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# On Shear Capacity of Reinforced Concrete Deep Beams with Rectangular Web Openings

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## Abstract

Based on an experimental study, this study provides an equation to describe the shear strength of high-strength concrete deep beams with rectangular openings and without web reinforcements. Twenty-four concrete deep beams were tested with the variables of concrete strength, size of web opening, and shear span-to-depth ratio. The proposed equation is expressed as the sum of the shear strength provided by longitudinal bars and concrete. It is illustrated that the proposed equation predicts the load-carrying capacity of the deep beams more properly than the experimental equations proposed by other researchers.

*Keywords: Deep Beams, Web Opening, High-strength Concrete, D-region, Shear Strength*

## 1. INTRODUCTION

A reinforced concrete deep beam is the structural member in D-region which is caused by a force discontinuity or a geometric discontinuity. Deep beams exhibit very complicated stress state by strut-tie action which is not applicable to the classical elastic theory of bending. The shear capacity of deep beam with rectangular web opening for the passage of utility ducts and pipes is abruptly deteriorated due to the loss of cross section, and diagonal cracks due to stress concentration at the corners of opening provide a bad influence on the durability of concrete structures. Thus, it is important to predict the actual shear capacity of deep beam to be deteriorated due to web openings and to properly make shear design such that the load-carrying capacity is improved.

However, there have been very limited researches and design codes concerned with the shear design of reinforced concrete deep beams with web openings. ACI 318-02 (2002) did not specify the shear design of deep beams with web openings, and AIJ (1988) only specifies the design of deep beams with circular web openings.

In the experimental studies to consider the shear behavior of deep beams with the shear span-to-depth ratio less than 0.3, Kong and Sharp (1977, 1978) compared the shear behavior according to the size of rectangular web openings. They reported that the effect of openings on the ultimate shear strength of deep beams depends on the extent to which the openings intercept the load paths joining the loading points and the support reaction points, and on the locations of the interception. Continuous deep beams exhibit more disturbed stress state than simply supported deep beams. Ashour and Rishi (2000) studied on the shear behavior of continuous deep beams with rectangular openings located at the interior or exterior of shear span. They found that the ultimate failure modes and shear capacity depend primarily on the positions of web openings. Based on the experimental study with test variables like the size, shape, and position of web openings, Ray [6] proposed an experimental equation to determine the shear capacity of reinforced concrete deep beams with web openings.

Although high-strength concrete is a main parameter to affect the strength of concrete deep beams, most of experimental studies related to deep beams with web openings were restricted to concrete strength below 30 MPa. This study considered the size of web opening, shear span-to-depth ratio, and concrete strength as the factors to affect the shear behavior of deep beams with web openings. Based on the experimental work to consider these parameters, this study evaluates the shear capacity and behavior of reinforced concrete deep beams, and presents an equation to predict the load-carrying capacity. Through the comparison with other analytical results, the validity of the proposed equation is investigated.

## 2. EXPERIMENT

### 2.1 Specimens

The load-carrying capacity of reinforced concrete deep beam with web openings depends on the extent to which the openings intercepts the paths of compressive strut joining the loading points and the support reaction points. This indicates that the shear capacity of the beam is affected by shear span-to-depth ratio, size of web opening, and concrete strength related directly to the strength of compressive strut. Thus, this study selected them as test variables.

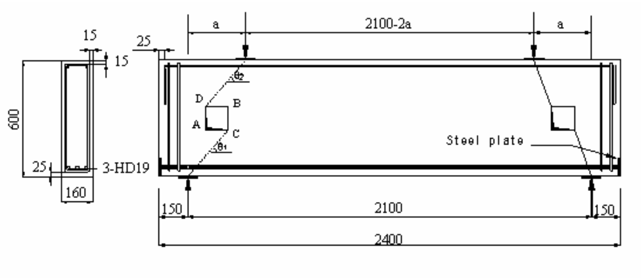


Figure 1. Section and reinforcement details of specimens (unit : mm)

Twenty-four simply supported deep beams of span, $l$, 2100 mm, overall depth, $h$, 600 mm, and width, $b$, 160 mm were tested under two-point top loading as shown by Fig. 1. Table 1 represents a summary of specimens. The longitudinal tension reinforcements consist of three 19 mm diameter high-strength deformed bars of 820 MPa yield strength to prevent the yielding of the bars before the ultimate shear failure. The longitudinal bars were perfectly anchored by 150 mm extra length to pass through the support point and welded with $160\times100\times10$ mm steel plate at both ends. Concrete cover of 30 mm thickness was adopted to prevent the splitting failure.

The shear span-to-depth ratios of the beams ranged from 0.5 to 1.5 and the size of web openings $(m_1 a\times m_2 h)$ was taken in the range $(0.25a\sim0.65a)\times(0.1h\sim0.2h)$. Compressive strength of concrete was designed within 24 ~ 82 MPa and was determined by testing a number of $100\times200$ mm cylinders cast with the reinforced concrete deep beams. Silica fume and fly ash of 12.5% per unit cement weight were added for manufacturing the high-strength concrete.

Existing researches reported that deep beams with web openings exhibit the critical load-carrying capacity when the center of web opening is located along the load path joining the loading point and the support reaction point. Accordingly, the web openings located at the critical positions for this study, and evaluated their shear capacity and behavior.

### 2.2 Testing

The beams were tested under two-point loading as shown by Fig. 2. Crack width was measured by six crack gages bonded on the expected diagonal crack surfaces. The bearing steel plates $160\times100\times20$ mm and rollers at the loading and supporting points were provided for preventing the bearing failure. Linear variable displacement transducers (LVDTs) were installed to measure the deflections at mid-span and the loading points. The test progress was monitored on a computer screen, and all load and deformation data were captured and stored in a diskette via a data logger. A schematic drawing of the test setup is shown in Fig. 2.

Table 1. Summary of specimens

| Specimens | $f'_c$ | $m_1 a$ | $m_2 h$ | $k_1 a$ | $k_2 h$ | $A_0/A_{sh}$ |
|---|---|---|---|---|---|---|
| L-5N* | 31.3 | - | - | - | - | 0.0 |
| L-5F3 | 23.5 | 150 | 180 | 75 | 210 | 0.15 |
| H-5N* | 52.9 | - | - | - | - | 0.0 |
| H-5F1 | | 150 | 60 | 75 | 270 | 0.05 |
| H-5F2 | | | 120 | | 240 | 0.1 |
| H-5F3 | | | 180 | | 210 | 0.15 |
| H-5T3 | | 75 | | 112.5 | | 0.075 |
| H-5S3 | | 195 | | 52.5 | | 0.195 |
| UH-5N* | 80.4 | - | - | - | - | 0.0 |
| UH-5F1 | | 150 | 60 | 75 | 270 | 0.05 |
| UH-5F2 | | | 120 | | 240 | 0.1 |
| UH-5F3 | | | 180 | | 210 | 0.15 |
| UH-5T3 | | 75 | | 112.5 | | 0.075 |
| UH-5S3 | | 195 | | 52.5 | | 0.195 |
| UH-7N* | | - | - | - | - | 0.0 |
| UH-7F3 | | 210 | 180 | 105 | 210 | 0.15 |
| UH-10N* | | - | - | - | - | 0.0 |
| UH-10F1 | | 300 | 60 | 150 | 270 | 0.05 |
| UH-10F2 | | | 120 | | 240 | 0.1 |
| UH-10F3 | | | 180 | | 210 | 0.15 |
| UH-10T3 | | 150 | | 225 | | 0.075 |
| UH-10S3 | | 390 | | 105 | | 0.195 |
| UH-15N* | | - | - | - | - | 0.0 |
| UH-15F3 | | 450 | 180 | 225 | 210 | 0.15 |

L-5F3 - ratio of opening depth($m_2$): 1(=0.1), 2(=0.2), 3(=0.3); ratio of opening width($m_1$): F(=0.5), T(=0.25), S(=0.65); shear span-to-depth ratio: 5(=0.5), 7(=0.7), 10(=1.0),15(=1.5):L(=23.5MPa), H(=52.9MPa), UH(=80.4 MPa)

* Letter N indicates the beams without web openings.

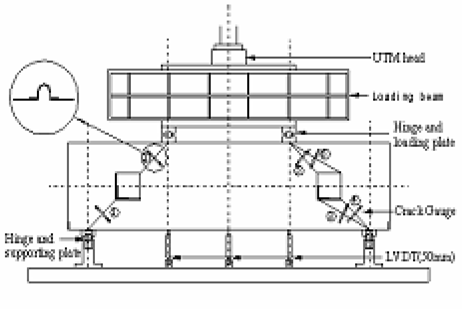


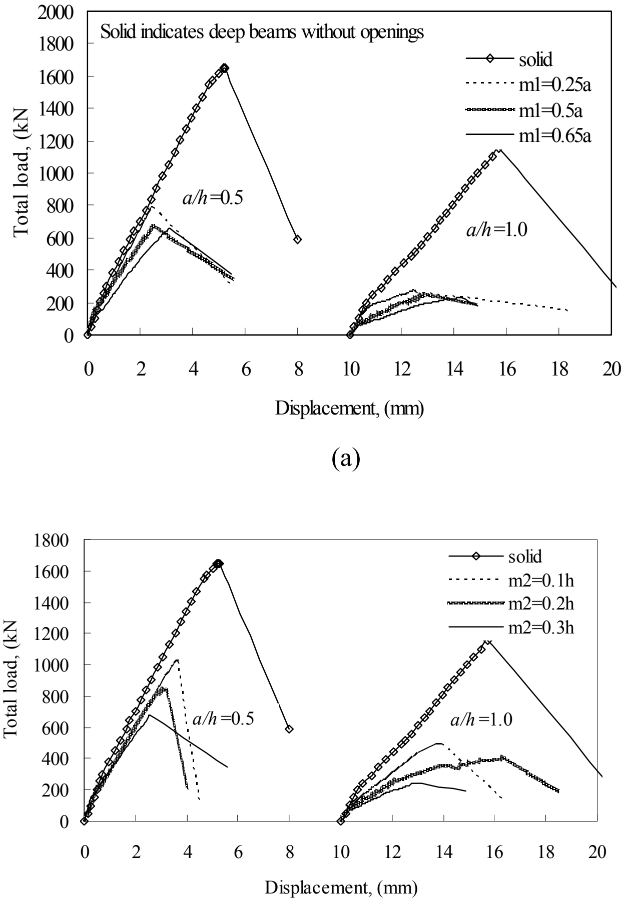
Figure 2. Test setup for concrete beams (unit : mm)

## 3. TEST RESULTS

### 3.1 Load-deflection

The maximum deflection occurred at different locations according to shear span-to-depth ratio. The maximum deflection of the beams of the shear span-to-depth ratios 0.5 and 1.0 occurred at the mid-span and the loading point near the opening, respectively. The location that the maximum deflection occurs tended to gradually move to the mid-span as the distance between two loading points decreases.

Figure 3 represents the load-deflection relation according to the size of web openings at the position that the maximum deflection is measured. It is shown that the maximum load decreases with the increase in shear span-to-depth ratio or size of openings, and the size of opening largely affects the mechanical characteristics of deep beams. At the concrete beams of shear span-to-depth ratio 0.5, the size of opening did not affect the initial rigidity, and the rigidity even after the appearance of an initial diagonal crack was not abruptly de-teriorated. However, at the concrete deep beams of shear span-to-depth ratio 1.0, the initial rigidity decreased as the size of opening increases, and the deflection abruptly increased after the occurrence of the diagonal cracks joining the loading points and the corners A or B of the opening shown in Fig. 1. It can be interpreted that the difference of crack patterns is because the load to be transmitted by the compressive strut with the increase in shear span-to-depth ratio is more widely dispersed out of the compressive strut.



(a)

(b)

Figure 3. Load-deflection curves; (a) the variations according to opening width, (b) the variations according to opening height

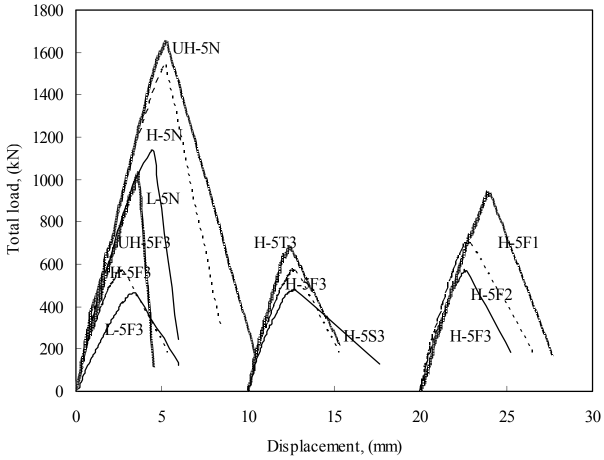


Figure 4. Load-deflection curves according to concrete strength

Figure 4 represents the load-deflection relation according to concrete strength. It can be observed that the maximum load-carrying capacity increases with the increase in concrete strength regardless of the openings. However, the rigidity of the high-strength concrete deep beams after the maximum load abruptly decreased due to more brittle property with the increase in concrete strength.

Table 2 represents the load-carrying capacity at the occurrence of diagonal crack and the ultimate shear capacity. The initial flexural crack and diagonal crack at the beams without web openings appeared in the range 15~25 % and 40~50 % of the ultimate load, respectively. The crack of the beams with web opening initially appeared at the corners A or B of the opening, and was developed to the loading and support point. In the range 30~60% of the ultimate load, a number of vertical cracks occurred at the lower region of the opening. A number of vertical cracks in the upper region of the opening occurred, and they were developed to the corners C or D of the opening as the load increases. The flexural cracks at mid-span of the beams were rarely found.

Due to the stress concentration at the corners of the opening, a number of cracks at the upper and lower regions of the opening of the beams of shear span-to-depth ratio 0.5 were found. The primary diagonal cracks joining the corners C or D of the opening and the loading points occurred in the range of 80~100 % of the ultimate load, and the beams ultimately failed after the appearance of the primary diagonal cracks. It was observed that the load is gradually concentrated on the corners of the openings with the decrease in shear span-to-depth ratio, and the beams ultimately failed with the appearance of the primary diagonal cracks.

The beams of shear span-to-depth ratio 1.0 failed along the initial diagonal crack surface. And the beams with large size $m_1 a$ failed along the diagonal crack surface joining the corner C of the opening and the support point. Also, it is expected that the difference of the upper and lower crack patterns of the opening comes from the effects to strengthen the shear span by longitudinal tension bars as tension ties.

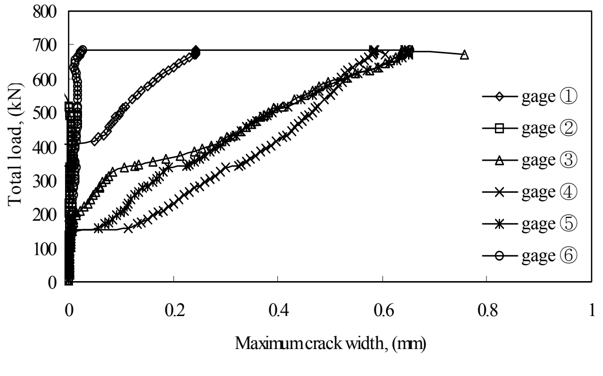
The loads at the appearance of maximum crack widths 0.3 mm and 0.4 mm were in the range of 40～60 % and 60～80 % of the ultimate loads, respectively. It was observed that the crack width does not depend on the size of the openings but the shear span-to-depth ratio.

Figure 5 exhibits the relation of the load and crack width according to shear span-to-depth ratio. In the figure, the gage number corresponds to the location of crack gage represented in Fig. 2. As the crack width gradually increases to 0.5 mm, it is observed that the load constantly increases without the deterioration of the rigidity.

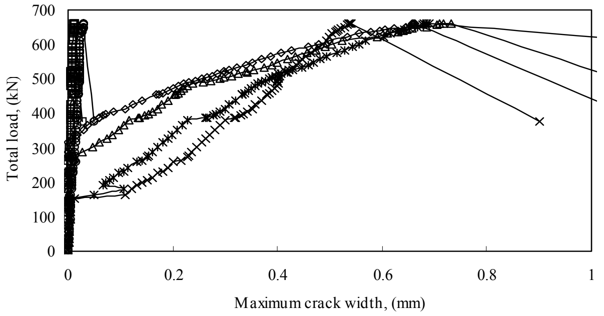
The maximum crack width increased with the increase in shear span-to-depth ratio. The failure of the beams of shear span-to-depth ratio 1.5 accompanied with the abrupt increase in crack width after the initial occurrence of diagonal crack. It is explained that this is because the load to be transmitted along the compressive strut is dispersed out of the width of the strut to determine the load-carrying capacity with the increase in shear span-to-depth ratio.

### 3.2 Effects of test variables

Figure 6 shows the shear strength at the initial crack and the ultimate shear strength according to shear span-to-depth ratio. The beams without web opening exhibited higher ultimate load than the beams with web opening $0.5a \times 0.3h$. The ultimate shear strength decreased with the increase in shear span-to-depth ratio.



(a)



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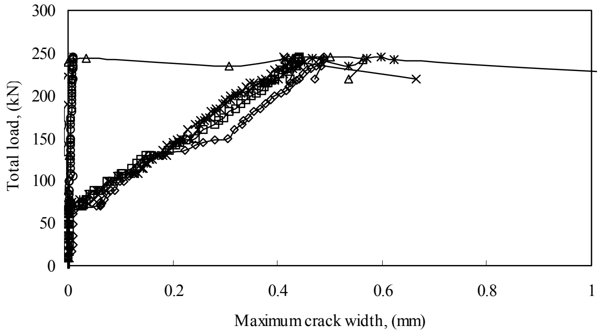
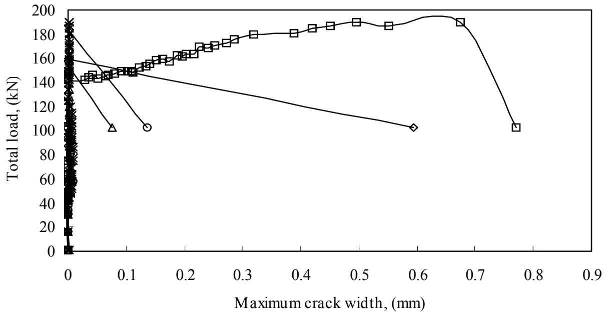


(c)



(d)

Figure 5. Load-crack width curves; (a) UH-5F3, (b) UH-7F3, (c) UH-10F3, (d) UH-15F3

As the shear strength decreases with the increase in shear span-to-depth ratio, it can be observed that the opening leads to more strength-deterioration rate with the increase in shear span-to-depth ratio as indicated in Fig. 6.

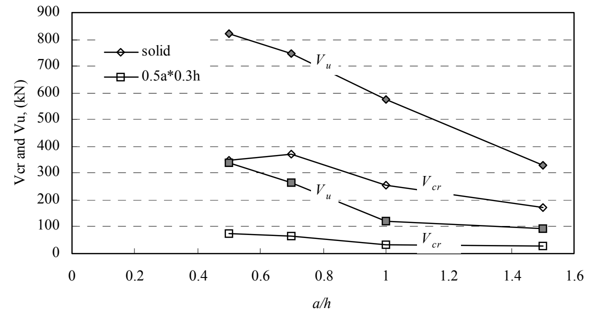


Figure 6. Shear strength according to shear span-to-depth ratio

### 3.3 A proposed method of analysis

Based on the existing experimental results and this experimental study, it was observed that the deterioration of the shear strength of deep beams with rectangular web opening is caused by the following reasons.

1) As the shear span-to-depth ratio increases, the load-carrying capacity of the beams adversely decreased. This phenomenon can be explained as the load to be transmitted along the compressive strut is widely dispersed out of the strut with the increase in shear span-to-depth ratio.

2) It can be recognized that the load-carrying capacity with the large opening size decreased because the load to be transmitted by compressive strut is more widely dispersed out of the width of the resisting strut and the stress is concentrated on the corners.

3) From these observations, it is observed that the load-carrying capacity can be improved by properly dispersing the load transmitted along the strut to minimize the stress concentration at the corners. To this aim, the shear reinforcements should be installed in the shear span region. Without the web reinforcements, the longitudinal bars play a role as an alternative to strengthen the shear span as well as the web openings.

It is necessary to quantitatively estimate the degree of the strength reduction before the design to strengthen web openings. This study evaluated the shear strength provided by concrete and longitudinal bars. The reinforcement effects by the longitudinal bars were approved by the following observations.

1) The ultimate failure of reinforced concrete deep beams was originated from the region between the opening and the bearing block region. With the diagonal cracks, the portion of the beam outside the diagonal cracks has tendency to rotate. These two observations indicate that the load-carrying capacity of the beams is enhanced by globally strengthening the shear span region including web openings.

2) The diagonal crack patterns of the upper and lower regions of the openings at the beams of shear span-to-depth ratio 1.0 are not symmetric.

It is also understood that the phenomenon appears the reinforcement effects of shear span by the longitudinal bars.

As indicated in the ACI 318-99 to specify the design guide for reinforced concrete deep beams, the nominal shear strength $v_n$ is computed from the nominal shear strength provided by concrete $v_c$, and shear reinforcement $v_s$. In this study, the longitudinal bars were also

Table 2. Experimental results and comparison

| Specimens | Experimental values | | Proposed values $V_u$ (KN) | | | $V_{\exp}/V_{pr0}$ | | |
|---|---|---|---|---|---|---|---|---|
| | $V_{cr}$ (KN) | $V_u$ (KN) | Kong | Ray | Eq.(1) | Kong | Ray | Eq.(1) |
| L-5N | 271.6 | 535.5 | | | | | - | |
| L-5F3 | 75.5 | 233.3 | 253.9 | 214.7 | 274.6 | 1.09 | 0.92 | 0.85 |
| H-5N | 294.1 | 770.6 | | | | | - | |
| H-5F1 | 124.5 | 466.7 | 378.4 | 472.6 | 447.4 | 0.81 | 1.01 | 1.04 |
| H-5F2 | 89.2 | 348.0 | 336.2 | 462.8 | 382.2 | 0.97 | 1.33 | 0.91 |
| H-5F3 | 71.1 | 288.2 | 291.2 | 447.1 | 301.7 | 1.01 | 1.55 | 0.96 |
| H-5T3 | 93.1 | 336.3 | 331.4 | 447.1 | 320.1 | 0.98 | 1.33 | 1.05 |
| H-5S3 | 45.1 | 236.3 | 269.6 | 447.1 | 290.6 | 1.14 | 1.90 | 0.81 |
| UH-5N | 34.8 | 823.5 | | | | | - | |
| UH-5F1 | 134.3 | 514.7 | 400.0 | 411.8 | 471.6 | 0.78 | 0.80 | 1.09 |
| UH-5F2 | 100.0 | 420.0 | 358.8 | 366.7 | 403.7 | 0.85 | 0.87 | 1.04 |
| UH-5F3 | 74.5 | 339.2 | 315.7 | 319.6 | 320.5 | 0.93 | 0.94 | 1.06 |
| UH-5T3 | 91.2 | 395.1 | 342.2 | 321.6 | 338.9 | 0.87 | 0.81 | 1.17 |
| UH-5S3 | 77.5 | 331.4 | 300.0 | 319.6 | 309.5 | 0.90 | 0.96 | 1.07 |
| UH-7N | 370.0 | 746.1 | | | | | | |
| UH-7F3 | 64.7 | 263.7 | 229.4 | 293.1 | | 0.87 | 1.11 | |
| UH-10N | 254.9 | 573.5 | | | | | | |
| UH-10F1 | 80.4 | 245.1 | 219.6 | 330.4 | 207.2 | 0.90 | 1.35 | 1.18 |
| UH-10F2 | 33.3 | 199.0 | 177.5 | 293.1 | 177.7 | 0.89 | 1.48 | 1.12 |
| UH-10F3 | 32.4 | 122.6 | 135.3 | 255.9 | 141.9 | 1.10 | 2.09 | 0.86 |
| UH-10T3 | 33.3 | 135.3 | 174.5 | 257.8 | 160.3 | 1.29 | 1.91 | 0.84 |
| UH-10S3 | 34.3 | 113.7 | 114.7 | 255.9 | 130.8 | 1.01 | 2.26 | 0.87 |
| UH-15N | 173.5 | 328.4 | | | | | | |
| UH-15F3 | 29.4 | 95.1 | 26.3 | 192.2 | | 0.28 | 2.03 | |
| Average | | | | | | 0.96 | 1.33 | 1.00 |
| Standard deviation | | | | | | 0.13 | 0.47 | 0.12 |

considered as web reinforcements to strengthen the web openings.

It was assumed that the longitudinal bars yielded and the opening is located at the critical position as prerequisites to obtain an analytical equation to describe the shear strength of the beams.

The ultimate shear strength of reinforced concrete deep beams is calculated by

$$v_n = v_c + v_s \qquad (1)$$

where $v_c$ and $v_s$ denote the shear strength provided by concrete and web reinforcements, respectively. Utilizing the experimental data and nonlinear multiple regres sion method, the shear strength provided by concrete excluding the opening region at reinforced concrete deep beams with web openings can be established as

$$v_c = \frac{1}{6}(f_c)^{0.63}\, bh\, (1-m2)e^{-[(a/h)-0.5]^{0.5}},\ a/h \geq 0.5 \qquad (2)$$

where the notation is as given at Appendix and the exponential term denotes the effects of the shear span-to-depth ratio. Depending on this experimental study, it was not easy to evaluate the effects of the shear span-to-depth ratio because of a few specimens. Equation (2) indicates that the strength provided by concrete depends on the concrete strength, the opening size $m_2\ h$, and the shear span-to-depth ratio.

The rest strength of the ultimate strength, $v_s$, is provided by the web reinforcements including the longitudinal bars. Utilizing the experimental data and nonlinear multiple regression method, the strength $v_s$ was derived as

$$v_s = 0.072\, A_s f_y (m_1 m_2)^{-0.58} e^{-[(a/h)-0.5]^{0.5}},\ a/h \geq 0.5 \qquad (3)$$

where the exponential term also denotes the effects of the shear span-to-depth ratio.

Inserting equation (2) and (3) into equation (1), the nominal strength of the deep beams with web opening can be calculated. Table 2 exhibits the comparison of the experimental results, the analytical results by Ray and Kong, and equation (1). As shown by this comparison, it can be certified that the proposed equation (1) predicts the ultimate shear strength more properly than the other proposed equations. However, it is observed that the shear strength of the beams of $a/h = 1.0$ is dispersed to the similar degree as Kong's results. It is evaluated that the forces along the strut to join the loading point and the support reaction are more widely dispersed out of the strut with the increase in shear span-to-depth ratio. In this viewpoint, it is necessary to investigate the effects of the shear span-to-depth ratio through a lot of data and experiments.

## 4. CONCLUSIONS

The experimental results to be tested with the variables of concrete strength, size of web opening, and shear span-to-depth ratio are summarized as follows:

1) The test variables considered in this study affect the shear strength of deep beams with web openings.

2) An experimental equation to determine the shear capacity of deep beams with web openings was provided and its validity was illustrated by the comparison of other analytical results.

3) The opening yields more strength-deterioration rate with the increase in shear span-to-depth ratio. The effects on the shear span-to-depth ratio shall be evaluated in the future work.

## ACKNOWLEDGEMENT

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## APPENDIX

$a$ : shear span length between the loading point and the support point(mm)
$d$ : effective depth of deep beam(mm)
$A_s$ : area of longitudinal bars(mm$^2$)
$b$ : width of beam section(mm)
$f_c$ : compressive strength of concrete cylinder (MPa)
$f_y$ : yield strength of a longitudinal bar (MPa)
$h$ : overall section depth
$k_1$, $k_2$ : coefficients to define the position of an opening
$l$ : span length between center-to-center of both supports
$m_1$, $m_2$ : coefficients to define the size of an opening
$v_c$ : shear strength provided by concrete (KN)
$v_n$ : ultimate shear strength (KN)
$v_s$ :shear strength provided by web reinforcements (KN)

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