) \cdot \frac{p}{\Delta}$$

$$N_5 = N - 3 = \left(\frac{x_2}{x_3^2} + \frac{4x_2}{x_3^2} + \frac{3\sqrt{2}x_2}{x_1x_3} + \frac{1}{2x_1}\right) \cdot \frac{p}{\Delta}$$

$$N_6 = \left(\frac{x_2}{x_1^2} + \frac{8x_2}{x_3^2} + \frac{5\sqrt{2}x_2}{x_1x_3} + \frac{\sqrt{2}}{x_3}\right) \cdot \frac{p}{\Delta}$$

$$N_7 = \left(\frac{\sqrt{2}x_2}{x_1^2} + \frac{2x_2}{x_1x_3} + \frac{5\sqrt{2}}{2x_1} + \frac{\sqrt{2}}{2x_2} + \frac{6}{x_3}\right) \cdot \frac{p}{\Delta}$$

$$N_8 = \left(\frac{2\sqrt{2}x_2}{x_1^2} + \frac{4x_2}{x_1x_3} + \frac{7\sqrt{2}}{2x_1} + \frac{\sqrt{2}}{2x_2} + \frac{4}{x_3}\right) \cdot \frac{p}{\Delta}$$

$$N_9 = -\left(\frac{\sqrt{2}x_2}{x_1^2} + \frac{4\sqrt{2}x_2}{x_3^2} + \frac{6x_2}{x_1x_3} + \sqrt{2}2x_1\right) \cdot \frac{p}{\Delta}$$

$$N_{10} = -\left(\frac{2\sqrt{2}x_2}{x_1^2} + \frac{8\sqrt{2}x_2}{x_3^2} + \frac{12x_2}{x_1x_3} + \frac{5\sqrt{2}}{2x_1} + \frac{8}{x_3}\right) \cdot \frac{p}{\Delta} \quad (3)$$

Where

$$\Delta = \frac{2x_2}{x_1^2} + \frac{4x_2}{x_3^2} + \frac{4\sqrt{2}x_2}{x_1x_3} + \frac{3}{x_1} + \frac{1}{2x_2} + \frac{3\sqrt{2}}{x_3}$$

2.3 Constraints

(1) Strength Constraints⁵⁾.

$$\frac{N_2}{\varphi_{2x_1}} \leq f \leq \frac{N_4}{\varphi_{4x_1}} \leq f \leq \frac{N_6}{\varphi_{6x_2}} \leq f \leq \frac{N_8}{\varphi_{8x_3}} \leq f \leq \frac{N_{10}}{\varphi_{10x_3}} \leq f \quad (4)$$

Where, f is the tensile or compression strength of steel; φ is strength reduction factor.

(2) Width thick ratio constraints of the bars.

$$\frac{D_i}{t_i} \leq 100 \quad (i = 1, 2, 3) \quad (5)$$

(3) Constraints of minimum and maximum bar size.

$$\left. \begin{aligned} D_i \geq 32, \quad t_i \geq 2.5mm \\ D_i \leq 630, \quad t_i \leq 16mm \end{aligned} \right\} \quad (i = 1, 2, 3) \quad (6)$$

(4) Constraints of deflection.

$$\frac{\delta}{l} \leq \frac{1}{400} \quad (7)$$

Where, δ is deflection of point C.

(5) Slenderness ratio constraints of the bars.

$$\left. \begin{aligned} \frac{l}{r_1} = \frac{4l}{\sqrt{D_1^2 + (D_1 - 2t_1)^2}} \leq 150 \\ \frac{l}{r_2} = \frac{4l}{\sqrt{D_2^2 + (D_2 - 2t_2)^2}} \leq 250 \\ \frac{l}{r_1} = \frac{4\sqrt{2}l}{\sqrt{D_3^2 + (D_3 - 2t_3)^2}} \leq 150 \end{aligned} \right\} \quad (8)$$

2.4 Selection of parameter

In this paper, we consider that genetic algorithm should use bigger crossover probability and lesser mutation probability in the frequent crossover operation, generally the amount of the crossover probability larger than 0.7, the mutation probability larger than 0.01 can guarantee the answer's diversity, and avoid getting into the local optimization^{4),6),7),8)}. Therefore, this numerical example is chosen the parameter as follows: the population size $\text{pop_size} = 200$, crossover probability $P_c = 0.9$, mutation probability $P_m = 0.15$, the maximum number of iterations $\text{maxgens} = 2000$. GEN and CHENG method used in control of the punishment function, the adjustment of punishment force is regulated by constraint disobeying degree $P(x)$ and the effective factor of modulus in the iterative process. It is supposed that x is a chromosome in current colony, $P(t)$, so the punishment function is created as

follows^{9,10)}:

$$\text{punishment}(x) = 5 + p(x) \times \text{modulus}$$

$$p(x) = 1 - \frac{1}{m} \sum_{i=1}^m \left(\frac{\Delta b_i(x)}{\Delta b_i^{\max}} \right)^\alpha$$

$$\text{modulus} = 4 - \frac{\text{gen}}{\text{max gens}} \times 3$$

$$\Delta b_i(x) = \max\{0, g_i(x) - b_i\}$$

$$\Delta b_i^{\max} = \max\{\varepsilon, \Delta b_i(x); x \in P(t)\} \quad (9)$$

Where, $\Delta b_i(x)$ is constraint i 's disobeying degree in x ; Δb_i^{\max} is constraint i 's maximum disobeying degree in current colony; α is the parameter to adjust the punishment sternness; ε is the small positive number to avoid getting rid of zero; gen is the current iteration number.

According to the reference⁶⁻⁸⁾, this paper, compared with classical GA's, has some improvements as follows:

(1) Using real number coding can save the time of the operation, and enhance precision of the results at the same time, so can promote the convergence availablely.

(2) The establishment of elite modal insure that excellent individual of previous generation inherit into current population evolution. If excellent individual of Cenozoic precede its parents, the individual is reserved, otherwise reserve its parents, made it get into next generation and population evolution.

(3) Using of non-uniformity mutation brought the obvious mutation at earlier stage and stable mutation in the later stage, this benefited the convergence of the best result.

(4) Adjusting punish power for punish function via level of the disobeying constraint and iteration times decided.

(5) To avoid GA's drawback of converging to local optimization easily, by the limit value of each variable was changed respectively and the genetic operation was performed two times, so the program could work more efficiently and obtained more precise results.

By improving the classical GA's, this paper presented optimum design of ten bars truss with GA's, and the results are shown as Table 1. According to the results of optimum design and linking structural demand of practical engineering, the dimensions of the bars can be chosen as follows: $D_1 = 530mm$, $D_2 = 127mm$, $D_3 = 351mm$; $t_1 = 9.0mm$, $t_2 = 4.0mm$, $t_3 = 9.0mm$. The weight of the ten bars truss is $W = 8337.9kg$.

The optimum design results of the ten bars truss shows that the change trends of fitness function average, standard deviation and excellent individual fitness value can be found by monitoring operation process of the objective function, and this will make fitness more excellent. The choice of design variables is stability on the range of criterion and it shows that optimum design converge global best excellent solution. If there are too discrepant solutions, it can be concluded that the objective function is complicated and has diversity of solution¹¹⁾. Otherwise, the operation results can be regarded as basis of modifying punishment function, and by changing deviation adjustment coefficient of the deviation criterion and maximum number of iterations, in this way should cost calculation time and engross CPU, we can achieve more precisions optimize value.

3. Conclusion

GA's can overcome weakness of traditional optimization method, provide new thoughts for the problem of many difficult global optimizations, and using GA's occupy

a evident superiority for optimum design of the truss structures. For objective function, no matter it is continuous or not, linear or nonlinear, whether or not derivative can be all solved by GA's, and also no restriction for constraints. So the constraints of stress and strain, size even frequency change, stability and so on can be all dealt with conveniently. Via continuous theory study, GA's has wide application prospects in aspect of structural optimum design.

By simulating evolution process of nature biology of a kind self-organize, self-adapt artificial intelligence, this paper established continuous structural optimization model for ten bars cantilever truss, and obtained satisfactory result of optimum design, this further explained that structural optimization is practicable with GA's. Besides, in coding of design variables, real number coding is used to improve operation speed and precision, and the classical GA's is improved by changing limit of design variable value, and using the elite models¹²⁾ and non-uniformity mutation operation. Therefore in this paper the global optimum solution can be searched for by obvious mutation at earlier stage and stable mutation in the later stage

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Table 1 Six times operation results of optimum design of ten bars cantilever truss

Times	Iterative Times	D_1 / mm	D_2 / mm	D_3 / mm	t_1 / mm	t_2 / mm	t_3 / mm	W / kg
1	1852	568.07	123.85	380.99	6.76	2.56	7.69	7225.6
2	1896	594.01	123.16	350.74	6.43	2.62	8.42	7225.9
3	1789	571.19	130.48	368.12	6.66	2.60	8.02	7225.6
4	1982	526.78	137.27	330.27	7.31	2.61	8.90	7226.3
5	1942	591.24	125.39	370.01	6.47	2.54	7.95	7226.1
6	1580	610.66	126.95	363.98	6.23	2.64	8.12	7225.6

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