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Comparisons of Index Numbers: An Application to Sawmills and Planing Mills Industry of U.S.

SoEun Ahn*

Research Fellow, Korea Environment Institute,
613-2 Bulgwang-dong, Eunpyeong-gu, Seoul 122-706, Korea

Abstract : The purpose of this paper is to investigate index numbers by conducting various comparisons among the widely used index formulas. The comparison is considered in three ways; 1) divergences in the magnitudes of index numbers due to the use of different formulas (Laspeyres, Paasche, Fisher, and Tornqvist); 2) the effect of selection of base year (fixed-year base vs. chain-type); 3) the degree of approximation of indirect to direct quantity index. The empirical application is to sawmills and planing mills industry of U.S. using a national time series data covering years of 1948-2000. The results show that the differences between Laspeyres and Paasche index can be substantial in some cases while the difference between Fisher and Tornqvist index is minimal. We also confirm that the selection of base year can cause significant divergences, especially when the variables undergo rapid price or quantity changes over time. We find that indirect quantity index approximates direct quantity index reasonably well in U.S. sawmill industry.

Key words **:** *Laspeyres index, Paasche index, Fisher index, Tornqvist index, chain-type index*

Introduction

Index numbers have been common tools to measure the changes of various economic variables, thus, have a well-documented history in the literature. Conceptually index numbers can be defined as a real number that gauges changes in a set of related variables from a reference period (Coellie *et al.*, 1998). Various index formulas have been developed over the years, and the properties of index numbers and the relationship between index numbers and economic theory have been extensively studied. The most popular index number formulas include Laspeyres, Paasche, Fisher, and Tornqvist formulas.

The purpose of this paper is to examine index numbers by conducting various comparisons among the widely used index formulas. First, we compare the divergences in the magnitudes of index numbers derived from different formulas. We calculate index numbers using Laspeyres, Paasche, Fisher, and Tornqvist formulas with the same data and investigate if there are significant differences among them. Theoretically, Fisher and Tornqvist index numbers are supposed to be very similar in magnitudes since they are both superlative, thus, exact to flexible aggregator functional forms. However, we expect some discrepancies between Laspeyres and Paasche index numbers because they depict two extremes in constructing their weights in formulas.

Second, we compare direct and indirect quantity index numbers. The "direct" quantity index is computed with observed actual quantity levels while "indirect" quantity index is generated by deflating the values (price times quantity) with the corresponding price index where actual quantity data are not available. Considering, in practice, that the most available form of data are values not actual quantity levels, it is important to guarantee that indirect index closely follow direct index numbers. We construct both direct and indirect quantity index numbers using the data for lumber output and examine how closely the indirect quantity index approximates the direct quantity index.

Lastly, we compare "fixed-year base" and "chain-type" index numbers. Fixed-year base index numbers use a particular year as a reference year. Chain-type index numbers first calculate annual changes where index of current year t is calculated based on previous year t-1 as a reference year, and then the annual changes are multiplied to measure the changes over a given period. Chain-type index numbers are preferred in many

*Corresponding author
E-mail: seahn@kei.re.kr

cases such as productivity studies since they measure changes over consecutive years, mitigating the influence of fixing a particular year as a base throughout the entire analysis period.

Our empirical application is to sawmills and planing mills industry of U.S. (1987 Standard Industry Classification: SIC242) using a national time series data covering years of 1948-2000. The data include a single output, lumber, and four inputs: labor, wood, energy, and capital. Next section lays out recipes to construct the most widely used price and quantity index numbers and discusses their properties. Empirical application of index numbers and results are presented in the following sections.

Index Number Formulas and Theory

1. Price index formulas

We begin with price index numbers, and two important formulas, Laspeyres and Paasche index, are defined as below:

$$P^L(\mathbf{p}_s, \mathbf{p}_t, \mathbf{x}_s, \mathbf{x}_t) \equiv \mathbf{p}_t \cdot \mathbf{x}_s / \mathbf{p}_s \cdot \mathbf{x}_s \tag{1}$$

$$P^P(\mathbf{p}_s, \mathbf{p}_t, \mathbf{x}_s, \mathbf{x}_t) \equiv \mathbf{p}_t \cdot \mathbf{x}_t / \mathbf{p}_s \cdot \mathbf{x}_t \tag{2}$$

where $\mathbf{p}_s$ and $\mathbf{p}_t$ are vectors of prices in reference period s, and current period t, respectively. $\mathbf{x}_s$ and $\mathbf{x}_t$ are corresponding vectors of quantity. $\mathbf{p} \cdot \mathbf{x}$ is a inner product of price and quantity vectors. It is clear that Laspeyres and Paasche price index numbers, in some way, represent extremes in constructing the weights in formulas. Laspeyres price index (1) emphasizes on the quantities of base period, and Paasche index (2) on the quantities of current period. Laspeyres and Paasche index numbers are important because theoretically they provide the upper and lower bounds of true index numbers (Coelli *et al.*, 1998). A natural alternative is a combination of these two index numbers resulting Fisher price index, which is a geometric mean of Laspeyres and Paasche index:

$$P^F \equiv \left[\frac{\mathbf{p}_t \cdot \mathbf{x}_s}{\mathbf{p}_s \cdot \mathbf{x}_s} * \frac{\mathbf{p}_t \cdot \mathbf{x}_t}{\mathbf{p}_s \cdot \mathbf{x}_t}\right]^{1/2} \tag{3}$$

Another important price index is Tornqvist formula, which is defined as a weighted geometric mean of price relatives where weights are simple mean of the value shares of i ($i = 1, \ldots, n$) commodity in a group at base period s and current period t:

$$P^T \equiv \prod_{i=1}^{n} \left[\frac{p_{it}}{p_{is}}\right]^{\frac{w_{is}+w_{it}}{2}} \tag{4}$$

2. Direct and indirect quantity index formulas

The quantity index numbers of Laspeyres, Paasche, Fisher, and Tornqvist can be defined as below by simply interchanging price and quantities in (1) to (4):

$$Q^L(\mathbf{p}_s, \mathbf{p}_t, \mathbf{x}_s, \mathbf{x}_t) \equiv \mathbf{p}_s \cdot \mathbf{x}_t / \mathbf{p}_s \cdot \mathbf{x}_s \tag{5}$$

$$Q^P(\mathbf{p}_s, \mathbf{p}_t, \mathbf{x}_s, \mathbf{x}_t) \equiv \mathbf{p}_t \cdot \mathbf{x}_t / \mathbf{p}_t \cdot \mathbf{x}_s \tag{6}$$

$$Q^F \equiv \left[\frac{\mathbf{p}_s \cdot \mathbf{x}_t}{\mathbf{p}_s \cdot \mathbf{x}_s} * \frac{\mathbf{p}_t \cdot \mathbf{x}_t}{\mathbf{p}_t \cdot \mathbf{x}_s}\right]^{1/2} \tag{7}$$

$$Q^T \equiv \prod_{i=1}^{n} \left[\frac{x_{it}}{x_{is}}\right]^{\frac{w_{is}+w_{it}}{2}} \tag{8}$$

When actual quantity data are available, Laspeyres, Paasche, Fisher, and Tornqvist index numbers can be created with (5)-(8), respectively. In practice, however, the most available data are the values of output or inputs (quantity times price; e.g. receipts of output or expenditures of input), not the distinct series of quantity levels. The task now is to decompose changes in values between two periods into the effect of price and quantity changes. A typical approach to generate quantity index using the information on values is to deflate the values with an appropriate price index. Quantity index numbers created by this technique are labeled as "indirect" quantity index in contrast to "direct" quantity index computed with actual output or input levels.

A construction of indirect quantity index numbers relies on the premise that value change can be solely decomposed into price and quantity changes. If the premise holds, the value index can be represented as a product of price and quantity index numbers:

$$\mathbf{p}_t \cdot \mathbf{x}_t / \mathbf{p}_s \cdot \mathbf{x}_s = P(\mathbf{p}_s, \mathbf{p}_t, \mathbf{x}_s, \mathbf{x}_t) * Q(\mathbf{p}_s, \mathbf{p}_t, \mathbf{x}_s, \mathbf{x}_t) \tag{9}$$

Once price index with any formula is given, then we can derive indirect quantity index number using the relationship in (9). It can be algebraically shown that there is a dual relationship between price index and resulting indirect quantity index. Substituting Laspeyres price index formula (1) into (9) and representing the equation (9) in terms of quantity index yields:

$$Q(\mathbf{p}_s, \mathbf{p}_t, \mathbf{x}_s, \mathbf{x}_t) = \frac{\mathbf{p}_t \cdot \mathbf{x}_t / \mathbf{p}_s \cdot \mathbf{x}_s}{\mathbf{p}_t \cdot \mathbf{x}_s / \mathbf{p}_s \cdot \mathbf{x}_s} = \frac{\mathbf{p}_t \cdot \mathbf{x}_t}{\mathbf{p}_t \cdot \mathbf{x}_s} = Q^P \tag{10}$$

Thus, Paasche quantity index is generated using Laspeyres price index as a value deflator. With the similar context, we can show that Paasche price and Laspeyres quantity index numbers are dual to each other:

$$Q(\mathbf{p}_s, \mathbf{p}_t, \mathbf{x}_s, \mathbf{x}_t) = \frac{\mathbf{p}_t \cdot \mathbf{x}_t / \mathbf{p}_s \cdot \mathbf{x}_s}{\mathbf{p}_t \cdot \mathbf{x}_t / \mathbf{p}_s \cdot \mathbf{x}_t} = \frac{\mathbf{p}_s \cdot \mathbf{x}_t}{\mathbf{p}_s \cdot \mathbf{x}_t} = Q^L \tag{11}$$

In practice, producer price index (PPI) numbers complied by Bureau of Labor Statistics (BLS) have been almost exclusively used as the value deflators to create indirect quantity index numbers. BLS adopts fixed-year base Laspeyres method (chapter 14, BLS Handbook of Methods) to compute PPI, which yields, in turn, Paasche indirect quantity index numbers by their dual relationship. Therefore, the selection of a value deflator has an important implication on what indirect quantity index numbers would be generated.

Among the four index formulas introduced above, only Fisher index has the property of self-duality. The property of self-duality (sometimes referred to as factor reversal test) warrants that the product of price and quantity index numbers computed with the same formulas equals to the value ratio (i.e. price and quantity index numbers in (9) are derived from the same formulas). The self-duality of Fisher index can be shown as:

$$Q(\mathbf{p}_s, \mathbf{p}_t, \mathbf{x}_s, \mathbf{x}_t) = \frac{\mathbf{p}_t \cdot \mathbf{x}_t / \mathbf{p}_s \cdot \mathbf{x}_s}{\left[\frac{\mathbf{p}_t \cdot \mathbf{x}_s / \mathbf{p}_t \cdot \mathbf{x}_t}{\mathbf{p}_s \cdot \mathbf{x}_s / \mathbf{p}_s \cdot \mathbf{x}_t}\right]^{1/2}} = [Q^P * Q^L]^{1/2} \quad (12)$$

The third term in (12) is obtained using results in (10) and (11). Self-duality of Fisher index indicates that Fisher price and quantity index numbers together decompose value index exactly.

It is well established that each quantity index number is consistent with a particular aggregator function (i.e. production function) in production theory of economics. For example, Laspeyres quantity index is consistent with Leontief aggregator function while Paasche index is with linear aggregator function. Tornqvist index is exact to translog aggregator function whereas Fisher is to a quadratic aggregator function. Diewert (1976) name index numbers "superlative" if they are exact to flexible aggregator functional forms. An aggregator functional form is said to be flexible if it can provide a second order approximation to an arbitrary twice-differentiable linearly homogeneous function.

One implication of this definition is that all superlative index numbers are likely to be very similar in the magnitudes because they approximate any arbitrary twice-differentiable function. Both Tornqvist and Fisher index numbers are superlative. Later, Diewert (1992) used a test (or axiomatic) approach to index number theory and discovered Fisher index satisfied more tests than any other index numbers, thus, he recommended the use of Fisher index numbers in the applications.

3. Fixed-year base versus chain type index numbers

Another taxonomy of index numbers is "fixed-year base" versus "chain-type" index numbers. As described above, index numbers measure the changes in a set of related variables from a reference year. Index constructed in reference to a particular year is named "fixed-year base" index numbers. An alternative to fixed-year base index is "chain-type" index numbers. The chain type index numbers first calculate annual changes where index numbers of current year t are derived based on the previous year t-1 as reference year. These annual changes are, then, multiplied to represent the changes over a given time period. As formally stated:

$$I(0, t) = I(0, 1) * I(1, 2), \ldots, * I(t-1, t) \quad (13)$$

where, $I(t\text{-}1, t)$ is any index number computed for year t with the reference year t-1. The comparison between year t and year 0 can be done by the product of chained index numbers computed in (13).

Fixed-year base index is easy to compute and require less information (e.g. value weights for fixed-year base Laspeyres index numbers remain the same for all periods). However, chain-type index numbers are recommended in practice, particularly in respect to productivity analysis. Since chain-type index numbers only concern changes over consecutive years, they measure relatively smaller changes, implying some of the approximations involved in the derivation of theoretically meaningful index numbers are more likely to be held (Coelli *et al.*, 1998).

Application to sawmils and Planing Mills Industry of U.S.

Our application is to sawmills and planing mills industry of U.S. (1987 Standard Industry Classification System: SIC242) and nation-wide annual time series data covering years of 1948-2000 are constructed. The data include a single output, lumber, and four inputs: labor, wood, energy, and capital.

The main sources of the data are Census of Manufactures (CM) for census years and Annual Survey of Manufactures (ASM) for non-census years published by Bureau of the Census. The Census of Manufactures reports various statistics of an industry or a group of industry and is conducted in every 5 years. ASM provides sample estimates of statistics for all manufacturing industry between census years. Both CM and ASM use SIC system for the definition of industry. SIC system has been redefined over the years, and major changes related to sawmills and planing mills industry occurred in 1958 and 1967. In 1997 Bureau of the Census introduced new industry classification system, North American Industry Classification System (NAICS), and replaced SIC system.

For this study, we select the industry definition based on SIC system defined in 1987, and all values from CM and ASM have been adjusted according to the regrouping of subcategories of industry in 1958, 1967 and 1997.

The remainder of the section describes the construction of each variable prepared in this study.

1. Lumber output

We assemble lumber prices and productions by species groups (softwood and hardwood) and regions from various sources including USDA Forest Service (1988), Adams (2003), and Timber Mart South. For the softwood, data are relatively well documented, and we are able to collect the data on prices and regional productions over the analysis period. Prices are aggregated with weights on regional productions to generate a national softwood lumber price series.

For the hardwood, however, we realize that the lumber prices vary greatly by species and regions, and observe time series data dates back to only 1965. Due to the lack of price series covering the whole period of the analysis and diversified price ranges by species and regions, it is important to recover a consistent national price series of hardwood lumber. To retrieve the nominal price series of hardwood lumber for the data-missing years, after a careful consideration, we decide to use PPI of hardwood lumber (1982=1.00) in which the observed lumber price in 1982 is multiplied by PPI for each year restoring nominal prices before they are converted into index numbers.

We find that the recovered prices closely follow the actual hardwood lumber price series between year 1965 and 2000 where the observed hardwood lumber prices are available. Constructed lumber prices of softwood and hardwood series are multiplied by the corresponding productions by species groups to calculate total value of output and value shares to be used in aggregation. Using the formulas (1) to (4), we compute price index numbers.

For the comparison of indirect and direct index numbers, two sets of quantity index numbers are calculated for the lumber. First, we derive four indirect output quantity index numbers by deflating total output values (values of shipment) of the industry with the corresponding price index computed above. The values of shipment count the received net selling values of all products as well as all miscellaneous receipts. Data on values of shipment are collected from CM and ASM over the analysis period. In addition, the direct quantity index numbers for the output are calculated based on formula (5) to (8).

2. Labor input

CM and ASM report various information on labor use of industry including total number of employees, total payroll for all employees, total number of production workers, total wages paid for production workers, total number of hours of production workers. The total payroll does not necessarily represent the total labor cost to employers because it does not include fringe benefits such as social security payment. Information on the payment of fringe benefit is also available in ASM, however, the data are available from 1967. Since true labor cost include both payroll and fringe benefit payment, we adjust total payroll using the ratio of payroll to total labor cost.

Labor is divided into the production and non-production labor. The implicit price of production labor per hour is calculated by dividing the adjusted total wages by the total number of production hours worked. To compute the total number of non-production hours worked, we subtract total number of production workers from total number of employees and assume 2000 hours per worker and per year. Subtracting the adjusted total wages from the adjusted total payroll returns the total labor cost of non-production labor. The implicit price of non-production labor is, then, computed by dividing the total non-production labor cost by total number of non-production hours worked. An aggregated price index numbers of labor are obtained by combining production and non-production labor using their weights on total labor cost. We derive indirect quantity index numbers of labor by deflating labor cost by the corresponding price index.

3. Energy input

Separate series of electricity and fuel cost in the industry SIC 242 is available in CM and ASM from 1967. Between 1958 and 1967, only total cost of energy (electricity and fuel combined) is reported in ASM, however, the disaggregated cost of energy is obtainable in the census years for the industry during this period. We split the total cost of energy into cost of electricity and fuel using the ratio of each to total cost in the census year nearby to retrieve the separate cost series between 1958 and 1967. Before 1958, data on the cost of energy is available only at higher aggregated industry level, Lumber and Wood Product (SIC 24). We calculate the ratios of electricity and fuel cost of SIC 242 to the counterparts of SIC 24 in 1958 Census of Manufactures. The computed ratios are, then, applied to the costs of electricity and fuel of SIC 24 for the years before 1958 to recover the cost series of SIC 242. Quantities of electricity used for the industry are also attainable from CM and ASM in the same years described above and are estimated with the same technique illustrated above to recover the data for the missing years.

Implicit price of electricity is calculated by dividing the cost of electricity by the total quantity of electricity used. Prices of fossil-fuel-composite[1)] are used as annual average price of fuel and are obtained from Annual Energy Review published by Energy Information Administration. A single price index of energy is determined by aggregating electricity and fuel with the weight of each cost to total energy cost.

4. Wood input

CM and ASM provide the cost of materials used in the industry to produce the output. The cost of materials include the total delivered cost of raw materials, parts, and supplies put into production or used for repair and maintenance, along with purchased electric energy and fuels consumed for heat and power, and contract work done by others of the plant. Given that the high percentage of raw materials put into the production of sawmills and planing mills industry, non-energy cost of materials (i.e. the remainder of costs after subtracting cost of energy from the total material costs) is likely to reflect the cost associated with purchasing wood input. We use non-energy cost of materials as wood input cost of the production.

Since historical saw-timber price (e.g. delivered prices of log to sawmills) series are not available except recent years to our knowledge, the sum of stumpage price and logging and haul cost is used as a proxy for the price of saw-timber. We gather stumpage prices and logging and haul cost by species groups (softwood and hardwood) and regions from various sources including Adams (2003) and Timber Mart South. As in the case of historical lumber price series, the data on softwood are relatively well established compared to hardwood. We are able to assemble the stumpage prices and logging cost by regions along with saw-timber harvest levels. Prices are aggregated with weights on regional harvest levels to generate a national softwood saw-timber price series.

For hardwood, we gather the stumpage prices and logging cost by regions for 1965-2000. Information, however, on regional harvests is not available, thus, we use regional lumber productions as weights to aggregate regional stumpage prices and logging cost given that hardwood saw-timber harvest level is likely to be highly correlated with regional lumber productions. We employ the same technique used to construct the hardwood lumber price series above to generate the hardwood saw-timber price series for the years where the data area not available. Saw-timber prices of softwood and hardwood, then, are aggregated to construct a single saw-timber price series.

5. Capital input[2)]

The distinguishing feature of capital input as a factor of production is that the contribution of services of durable goods lasts over the years with a depreciation depending on the age of durable goods. Capital input is defined as the flow of service from physical assets, and quantity index of capital input are measured in three stages. The first stage is to estimate capital stock in constant dollars for asset groups (i.e. equipment and structure) using perpetual inventory method (PIM). In the second stage, the rental prices (user cost) of asset groups are estimated and expressed in rates per constant dollar of productive capital stock. Lastly, the rental prices of equipment and structure are multiplied by its corresponding stock estimates to obtain the total capital cost, and cost share of each asset group is computed accordingly. Stock estimates of equipment and structure are combined to generate an aggregated capital quantity index numbers using the cost shares computed above.

Results and Discussion

As explained in the previous section, we generate the separate series of price and quantity index numbers for the single output, lumber, and 4 inputs: labor, energy, wood, and capital. All index numbers are created based on the year 1996. Due to the limited space, we report the most results using the figures instead of laying out all index numbers in tables[3)].

We, first, discuss the divergences among index numbers constructed from different formulas. As expected, we observe that Laspeyres and Paasche form the upper and lower bounds and Fisher and Tornqvist index numbers fall in between two index numbers throughout the entire exercise. We find some discrepancies between Laspeyres and Paasche index numbers, and the differences are quite significant in some instances, especially in the case of energy input (Table 1). The divergence between Laspeyres and Paasche price index is due to the variation of price relatives between current and base year, indicating that price of energy input has been changed significantly over the years. To contrast, we discover that the magnitudes of Fisher and Tornqvist index numbers are very similar, almost identical in many cases, including energy input, in our practice. This result implies that Fisher and Tornqvist index numbers, both known as superlative, indeed approximate any arbitrary

[1)]Derived by multiplying the price per Btu of each fossil fuel by the total Btu content of the production of each fossil fuel, then, dividing this accumulated values of total fossil fuel production by the accumulated Btu content of total fossil fuel production.

[2)]Full description of the treatment of capital input including the use of perpetual inventory method (PIM) and estimation of user cost is available in Ahn (2003).

[3)]Readers who are interested in actual index figures can obtain the results in tabular format upon the request from the author.

Table 1. Price index numbers for energy input.

Year	Fixed-Year Base Index (1996=1)				Chain-Type Index (1996=1)			
	Laspeyres	Paasche	Fisher	Tornqvist	Laspeyres	Paasche	Fisher	Tornqvist
1948	0.2186	0.1571	0.1853	0.1856	0.1472	0.1782	0.1620	0.1619
1949	0.2148	0.1613	0.1862	0.1864	0.1503	0.1817	0.1652	0.1652
1950	0.2097	0.1604	0.1834	0.1836	0.1490	0.1802	0.1639	0.1638
1951	0.2600	0.1794	0.2160	0.2177	0.1613	0.1975	0.1785	0.1785
1952	0.2589	0.1789	0.2153	0.2169	0.1610	0.1971	0.1781	0.1781
1953	0.2407	0.1792	0.2077	0.2087	0.1593	0.1944	0.1760	0.1760
1954	0.2224	0.1822	0.2013	0.2020	0.1571	0.1899	0.1727	0.1727
1955	0.2219	0.1765	0.1979	0.1986	0.1542	0.1861	0.1694	0.1694
1956	0.2370	0.1851	0.2095	0.2104	0.1622	0.1958	0.1782	0.1782
1957	0.2473	0.1972	0.2208	0.2217	0.1716	0.2070	0.1885	0.1885
1958	0.2473	0.1924	0.2181	0.2191	0.1687	0.2034	0.1852	0.1852
1959	0.2462	0.1910	0.2169	0.2178	0.1683	0.2029	0.1848	0.1848
1960	0.2435	0.1882	0.2141	0.2152	0.1644	0.1983	0.1805	0.1806
1961	0.2366	0.1909	0.2125	0.2133	0.1650	0.1987	0.1811	0.1811
1962	0.2443	0.1933	0.2173	0.2183	0.1677	0.2019	0.1840	0.1841
1963	0.2289	0.1805	0.2033	0.2039	0.1595	0.1924	0.1752	0.1752
1964	0.2242	0.1806	0.2012	0.2019	0.1580	0.1905	0.1735	0.1735
1965	0.2223	0.1803	0.2002	0.2009	0.1574	0.1897	0.1728	0.1728
1966	0.2195	0.1821	0.1999	0.2006	0.1565	0.1884	0.1717	0.1717
1967	0.2034	0.1814	0.1921	0.1925	0.1505	0.1796	0.1644	0.1644
1968	0.2033	0.1841	0.1935	0.1938	0.1525	0.1819	0.1666	0.1666
1969	0.2033	0.1845	0.1937	0.1939	0.1545	0.1846	0.1689	0.1689
1970	0.2046	0.1903	0.1973	0.1974	0.1595	0.1907	0.1744	0.1744
1971	0.2418	0.2148	0.2279	0.2283	0.1798	0.2150	0.1966	0.1967
1972	0.2105	0.2054	0.2079	0.2080	0.1688	0.1953	0.1816	0.1816
1973	0.2345	0.2300	0.2322	0.2322	0.1893	0.2191	0.2037	0.2037
1974	0.2681	0.2841	0.2760	0.2767	0.2474	0.2681	0.2575	0.2575
1975	0.3389	0.3543	0.3466	0.3472	0.3099	0.3361	0.3228	0.3227
1976	0.3825	0.3964	0.3894	0.3899	0.3480	0.3776	0.3625	0.3624
1977	0.4451	0.4560	0.4505	0.4509	0.4028	0.4375	0.4198	0.4197
1978	0.5077	0.5208	0.5142	0.5146	0.4579	0.4970	0.4771	0.4769
1979	0.5963	0.6181	0.6071	0.6079	0.5439	0.5902	0.5666	0.5664
1980	0.7593	0.7876	0.7733	0.7754	0.7059	0.7612	0.7330	0.7328
1981	0.8905	0.8443	0.8671	0.8664	0.8417	0.8775	0.8594	0.8588
1982	0.9263	0.8849	0.9054	0.9047	0.8796	0.9168	0.8980	0.8974
1983	0.9327	0.9065	0.9195	0.9198	0.8887	0.9253	0.9068	0.9062
1984	0.9770	0.9399	0.9583	0.9572	0.9355	0.9771	0.9561	0.9554
1985	0.9982	0.9699	0.9840	0.9831	0.9642	1.0067	0.9852	0.9845
1986	0.8832	0.8831	0.8831	0.8831	0.8763	0.8920	0.8841	0.8841
1987	0.8972	0.8950	0.8961	0.8961	0.8901	0.9053	0.8977	0.8976
1988	0.8730	0.8725	0.8728	0.8728	0.8701	0.8799	0.8749	0.8750
1989	0.9200	0.9201	0.9201	0.9201	0.9174	0.9269	0.9221	0.9222
1990	0.9529	0.9519	0.9524	0.9524	0.9495	0.9589	0.9542	0.9542
1991	0.9439	0.9434	0.9436	0.9436	0.9419	0.9482	0.9451	0.9451
1992	0.9991	0.9970	0.9980	0.9980	0.9959	1.0023	0.9991	0.9992
1993	0.9932	0.9908	0.9920	0.9920	0.9903	0.9968	0.9935	0.9936
1994	0.9590	0.9517	0.9553	0.9554	0.9547	0.9596	0.9572	0.9572
1995	0.9389	0.9338	0.9364	0.9364	0.9338	0.9389	0.9364	0.9364
1996	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1997	0.9870	0.9869	0.9870	0.9870	0.9870	0.9869	0.9870	0.9870
1998	0.9409	0.9283	0.9346	0.9349	0.9399	0.9292	0.9345	0.9347
1999	0.9502	0.9462	0.9482	0.9482	0.9565	0.9436	0.9500	0.9503
2000	1.0769	1.0706	1.0737	1.0737	1.1015	1.0609	1.0810	1.0806

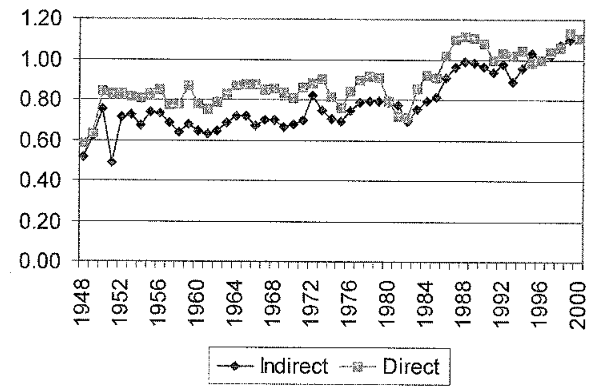

Figure 1. Comparison between direct and indirect quantity index of lumber.

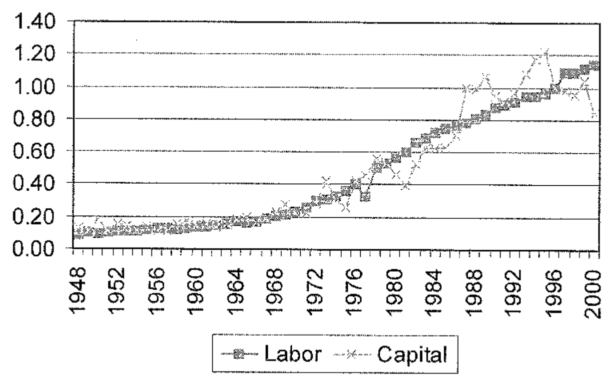

Figure 2. Chain-type Fisher price index for labor and capital.

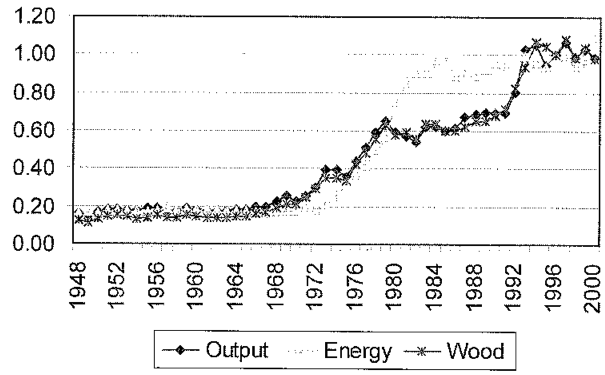

Figure 3. Chain-type Fisher price index for wood, energy, and lumber.

twice-differentiable linearly homogeneous functions.

The comparison between direct and indirect quantity index numbers of lumber output can be observed in Figure 1. Although indirect index numbers, throughout the whole periods, underestimate the direct index numbers in magnitudes, they closely follow the trends of direct index, confirming that indirect quantity index numbers represent the trends of actual quantity changes reasonably well. This result is promising given that researchers are likely to depend on the decomposition of values in application of index numbers due to the lack of actual data on quantity levels.

The results from the comparison between fixed-year base and chain-type index reveal that index numbers can be influenced by the selection of a base year. The most dramatic case is, again, the energy input (Table 1). As shown in Table 1, fixed-year base Laspeyres price index of energy in 1948 is 0.2186 while the chain-type Laspeyres counterpart is 0.1472. Notice that the differences become larger as index numbers move away from the base year 1996, indicating the significant influence of fixing the base year for entire analysis period. Alternatively, we detect comparatively small differences between fixed and chain type index numbers nearby 1996, while the divergences are consequential in early years.

Chain-type index is conceptually preferable because its construction involves the comparison over only consecutive periods, measuring smaller changes. A disadvantage, however, of chain-type index is that it requires to revise the base year annually. Most published index numbers (e.g. producer price index) employ the fixed year base Laspeyres formula since it demands the minimal information (i.e. value weights remain the same for all years and the reference year is fixed).

All price index numbers, in the form of chain-type Fisher index, are shown in Figure 2 (capital and labor inputs) and in Figure 3 (wood and energy inputs and lumber output). All quantity index numbers are represented in Figure 4 (capital and labor inputs) and in Figure 5 (wood and energy inputs and lumber output).

Regarding price index numbers, U.S. sawmill industry in general shows the moderate, yet consistent increasing trends for all inputs until early 1970s and, then, rather rapid increases until mid-90s (Figure 2 and 3). After mid-90s, the prices are stabilized. Exceptions are energy and capital inputs even if they follow the general trends. Energy prices experience dramatic increases in 1970s and 1980s compared to other inputs. After the big boosts, energy prices are settled down displaying a little variation. This explains the significant differences among index formulas and between fixed base and chain-type index numbers for the energy.

Regarding quantity index numbers, the labor quantity index numbers present a mirror image with the corresponding price index (Figure 2 and 4), implying that the industry has decreased labor uses due to the ever-increasing prices of labor over the years. The most likely substitute of labor is capital, and the evidence of substitutability between labor and capital can be examined in Figure 4. Up to year 1980, the labor input uses have diminished significantly while the capital uses show rather steady increases over the years, indicating

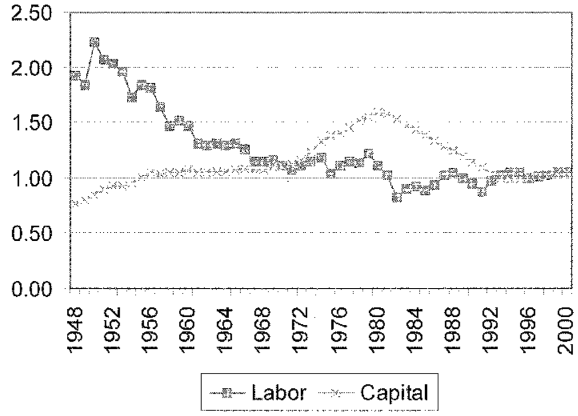

Figure 4. Chain-type Fisher quantity index for labor and capital.

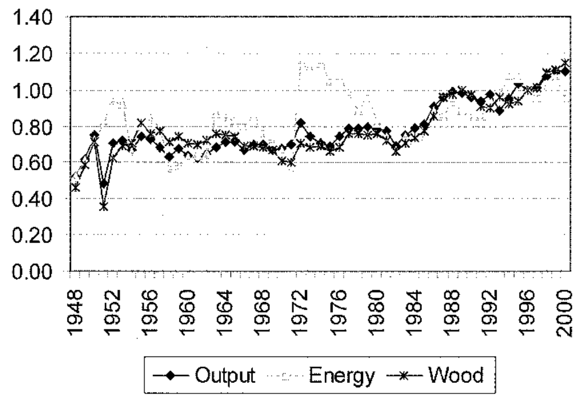

Figure 5. Chain-type Fisher quantity index for wood, energy, and lumber.

that substitution between labor and capital may not as strong as one can expect in the U.S. sawmill industry.

From the results of the analysis, we summarize that there could be discrepancies among index number formulas, and the differences between Laspeyres and Paasche price index can be substantial while the difference between Fisher and Tornqvist index is minimal. We also confirm that the selection of base year can cause significant divergences, especially when the variables undergo rapid price or quantity changes over time (energy input in our exercise). We find, at least in our study, that indirect quantity index approximates direct quantity index reasonably well.

Overall, our study supports the use of chain-type Fisher index numbers in the application because chain-type Fisher index is turn out to be more desirable in many respects than other index numbers considered in this exercise. This conclusion, however, is rather expected because Fisher index has proven to be the most theoretically sound index numbers in the literature, and this study provides an empirical verification.

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