

# Modeling of Dynamic Loads Due to Pedestrian Walking

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## Abstract

Walking loads are influenced by various parameters so that they need to be measured considering such parameters. Walking frequency(rate) is experimentally investigated as the most important parameter in determining the walking load expressed with dynamic load factor. This study focuses on the derivation of continuous walking load-time functions at any walking frequency ranging from 1.30Hz to 2.70Hz. Experiments were conducted to obtain time-histories of walking loads at the increment of 0.1Hz, which are decomposed into harmonic loads by the Fourier transformation. The polynomial load-time functions are proposed representing the relationship between harmonic coefficients and walking frequencies, thereby easily formulating walking load-time histories for dynamic load factor with various walking frequencies.

*Keywords : walking load, walking frequency(rate), Fourier transformation, equivalent nodal force, computational time*

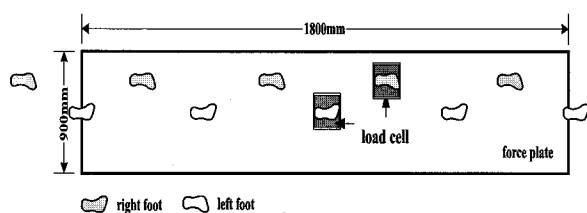
## 1. Introduction

In these days, high strength and composite materials make it possible to use lightweight structural members such as beams and slabs, which may cause excessive deflection and vibration. The design of such structural members is governed by the deflection and vibration, because the serviceability of structures has been taken into consideration more significantly than in the past. Therefore, the serviceability of structures is required to maintain the structural behaviors such as deflection and acceleration within specified values. Structures with long span and flexible floor system, such as assembly rooms and shopping malls, have low natural frequencies of floor vibrations and thus have the possibility of experiencing excessive vibrations induced by resident movements. And it is now widely accepted that walking load is one of the major sources of floor vibration disturbance. These excessive vibrations in-

duced by walking load may deteriorate structural function, cause falling down of equipments or cladding, and even make occupants uncomfortable. Hence, floor vibration is becoming a serious problem in the design of building structures. Since walking load is known as a main source inducing floor vibration, lots of works have been performed on the investigation of walking loads. Harper, Warlow and Clarke experimentally measured walking loads and analyzed their characteristics<sup>[1]</sup>. Galbraith and Barton conducted a series of laboratory experiments to find key parameters that influence walking loads<sup>[2]</sup>. Tuan and Saul defined various types of in situ movements and derived descriptive parameters from statistical and spectral studies of the load samples<sup>[3]</sup>. Rainer and Pernica presented the contribution of each harmonic to the total dynamic forces based on the measured walking and running loads<sup>[4]</sup>. Bachmann and Ammann derived the load-time functions resulting from various human motions<sup>[5]</sup>. Ebrahimipour and Sack described dynamic oc-

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cupant loads analytically using data obtained from small groups of people<sup>[6]</sup> and Ebrahimpour et. al. determined the load factors for various crowd sizes with unprompted and prompted walking<sup>[7]</sup>. Kerr and Bishop clearly defined the differences between human induced loading on a floor with that generated while ascending or descending a staircase<sup>[8]</sup>. Since walking loads are influenced by various parameters, their measurement and analysis should be associated with those parameters. The walking loads or load-time functions investigated in previous studies are based on the specific walking frequencies for in situ or walking movement. Therefore, they are not appropriate for human movement with different walking frequency(rate). It is required to obtain load-time functions for walking over a wide band of walking frequencies and also derive continuous load-time functions, which is used at any walking frequency. Walking frequencies are generally regarded as slow walking, normal walking and fast walking whose ranges are less than 1.7Hz, 2.0Hz, and 2.3Hz, respectively<sup>[9]</sup>. This study classified slow, normal, and fast walking in detail as 1.30Hz~1.65Hz, 1.60Hz~1.95Hz and 1.90Hz~2.70Hz, respectively. Experiments are conducted at every incremental step of 0.1Hz within the range to obtain load-time functions for walking. Walking loads are directly measured by using a walkway in which two load cells are equipped. Walking load-time functions for dynamic load factors are found by obtaining harmonic load components from decomposition of measured walking loads and by investigating and curve-fitting the relationship between walking frequencies and harmonic loads.



<Fig. 1> Walkway with load cells

## 2. Measurement of walking load

Walking loads with various parameters are directly measured by using a walkway equipped with load cells. As shown in Fig. 1, the size of the walkway is 5400mm by 900mm and the thickness of 24mm. The thickness of the walkway is the same as that of the load cell. The walkway provides a 2.5 meter "lead in" and a 1.8 meter "lead out" to the load cells so that pedestrians can walk comfortably before and after they make contact with the load cells. The measured walking loads are transformed as shown in Fig. 2.

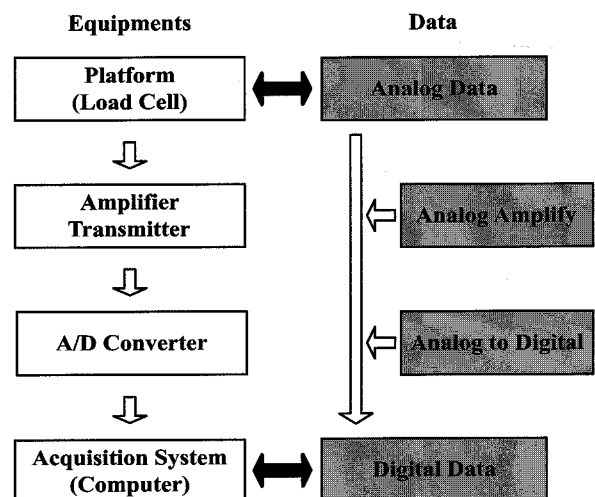


Fig. 2 Walking loads data transformation

The digitizing rate chosen for gathering load-time history data is 500Hz. Load-time histories from fourteen pedestrians, who are four females and ten males, are recorded. To overcome small subject number, we selected them with different sex and various weight. Each individual was asked to walk three times according to a specified walking frequency: 15 sets of walking frequencies are applied from 1.30Hz to 2.70Hz with increment of 0.1Hz. Therefore, as many as 672 tests were conducted to obtain the measure data for maintaining data credibility: 15 sets×14 individuals×3+additional 42 tests. Weights of the pedestrians are ranged from 50kgf to 103kgf as shown in Table 1.

<Table 1> Pedestrian weight and sex

Pedestrian	Sex	Weight (kgf)	Pedestrian	Sex	Weight (kgf)
A	Female	50	I	Male	72
B	Female	51	J	Male	74
C	Female	53	K	Male	78
D	Female	57	L	Male	84
E	Male	62	M	Male	89
F	Male	63	N	Male	90
G	Male	65	O	Male	95
H	Male	70	P	Male	103

Electronic metronome is used to keep the pedestrians walking with a specific walking frequency. As stride lengths are changed according to walking frequencies, the pedestrian steps are marked on the walkway and the distance between the two load cells is adjusted to keep the pedestrians walking with the constant stride length. The data acquisition system consists of load cells, a data transmitter, an A/D converter and a microcomputer as shown in Fig. 3.

Load cells are of slim type with the size of 300 mm by 300mm and the thickness of 25mm. Effective loading capacity of the load cell is 400kgf and maximum loading capacity is 600kgf. Transmitters are available for data transmission without a charge amplifier. And the surge absorber is a built-in transmitter to prevent transmission error induced by accident as the analog data is transmitted. Output voltage traces are converted into digital data by the A/D converter that has 200,000 digitizing rate and 12 bits resolution at maximum. In this study, walking loads are expressed as dynamic load factors (DLF) that are defined as the ratio of the dynamic load induced by walking to the pedestrian weight.

Sequence of stepping on the load cell are shown in Fig. 4. Heel strikes on the load cell and then toe-lift up.

### 3. Decomposition of measured walking load

This section begins with transforming the measured



(a) Walking on the walkway



(b) Stepping on the load cells

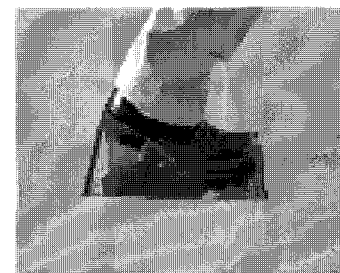
<Fig. 3> Measuring of dynamic load due to pedestrian walking



(a) Hee striking



(b) Foot contact fully



(c) Toe-lift up

<Fig. 4> Sequence of stepping on the load cell

walking load data for DLFs into a series of harmonic loads to find their frequency contents, and then derive the walking load functions expressed as simple polynomial equations that accurately describe the measured walking loads<sup>[8]</sup>.

**3.1 Decomposition of measured walking loads by Fourier transformation**

Measured walking loads for DLFs,  $F(t)$ , can be expressed as the sum of a constant DLF and a series of harmonic DLFs as follows;

$$F(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n \frac{2\pi}{T} t) + \sum_{n=1}^{\infty} b_n \sin(n \frac{2\pi}{T} t) \tag{1}$$

where the constants  $a_0$ ,  $a_n$ , and  $b_n$  are Fourier coefficients, which are obtained using the following Eqs. (2), (3) and (4), respectively.

$$a_0 = \frac{1}{T} \int_0^T F(t) dt \tag{2}$$

$$a_n = \frac{2}{T} \int_0^T F(t) \cos(n \frac{2\pi}{T} t) dt \tag{3}$$

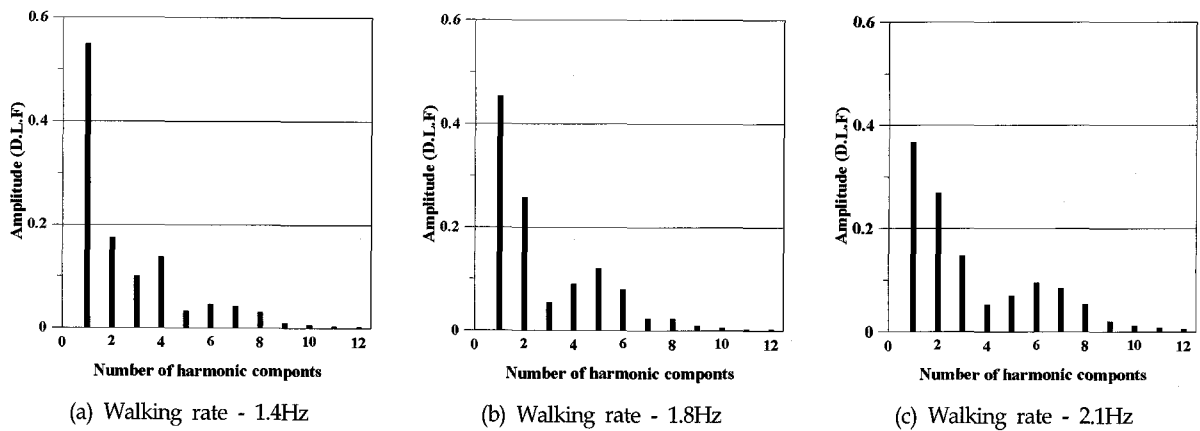
$$b_n = \frac{2}{T} \int_0^T F(t) \sin(n \frac{2\pi}{T} t) dt \tag{4}$$

where,  $n$  is the order of harmonic component and  $T$  is the walking duration.

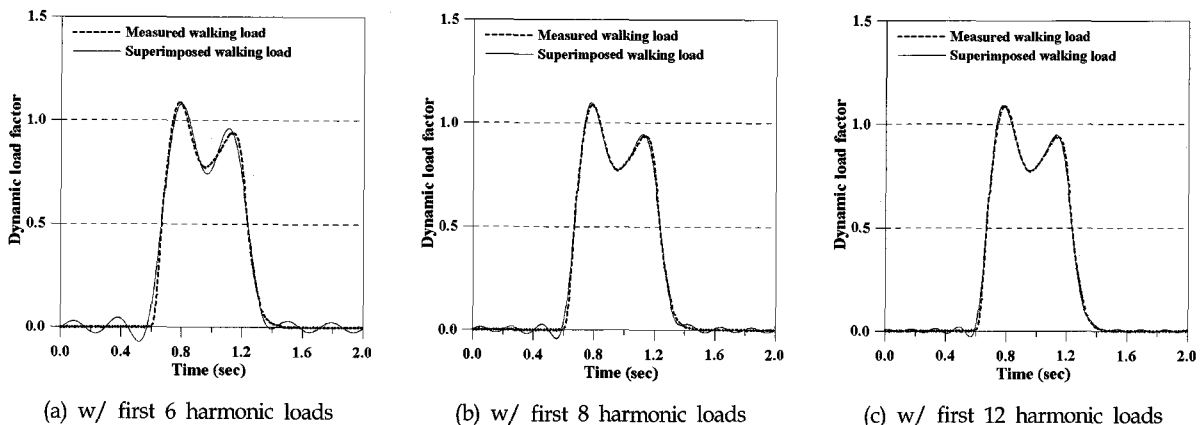
The amplitude of harmonic loads with their frequency contents are presented in Fig. 5, The amplitude over the 9th component is relatively smaller than below 8th components.

**3.2 Superposition of harmonic loads**

Measured walking loads for DLFs are compared with the approximate ones that are obtained as the



<Fig. 5> Amplitude of harmonic loads

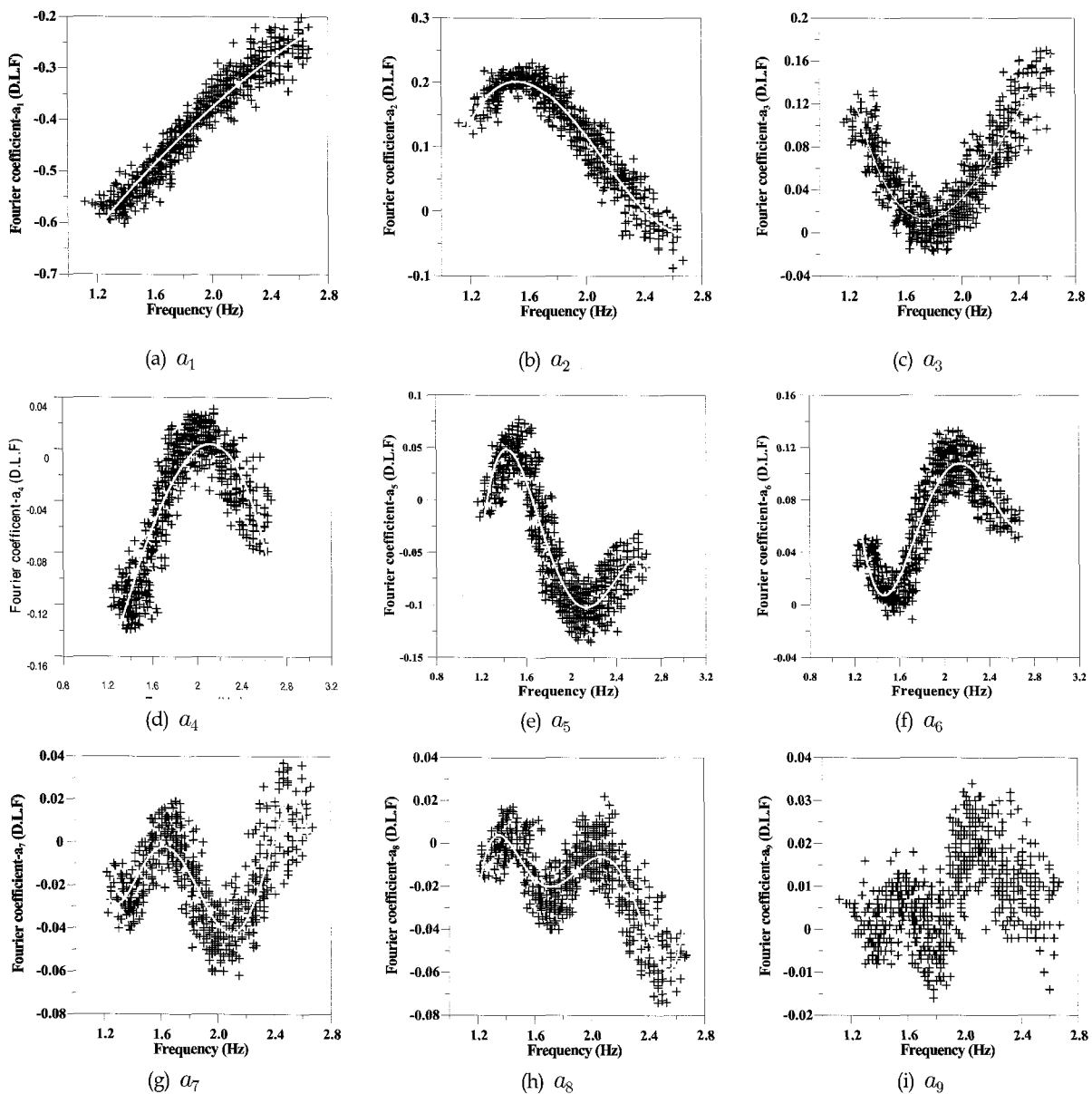


<Fig. 6> Superimposing of harmonic loads

combination of selected sequential harmonic components. Figures 6(a), 6(b) and 6(c) compares the measured walking loads for DLFs to the approximate ones obtained using the first 6, 8, and 12 harmonic components, respectively. Approximate loads become close to the measured ones, as more harmonic components are included. It is predicted that the measured walking loads can be depicted well by superposing the first 8 harmonic components based on this observation.

### 3.3 Fourier coefficients for various walking frequencies

All measured walking loads for the various walking frequencies are decomposed into the combination of harmonic loads. Then, Fourier coefficients are investigated; for example the first, second and ninth Fourier coefficients are plotted as shown in Figs. 7 and 8. Since the ninth Fourier coefficients denoted as  $a_9$  and  $b_9$  are scattered, it is difficult to define the polynomial equations of Fourier coefficients with walking frequencies by curve-fitting technology. On



<Fig. 7> Fourier coefficient( $a_n$ ) - frequency domain

the other hand, the first and second Fourier coefficients such as  $a_1, b_1, a_2$  and  $b_2$  can be curve-fitted into the corresponding polynomial functions, because they show consistent patterns.

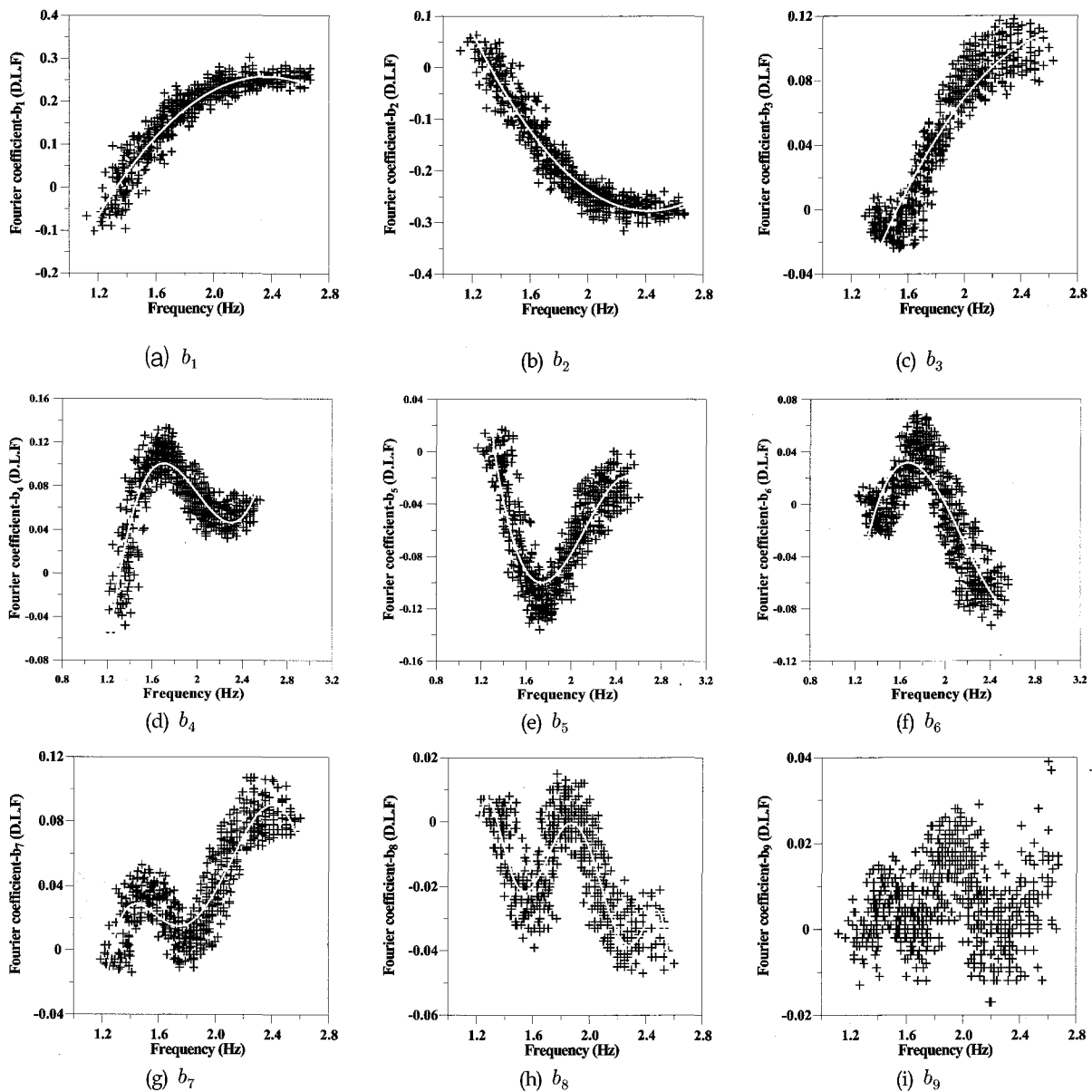
As dynamic loads made by superposing the first 8th harmonic loads are very similar with the measured walking loads, harmonic load components over the 9th component are not considered in modeling of walking loads. Table 2 and 3 presents the functional relation between the Fourier coefficients and the walking frequencies. As expected, for the lower

order of Fourier coefficients, the corresponding order of the polynomial functions is also low.

### 4. Modeling of walking loads

#### 4.1 Generating walking loads by using Fourier coefficient for DLFs

The measured walking loads for DLFs are decomposed into harmonic loads, and then the polynomial functions for DLFs of walking load depicting these harmonic loads with walking frequencies are derived. Therefore, given a pedestrian's walking



(Fig. 8) Fourier coefficient( $b_n$ ) - frequency domain

frequency, corresponding walking loads for DLFs are easily generated by using polynomial functions shown in Table 2 and Table 3.

Walking loads with various walking frequencies from 1.30Hz to 2.70Hz, generated by using the proposed walking load-time functions are compared with the measured walking loads. They are compared each other at the walking frequencies of 1.4Hz, 1.8Hz, 2.1Hz and 2.4Hz as shown in Fig. 9.

Two peaks and valley point of the generated walking loads show a little difference comparing to those of measured walking loads. But, the entire shapes are very similar each other. Components of higher order sinusoidal loads are found in generated

〈Table 2〉 Fourier coefficient  $a_n$

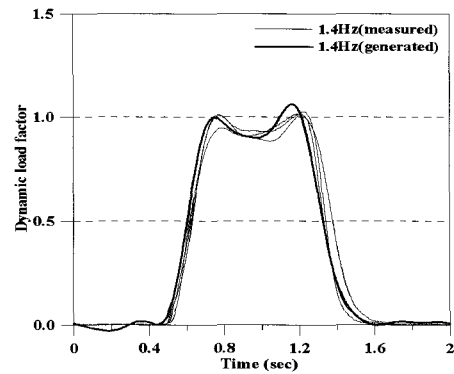
$$a_n(f) = Af^5 + Bf^4 + Cf^3 + Df^2 + Ef + F$$

	A	B	C	D	E	F
$a_0$	0.000	0.000	0.000	0.078	-0.447	0.831
$a_1$	0.000	0.000	0.000	-0.054	0.469	-1.100
$a_2$	0.000	0.000	0.283	-1.797	3.510	-1.970
$a_3$	0.000	0.000	-0.171	1.248	-2.793	1.999
$a_4$	0.000	0.000	-0.093	0.301	-0.025	-0.411
$a_5$	0.000	-0.788	6.449	-19.252	24.695	-11.461
$a_6$	0.000	0.566	-4.722	14.354	-18.770	8.936
$a_7$	0.000	-0.320	2.712	-8.374	11.154	-5.428
$a_8$	1.084	-10.399	39.197	-72.556	65.926	-23.524

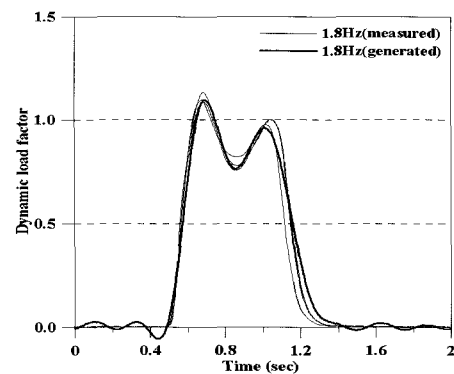
〈Table 3〉 Fourier coefficient  $b_n$

$$b_n(f) = Af^5 + Bf^4 + Cf^3 + Df^2 + Ef + F$$

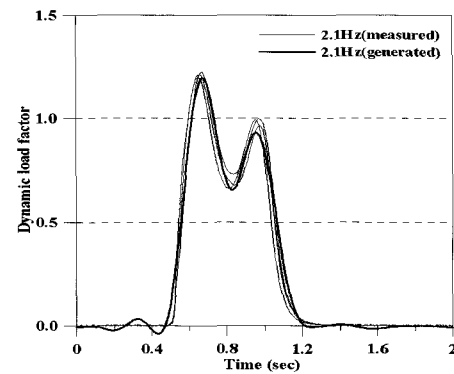
	A	B	C	D	E	F
	-	-	-	-	-	-
$b_1$	0.000	0.000	0.000	-0.237	1.121	-1.071
$b_2$	0.000	0.000	0.000	0.232	-1.122	1.081
$b_3$	0.000	0.000	0.000	-0.073	0.402	-0.444
$b_4$	0.000	0.000	0.506	-3.043	5.960	-3.725
$b_5$	0.000	0.000	-0.361	2.298	-4.718	3.052
$b_6$	0.000	0.000	0.252	-1.629	3.332	-2.164
$b_7$	0.000	-0.684	5.130	-14.114	16.923	-7.454
$b_8$	0.000	0.491	-3.756	10.571	-12.978	5.859



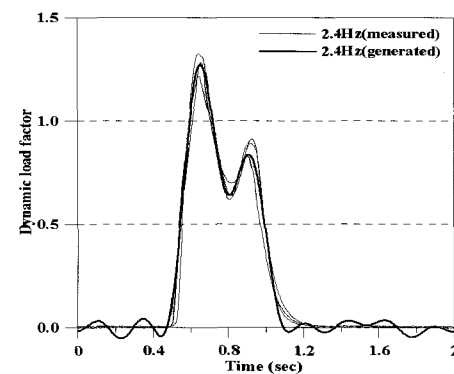
(a) Walking frequency-1.4Hz



(b) Walking frequency-1.8Hz



(c) Walking frequency-2.1Hz



(d) Walking frequency-2.4Hz

〈Fig. 9〉 Proposed walking loads and measured walking loads

walking loads unlike measured walking loads. This is because the harmonic components of the higher order than the eighth frequency are not included when generating walking loads. Despite of this, the walking loads that are generated by using the proposed functions are very similar to the measured walking loads-time histories. Therefore, the generated walking loads are applicable for vibration analysis of structures subjected to walking.

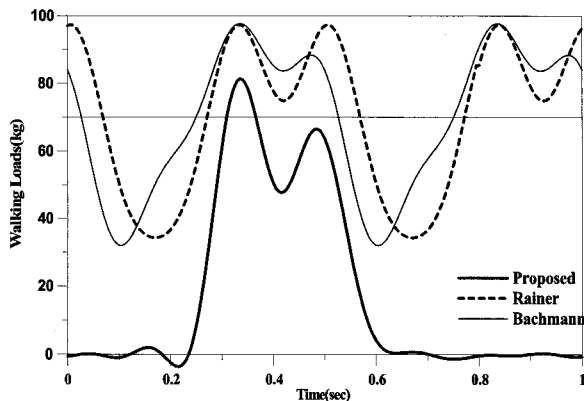
**4.2 Bachmann, Pernica & Reiner and proposed walking load model**

Walking load models that are proposed by Bachmann and Pernica & Reiner as Eq. (5) and (6) are mainly applied for analysis of floor vibration induced by walking load. These models that are limited in application of walking frequency represent walking load as harmonic loads. This proposed model that could be applied extensively in a walking frequency also represents walking load as harmonic loads.

Bachmann walking load model is given by Eq. 5

$$F(t) = G + \sum_{i=1}^3 G \alpha_i \sin(2\pi i f t - \phi_i) \quad (5)$$

Pernica & Reiner walking load model is expressed by:



(Fig. 10) Walking loads of Bachmann, Pernica & Reiner and proposed model

$$F(t) = P (1 + \sum_{i=1}^3 \alpha_i \sin(n_i 2\pi f t + \phi_i)) \quad (6)$$

Bachmann and Pernica & Reiner models result from the continuous walking. But this proposed model is produced by a unit walking. Walking loads of Bachmann, Pernica & Reiner and proposed model are represented in Fig. 10.

Walking loads of Bachmann and Pernica & Reiner models that are the result of the continuous walking are shown as periodic loads. These walking loads are larger than this proposed walking loads because of phase angle and lag of continuous walking. Proposed walking load of this paper is very similar to measured walking loads of by Ellingwood & Tallin and Kerr & Bishop.<sup>[8],[10]</sup>

**5. Conclusions**

In the present study, walking loads that are one of the sources of floor vibration are measured and their parametric studies are conducted. Parameters in experiment were left foot /right foot, walking frequency, surface condition of floor, footwear and walking posture. Results show that walking frequency and walking posture are important parameters. As this study focuses on the derivation of continuous walking load-time functions at any walking frequency ranging from 1.30Hz to 2.70Hz, experiments were conducted to obtain time-histories of walking loads at the increment of 0.1Hz. Measured walking loads are decomposed into a set of harmonic loads and they are analyzed to propose walking load-time functions expressed with simple polynomial form.

**References**

[ 1 ] Harper F.C., Warlow W.J., Clarke B.L. The Forces Applied to The Floor by The Foot in The Walking. Building Research Station. National Building Studies Research Paper No. 32. HMSO. London.



- U.K. 1961.
- [ 2] Galbraith F.W., Barton M.V. Ground Loading from Footsteps. The Journal of Acoustical Society of America 1970;48(5):1288-1292.
- [ 3] Tuan C.Y and Saul W.E. Loads Due to Spectator Movements. Journal of Structural Engineering ASCE 1985;111(2):418-4434.
- [ 4] Rainer J.H., Pernica G. Vertical Dynamic Forces from Footsteps. Noise and Vibration Section. Division of Building Research. National Research Council Canada. 1986. p.12-21.
- [ 5] Bachmann H., Ammann W. Vibrations in Structures Induced by Man and Machines. Structural Engineering Documents 3e. IABSE. 1987: 14-26.
- [ 6] Ebrahimpour A. and Sack R.L. Modeling of Dynamic Occupant Loads. Journal of Structural Engineering ASCE 1989;115(6):1476-1496.
- [ 7] Ebrahimpour, A., Hamam A. and Sack, R.L., Patten W.N. Measuring And Modeling Dynamic Loads Imposed by Moving Crowds. Journal of Structural Engineering ASCE 1996;122(12): 1468-1474.
- [ 8] Kerr S.C., Bishop N.W.M. Human Induced Loading on Flexible Staircases, Engineering Structures 2001;23(1):37-45
- [ 9] Wheeler J.E. Prediction and Control of Pedestrian Induced Vibration in Structures. Journal of Structural Engineering Division, ASCE 1982; 108(No. ST9):2045-2065
- [10] Ellingwood B, Tallin A. Structural Serviceability: Floor Vibrations. Journal of Structural Engineering Division, ASEC 1984;110(2):401-417.