# A Simplified Decoding Algorithm Using Symbol Transformation for Turbo Pragmatic Trellis-Coded Modulation

Euna Choi, Ji-Won Jung, Nae-Soo Kim, Young-Wan Kim, and Deock-Gil Oh

*ABSTRACT—This paper presents the application of a turbo coding technique combined with a bandwidth efficient method known as trellis-coded modulation. A transformation applied to the incoming I-channel and Q-channel symbols allows the use of an off-the-shelf binary/quadrature phase shift keying (B/QPSK) turbo decoder without any modifications. A conventional turbo decoder then operates on transformed symbols to estimate the coded bits. The uncoded bits are decoded based on the estimated coded bits and locations of the received symbols.*

*Keywords—Turbo code, pragmatic trellis coded modulation, constellation, symbol transformation.*

## I. Introduction

Turbo codes introduced in [1] have been shown to perform near the Shannon's channel capacity limit on additive white Gaussian noise channels. As a powerful coding technique, a turbo code offers great promise for improving the reliability of communication systems such as those based on the Digital Video Broadcasting standard for Return Channel via Satellite (DVB-RCS) technology [2]. These systems adaptively control the modulation (binary phase shift keying, quadrature PSK, 8-PSK, 16 quadrature amplitude modulation, etc.), the transmission rate, and the coding rate of an error-correcting code. However, the structure of a turbo decoder depends on the modulation scheme. Its decoding structure is different for a modulation scheme other than B/QPSK, requiring a larger decoder size or power consumption.

A single decoder would be more desirable to decode all modulation schemes in order simplify the hardware, minimize the power consumption, and to reduce the receiver cost. In this paper, a novel decoding procedure is proposed. This procedure allows the use of an off-the-shelf turbo decoder originally designed for a standard rate 1/2 turbo decoder over B/QPSK modulation to decode some trellis- coded modulation schemes over $2^m$-PSK/QAM constellations, with $m \geq 3$.

Wahlen and Mai [3] proposed to combine turbo coding with pragmatic trellis-coded modulation (PTCM), where the received 8-PSK symbols are directly inputted into the turbo decoder, causing a reduction of the Euclidean distance, which is one of the reasons for the performance degradation. This paper proposes a new method for a pragmatic turbo decoding algorithm using the coset symbol transformation, where the received 8-PSK signals are transformed into QPSK signals after some manipulations. The novel turbo TCM (TTCM) with two coded bits per symbol is based on a realization of the rate $n/(n+1)$ trellis-coded scheme using an off-the-shelf turbo decoder originally designed for a standard rate 1/2 turbo decoder for B/QPSK modulation.

## II. A New Decoding Algorithm of TTCM

Compared to an uncoded QPSK system, TCM is now a well-established method in digital communication systems

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Euna Choi (phone: +82 42 860 6655, email: eachoi@etri.re.kr), and Deock-Gil Oh (email: dgoh@etri.re.kr) are with Digital Broadcasting Research Division, ETRI, Daejeon, Korea.

Ji-Won Jung (email: jwjung@hanara.kmaritime.ac.kr ) is with the Department of Radio and Science Engineering, Korea Maritime University, Busan, Korea.

Nae-Soo Kim (email: nskim@etri.re.kr) is with Telematics & USN Research Division, ETRI, Daejeon, Korea.

Young-Wan Kim (email: ywkim@kunsan.ac.kr) is with the Department of Electronics and Information Engineering, Kunsan National University, Gunsan, Korea.

and is capable of achieving a coding gain of within a 3 to 6 dB range of the Shanon channel capacity for a trellis-coded 8-PSK system. The application of turbo codes to TCM has received much attention in the literature. For example, Hauro and others have proposed a new turbo trellis-coded modulation scheme [4]. In this paper, we propose to combine the turbo with a pragmatic TCM, creating what we call the turbo pragmatic TCM (TPTCM). In this section, we present a TPTCM with a rate 2/3. For the sake of clarity, only the case of 8-PSK modulation is considered, but the result can be easily generalized to M-PSK modulation for any M equal to a power of 2. Figure 1 shows the rate 2/3 TPTCM encoder/decoder structure. The encoder consists of two recursive systematic convolution (RSC) codes and one interleaver. The decoder consists of two off-the shelf turbo decoders (DEC1, DEC2) with a rate of 1/2, a phase sector quantizer (PSQ), a coset symbol transformer, a de-interleaver, and a re-encoder. The decoding procedure requires a standard turbo decoder without any modification to calculate the log-likelihood ratio, the forward/backward state metric, and the branch metric.

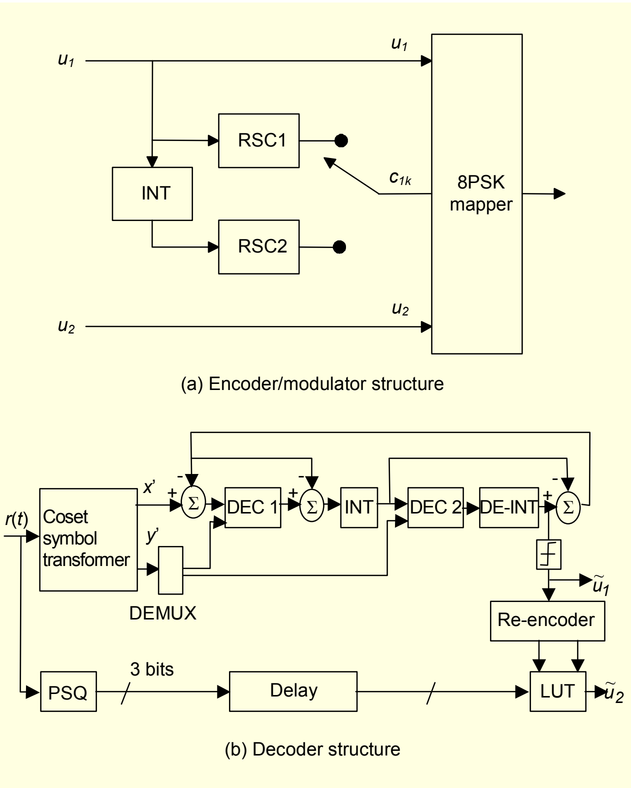



Fig. 1. Rate 2/3 turbo pragmatic TCM encoder and decoder structures.

## 1. Mapping of Bits to Signal

Two information bits ($u_1$, $u_2$) are encoded to produce three coded bits ($u_2$, $u_1$, $c_{1k}$), which are mapped onto 8-PSK signal points, where $c_{1k}$ is the output of the standard turbo encoder and is punctured. The signal points are labeled by a triplet ($u_2$, $u_1$, $c_{1k}$), as shown in Fig. 2.

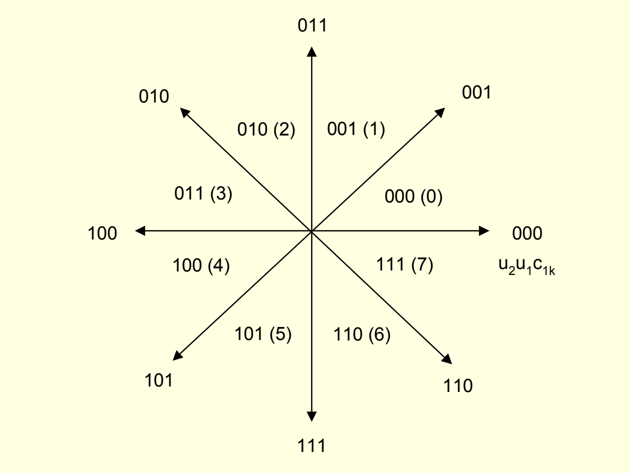



Fig. 2. Labels and phase information of an 8-PSK constellation.

## 2. Coset Symbol Transformer

Let ($x$, $y$) denote the I- and Q-channel values of the received 8-PSK symbols $r(t)$, and $\varphi$ denote the phase of the received signal. Then,

$$r(t) = x + jy,$$

$$\varphi = \tan^{-1}(y/x).$$

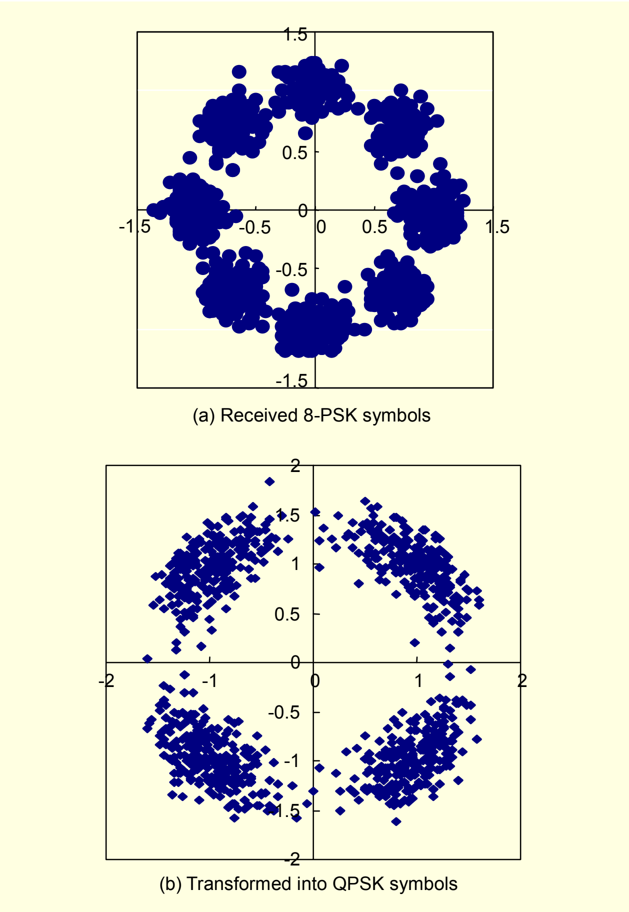
In order to use the turbo decoder with the rate 1/2, a transformation is applied such that the 8-PSK points are mapped into QPSK points labeled by ($u_1$, $c_{1k}$), as shown in Fig. 2. The $x'$ and $y'$ projections in the transformed QPSK constellation are obtained from the received 8-PSK symbols by

$$x' = \sqrt{2}\cos(2(\varphi + 5\pi/8)),$$
$$y' = \sqrt{2}\sin(2(\varphi + 5\pi/8)).$$

The scaling factor to project onto QPSK points with $(\pm 1, \pm 1)$ is $\sqrt{2}$, and $5\pi/8$ is the phase rotation constant to map into the QPSK point with ($u_1$, $c_{1k}$): $(11) \to \pi/4, (01) \to 3\pi/4,$ $(00) \to 5\pi/4$, and $(10) \to 7\pi/4$. Figure 3 shows an example of the received 8-PSK symbols being transformed into QPSK symbols at an $E_b/N_0$ of 12 dB.

## 3. Phase Sector Quantizer

The signal vector space is quantized to determine the locations of received symbols, and the PSQ in Fig. 1 is used for decoding the uncoded bits, $u_2$. The PSQ is designed with the assumption that the in-phase (I) and quadrature (Q) components of the received signals $r(t)$ are already converted to $q$ quantization bits. The circuit is shown in Fig. 4. Three comparators and two absolute value generations produce the three bits of phase information identifying one of the eight sectors. The three phase information bits provide information about the location of the received vector: $\phi_2$ and $\phi_3$ indicate the quadrant, and the remaining one bit indicates the location within the quadrant. They each assume a value of 1 or 0.



(a) Received 8-PSK symbols

(b) Transformed into QPSK symbols

Fig. 3. Transformed QPSK symbols at an $E_b/N_0$ of 12 dB.

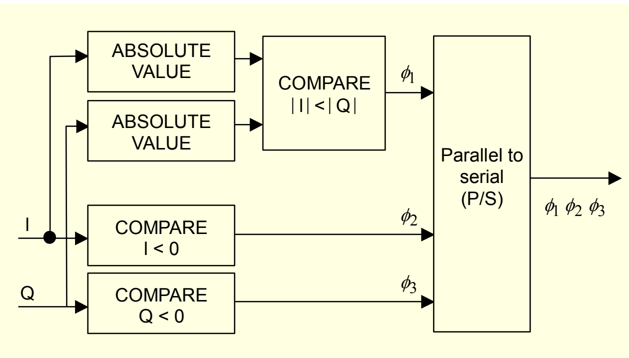


Fig. 4. Phase sector quantizer and soft decision mapping block.

The coset symbol transformer outputs $(x', y')$ are used by an iterative maximum a posteriori turbo decoder to produce estimates $\tilde{u}_1$ of $u_1$. The value $\tilde{u}_1$ is then encoded by the rate 1/2 turbo encoder to provide estimates $\tilde{u}_1$ and $\tilde{c}_{1k}$ of $u_1$ and $c_{1k}$, respectively. After the turbo decoder estimates the coded bits, it remains to determine the uncoded bit. This is accomplished by making a threshold decision. Using the estimated coded bits $(\tilde{u}_1, \tilde{c}_{1k})$ and phase information $(\phi_1, \phi_2, \phi_3)$, we can determine the uncoded bit, $\tilde{u}_2$ of $u_2$. Due to the structure of the turbo decoding algorithm, every decoder delays the data by a fixed number of symbol periods. The phase information bits $\phi_1, \phi_2, \phi_3$ must be delayed by the amount of the turbo decoding delay to synchronize with the reconstructed code sequence. A simple look-up table as shown in Table 1 can be used to estimate $\tilde{u}_2$. For example, if the re-encoded bits $(\tilde{u}_1, \tilde{c}_{1k})$ are (00) and the phase information $(\phi_1, \phi_2, \phi_3)$ is (111), then $\tilde{u}_2 = 0$.

Table 1. Look-up table for estimating $\tilde{u}_2$.

| Estimated coded bits $(\tilde{u}_1, \tilde{c}_{1k})$ | Phase information $(\phi_1, \phi_2, \phi_3)$ | Estimate $\tilde{u}_2$ |
|---|---|---|
| 00 | (000),(001),(110),(111) | 0 |
| | (010),(011),(100),(101) | 1 |
| 01 | (000),(001),(010),(111) | 0 |
| | (011),(100),(101),(110) | 1 |
| 10 | (000),(101),(110),(111) | 0 |
| | (001),(010),(011),(100) | 1 |
| 11 | (000),(001),(010),(011) | 0 |
| | (100),(101),(110),(111) | 1 |

## III. Simulation Results

Computer simulations are conducted to compare the performance of the turbo pragmatic TCM with that of the pragmatic TCM and with the published simulation results of the turbo TCM method in [4]. The following simulation results are plotted in Fig. 5 for comparable 2 bps/Hz systems: i) the uncoded QPSK; ii) the rate 2/3 pragmatic TCM with a single 64-state convolutional encoder; iii) the rate 2/3 turbo pragmatic TCM of Fig. 1 with two 16-state RSC encoders, a 500-bit random interleaver, and five decoding iterations; and iv) the rate 2/3 turbo TCM with two 16-state RSC encoders, 500-bit random interleaver, and five decoding iterations [4]. At a BER of $10^{-5}$, the TCPTCM achieves a 2 dB gain relative to the PTCM. Compared to [4], the proposed decoder exhibits a small loss of less than 0.2 dB.

## IV. Conclusion

In this paper, we presented the turbo pragmatic TCM with two coded bits per symbol, based on a realization of the rate n/(n+1) trellis-coded scheme using an off-the-shelf turbo decoder originally designed for a standard rate 1/2 turbo decoder for B/QPSK modulation. Compared to the conventional turbo TCM, although the proposed decoder exhibits a small loss of less than 0.2 dB, it requires less hardware, consumes less power, and reduces the receiver cost. The proposed approach may be extended to a variable coding rate (rate-5/6 and rate-8/9) depending on how many uncoded bits are assigned. Also, it can be used for $2^m$-QAM constellations with $m \geq 4$.

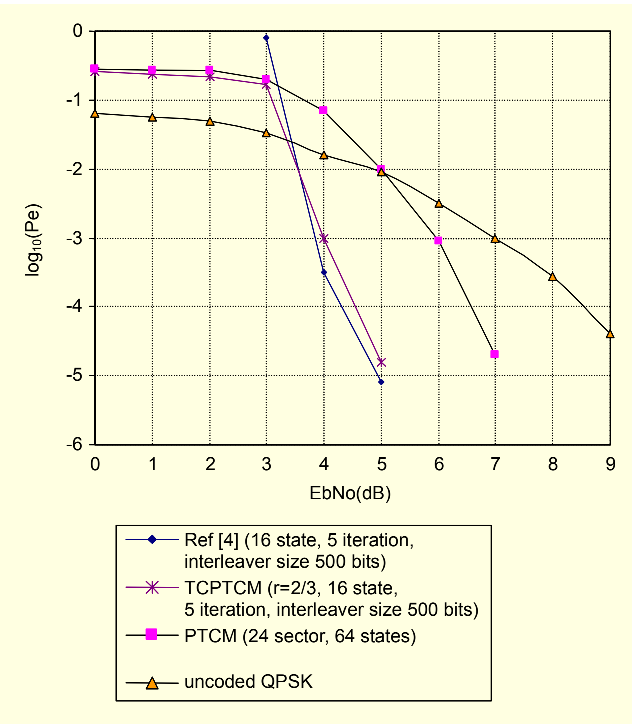



Fig. 5. Bit error rate performance comparison among the pragmatic TCM, the turbo-coded pragmatic, and the turbo TCM.

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