Sliced Multi-modulus Blind Equalization Algorithm

Shafayat Abrar and Roy A. Axford Jr.

Many multi-modulus blind equalization algorithms (MMA) have been presented in the past to overcome the undesirable high misadjustment exhibited by the well-known constant modulus algorithm. Some of these MMA schemes, specifically tailored for quadrature amplitude modulation (QAM) constellations, have also been proved to fix the phase offset error without needing any rotator at the end of the equalizer stage. In this paper, a new multi-modulus algorithm is presented for QAM signals. The contribution lies in the technique to incorporate the sliced symbols (outcomes of decision device) in the multi-modulus-based weight adaptation process. The convergence characteristics of the proposed sliced multi-modulus algorithm (S-MMA) is demonstrated by way of simulations, and it is shown that it gives better steady-state performance in terms of residual inter-symbol interference and symbol-error rate. It has also been shown that the proposed algorithm exhibits lesser steady-state misadjustment compared to the best reported MMA.

Keywords: Blind equalization, multi-modulus, constant modulus, carrier/channel phase-offset recovery, slicer, misadjustment.

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I. Introduction

In most digital communication systems, inter-symbol interference (ISI) occurs due to bandwidth limited channels or multipath propagation. Channel equalization is one of the techniques to mitigate the effect of ISI. Adaptive algorithms are used to initialize and adjust equalizer coefficients when a channel is unknown and possibly time-varying. Conventionally, an initial setting of the equalizer tap weights is achieved by a training sequence before data transmission.

However, when sending a training sequence is impractical or impossible, it is desirable to equalize a channel without the aid of a training sequence. Equalizing a channel without training mode is known as *blind equalization*. For example, an originally connected transmission route in a telephone network or a mobile radio system might be disconnected abruptly, making a rapid reconnection necessary to re-establish the link and minimize the outage loss. In the course of reconnection, the receiver has to recover every demodulation operation, including equalization, and is adapted to the newly connected channel without the help of a training sequence. Also in a multi-point network, by providing the capability of unsupervised training (or retraining) of a tributary receiver, the data throughput of the system may be enhanced.

Blind equalization of digital communication channels is a domain that has gained increased attention over the last two decades. A typical blind equalization setup is depicted in Fig. 1 where a baseband representation of a communication system is illustrated. The purpose of the blind equalization algorithm is to make the equalizer match the impulse response of the inverse of the communication channel, thus opening the eye of the communication system and allowing for a correct retrieval of the transmitted symbols. The performance of a blind equalization algorithm can be measured in many ways including for example, the convergence rate, the residual ISI, and symbol error rate (SER). The convergence rate is important, as it relates to the amount of time that service would be interrupted on the network during initialization, a change in the channel characteristics, or in the event that there is a significant interference in the channel being used. The SER relates to the equalizer's ability to yield, upon convergence, the correct alphabets. Adaptive equalization must, therefore, provide the best possible convergence-time without compromising the SER.

While the *constant modulus algorithm* (CMA) [1], a special case of Godard's family of blind equalization algorithms [2], is a famous candidate that could achieve desired convergence requirements, its respective cost function is only amplitude-dependent, and knowledge about the signal constellation is dismissed. For signal constellation exhibiting the constant modulus property, where all signal points have the same magnitude, the performance of CMA is reasonable. On the other hand, the CMA yields a degraded performance with a very poor SER for multi-level signals such as the quadrature amplitude modulation (QAM) signals because the CMA projects all signal points onto a single modulus [1].

In order to improve the performance of the CMA for QAM signals, a *multi-modulus* CMA, known as the decision-adjusted modulus algorithm (DAMA) [3]-[4], and radius directed (modulus) equalization (RDE) [5] have been proposed which employ multiple moduli rather than a single modulus. These algorithms allocate a modulus to each subset of signal constellation points depending on the equalizer output power (each subset is usually located on a circle). If there are n moduli on the constellation, then these algorithms can be realized as n-CMA, one for each of the known moduli. However, this multi-modulus CMA proved diverging even under a moderately distorted channel condition either due to a constellation size larger than 16-QAM [6] or simply due to a low signal-to-noise ratio (SNR) [7].

Another variant of the CMA is the dual-mode type CMA [7]-[9], which operates either in blind mode or in decision-directed (DD) mode depending on the error level exhibited by the equalizer output with respect to the region it lies in. The decision boundaries of these regions are usually chosen by bisecting adjacent moduli. However, these algorithms also become infeasible at larger QAM constellations and a low SNR. The main reason of this infeasibility is that, with higher constellations, the outer (circular) moduli come very close to each other, which narrows down the regions suitable for blind equalization. Some authors suggested to start with the CMA, and after gaining some initial convergence they suggested to switch to the DAMA [10] or the multi-modulus algorithm (MMA) [11].

Interestingly, the literature provides another class of multi-modulus algorithms - the *modified constant modulus algorithm* (MCMA), proposed independently by many authors [12]-[14]. This algorithm, instead of minimizing the dispersion of the magnitude of the equalizer output $y(n)$, minimizes the dispersion of real and imaginary parts, y_R and y_I, of $y(n)$ separately (embedded in its cost function). Recently, it was named *multi-modulus algorithm* (MMA) [15], [16]. Unlike CMA, the MMA cost function ignores the cross term $y_R\, y_I$ between the in-phase and quadrature components in the CMA cost function. As a result, the MMA cost function is not a two-dimensional cost function. It can be considered as the sum of two one-dimensional cost functions, which minimizes the dispersion of y_R and y_I around separate contours [17]. However, by considering the real and imaginary parts of the equalizer output in the cost function, it carries the information of the channel phase-distortion and the constellation orientation [18]. As far as the convergence of the MMA is concerned, Wesolowski showed that the stationary points of the MMA have a similar form to the stationary points of the CMA [19]. The MMA provides better convergence for higher QAM constellations compared to the CMA. However, it also exhibits a very high misadjustment in the steady state, though a much smaller one than that of the CMA.

In this paper, we propose a sliced multi-modulus algorithm (S-MMA) for application to digital transmission employing QAM signals. In the S-MMA, the cost function embeds the dispersion constant and the slicer output. The S-MMA cost function satisfies a number of desirable properties, including multiple-modulus, symmetry, and (almost) uniformity. The S-MMA cost function exhibits a much lower misadjustment compared to CMA and MMA. The performance evaluation of the proposed equalization approach is provided for a typical voice-band telephone channel using the transient and steady-state behavior of residual ISI and SER, respectively.

Brief overviews of the conventional CMA and MMA are given in sections III and IV, respectively. An analysis of the steady-state misadjustment exhibited by the MMA is provided in section V. The development of the proposed S-MMA is provided in section VI. Performance comparisons between the proposed technique and an existing effective equalization method are provided in section VII. Conclusions are drawn in section VIII.

II. System Model

Let $a(n)$ denote the transmitted symbol and $x(n)$ be the complex received signal, given as

$$x(n) = \sum_{i=0}^{K-1} h_i a(n-i) + v(n), \tag{1}$$

where $\{h\}$ is the complex baseband impulse response of the unknown channel of length K and v is the additive white Gaussian noise. The equalizer N-tap weight-vector and (tap) input-vector are respectively defined as $\mathbf{w}(n) = [w_0(n), w_1(n), \cdots, w_{N\text{-}1}(n)]^T$ and $\mathbf{x}(n) = [x(n), x(n\text{-}1),\cdots, x(n\text{-}N\text{+}1)]^T$. We define $y(n) = \mathbf{w}^T(n)\mathbf{x}(n)$ as the equalizer output and $\hat{a}(n)$ is the outcome of the decision device (slicer), computed as the closest constellation symbol to $y(n)$. The objective is to achieve an estimate of the actual transmitted signal $a(n)$ without using a training signal available at the receiver, such that $\hat{a}(n)=a(n\text{-}\Delta)$, where Δ is the bulk delay due to the channel-equalizer combined impulse response. In this case, the equalizer perfectly estimates the symbol that was transmitted Δ baud times earlier.

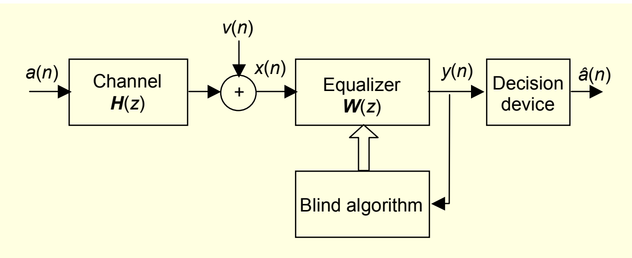

Fig. 1. Blind equalization in the baseband. $\boldsymbol{H}(z)$ and $\boldsymbol{W}(z)$ are z-transforms of channel and equalizer coefficients, respectively.

In blind equalization, the channel input $a(n)$ is unavailable, and thus different minimization criteria are explored. The crudest blind equalization algorithm is the DD scheme that updates the adaptive equalizer coefficients according to

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \mu(\hat{a}(n) - y(n))\mathbf{x}^*(n). \tag{2}$$

Under a high ISI, the convergence behavior of the DD equalizer is very poor. Better blind adaptive equalization algorithms are designed to minimize special non-mean square error cost functions that do not directly involve the input $a(n)$ while still reflecting the current level of ISI in the equalizer output. Define the mean cost function as

$$J(\mathbf{w}) = E[\Psi[y(n)]], \tag{3}$$

where $\Psi[\cdot]$ is a scalar function of the equalizer output and E denotes statistical expectation. $J(\mathbf{w})$ should be specified such that at its minimum, the corresponding $\mathbf{w}(n)$ results in a minimum ISI or mean square error equalizer. Using (3), the stochastic gradient descent minimization algorithm is easily derived as

$$\begin{aligned}\mathbf{w}(n+1) &= \mathbf{w}(n) - \mu\frac{\partial\Psi[y(n)]}{\partial\mathbf{w}^*(n)} \\ &= \mathbf{w}(n) - \mu\Psi'[y(n)]\mathbf{x}^*(n).\end{aligned}$$

Let ψ be the first derivative of Ψ, where ψ is often called the *error-function*. The resulting blind equalization algorithm can be written as

$$\mathbf{w}(n+1) = \mathbf{w}(n) - \mu\cdot\psi[y(n)]\mathbf{x}^*(n). \tag{4}$$

Thus, the design of the blind equalizer translates into the selection of a suitable function Ψ (or ψ) such that the local minima of $J(\mathbf{w})$ corresponds to a significant removal of ISI from the equalizer output $y(n)$. A necessary condition in the selection of ψ is that $E[y^*(n)\psi[y(n)]] = 0$.

III. Constant Modulus Algorithm

The CMA [1] is a stochastic gradient algorithm for the cost function

$$J_2 = \frac{1}{4}E\left(|y(n)|^2 - R\right)^2, \tag{5}$$

where R is the dispersion constant, defined as $E[|a|^4]/E[|a|^2]$ [2]. Notice that this cost function uses the second and the fourth-order statistics of the signals. The corresponding stochastic gradient algorithm is given by

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \mu y(n)(R - |y(n)|^2)\mathbf{x}^*(n), \tag{6}$$

where μ is the step-size parameter and the asterisk denotes the complex transpose. The CMA cost function enforces the equalizer output to lie on a circular contour. The cost function of the CMA doesn't contain any phase terms, so the CMA is not related to the carrier phase [20]. To remove the phase offset error, it either needs differential encoding or the need to add a rotator at the output of the equalizer. The rotator removes a possible phase offset error and facilitates a reliable switching from blind mode to DD mode.

IV. Multi-modulus Algorithm

Unlike the CMA, the MMA cost function penalizes the dispersion of the real and imaginary parts of $y(n)$ separately, which is given as

$$J = E[(y_R^2(n) - R_R)^2 + (y_I^2(n) - R_I)^2]. \tag{7}$$

The corresponding MMA tap updating algorithm is

$$\begin{aligned}&\mathbf{w}(n+1) \\ &= \mathbf{w}(n) + \mu[y_R(n)(R_R - y_R^2(n)) + j.y_I(n)(R_I - y_I^2(n))] \\ &\qquad \cdot\mathbf{x}^*(n),\end{aligned} \tag{8}$$

where $R_R = E[a_R^4]/E[a_R^2]$ and $R_I = E[a_I^4]/E[a_I^2]$ are defined respectively as the dispersion constants for the real and imaginary parts of the transmitted signal. Minimizing the cost function (7) can be interpreted as fitting the signal constellation onto a square. An advantage of the MMA over the CMA is that, upon convergence, it produces the correct constellation orientation, making a phase compensator unnecessary.

V. CMA and MMA with Dense Constellations

Though the CMA provides reliable initial convergence and is capable of reducing the ISI level to a significantly low level, it is not very effective in providing a good eye opening when the number of different symbols in the signal constellation becomes very large. The basic MMA algorithm also has difficulties with very dense constellations, but because of its flexibility, it can be modified to ease the opening of these constellations (as will appear in section VI).

In this section, we intuitively provide a brief explanation of why the CMA and MMA have difficulties in opening the eye of very large signal constellations. In steady-state, the mean of the correction term $E[\delta \mathbf{w}(n)] = E[\mathbf{w}(n+1) - \mathbf{w}(n)]$ in the CMA tap updating algorithm is zero, but its variance is generally not equal to zero. This results in tap fluctuations, which contribute tap adaptation noise to the output signal of the equalizer. Especially for QAM, due to the mismatch between the constellation and the cost function, the CMA update equation causes the adaptive weights to jitter (fluctuation noise) about their optimum settings even if the perfect equalization is achieved. Therefore, under any conditions, one would expect the misadjustment of the CMA should be very high for QAM. For a rigorous misadjustment analysis of the CMA, readers are referred to [6]. An indication of the amount of jittering about the stationary point can be obtained by calculating the variance of the i-th element of the update vector, which is given by $E\left[|\delta w_i(n)|^2\right]$ after convergence.

Rewrite the MMA weight-update rule (8) as follows:

$$\begin{aligned}\mathbf{w}(n+1) - \mathbf{w}(n) &= \delta\,\mathbf{w}(n) \\ &= \mu[y_R(n)(R_R - y_R^2(n)) + j.y_I(n)(R_I - y_I^2(n))] \\ &\quad \cdot \mathbf{x}^*(n). \end{aligned} \tag{9}$$

Specifically, the i-th element of the update vector is

$$\begin{aligned}\delta w_i(n) = \mu[y_R(n)(R_R - y_R^2(n)) + j.y_I(R_I - y_I^2(n))] \\ \cdot x^*(n-i). \end{aligned} \tag{10}$$

Now consider the jittering of the MMA weights after convergence. We wish to focus on the component of this jittering that is caused by the mismatch between MMA cost function and the QAM constellation. Since this mismatch exists regardless of the quality of equalization, we consider the case of perfect equalization under the further assumption that there is no thermal noise. In this case, we assume that

$$y(n) = a(n-\Delta). \tag{11}$$

Clearly Theorem 2.2 of [21] also applies; that is, the probability distribution of $y(n)$ matches that of the original transmitted constellation $a(n)$ and this allows statistical moments over the equalizer output to instead be taken over the constellation. The received signal sample, in the absence of noise, is given by

$$x(n-i) = \sum_k h_k a(n-i-k). \tag{12}$$

Using (11) and (12), we may write

$$\begin{aligned}\delta w_i(n) = \mu\big[&a_R(n-\Delta)\big(R_R - a_R^2(n-\Delta)\big) \\ &+ j.a_I(n-\Delta)\big(R_I - a_I^2(n-\Delta)\big)\big] \\ &\cdot \sum_k h_k^* a^*(n-i-k). \end{aligned} \tag{13}$$

Knowing that $E[a(n)]=0$ (zero-mean), $E[a^2(n)]=0$ (symmetries about in-phase and quadrature axes), $E[a^*(i)a(j)]=E[|a(m)|^2]\ \delta_{ij}$ (stationary and uncorrelated), $R_R = E[a_R^4]/E[a_R^2]$ and $R_I = E[a_I^4]/E[a_I^2]$, we can easily find that the expectation of (13) is zero; that is, $E[\delta w_i(n)]=0$. However, the variance of the jittering phenomenon, which can be measured from $E[|\delta w_i(n)|^2]$, is non-zero and can be computed as follows:

$$\begin{aligned} &E\left[|\delta w_i(n)|^2\right] \\ &= \mu^2 \sum_k \sum_l h_k h_l^* E\big[a^*(n-i-l)a(n-i-k) \\ &\qquad \cdot \{a_R^2(n-\Delta)(R_R - a_R^2(n-\Delta))^2 \\ &\qquad + a_I^2(n-\Delta)(R_I - a_I^2(n-\Delta))^2\}\big]. \end{aligned} \tag{14}$$

Again using the assumptions regarding the transmitted sequence stated above, only the $k = l$ terms in the double sum make non-zero contributions. Therefore,

$$\begin{aligned} &E\left[|\delta w_i(n)|^2\right] \\ &= \mu^2 \sum_k |h_k|^2 E\big[|a(n-i-k)|^2 \\ &\qquad \cdot \{a_R^2(n-\Delta)(R_R - a_R^2(n-\Delta))^2 \\ &\qquad + a_I^2(n-\Delta)(R_I - a_I^2(n-\Delta))^2\}\big]. \end{aligned} \tag{15}$$

Separating out the $k = \Delta - i$ term, we have

$$
\begin{aligned}
&E\left[\left|\delta w_i(n)\right|^2\right] \\
&= \mu^2 \left|h_{\Delta-i}\right|^2 E\Big[\left|a(n-\Delta)\right|^2 \\
&\qquad \cdot \left\{a_R^2(n-\Delta)(a_R^2(n-\Delta)-R_R)^2 \right. \\
&\qquad\quad \left. + a_I^2(n-\Delta)(a_I^2(n-\Delta)-R_I)^2\right\}\Big] \\
&+ \mu^2 \sum_{k\neq\Delta-i} \left|h_k\right|^2 E\Big[\left|a(n-i-k)\right|^2 \\
&\qquad \cdot \left\{a_R^2(n-\Delta)(R_R - a_R^2(n-\Delta))^2 \right. \\
&\qquad\quad \left. + a_I^2(n-\Delta)(R_I - a_I^2(n-\Delta))^2\right\}\Big].
\end{aligned} \tag{16}
$$

Using the statistical independence of successive symbols yields

$$
\begin{aligned}
&E\left[\left|\delta w_i(n)\right|^2\right] \\
&= \mu^2 \left|h_{\Delta-i}\right|^2 E\Big[\left\{a_R^2(m)+a_I^2(m)\right\} \\
&\qquad \cdot \left\{a_R^2(m)(a_R^2(m)-R_R)^2 + a_I^2(m)(a_I^2(m)-R_I)^2\right\}\Big] \\
&\quad + \mu^2 E\left[a_R^2(m)+a_I^2(m)\right] \\
&\qquad \cdot E\left[a_R^2(m)(R_R-a_R^2(m))^2 + a_I^2(m)(R_I-a_I^2(m))^2\right] \sum_{k\neq\Delta-i} \left|h_k\right|^2 .
\end{aligned} \tag{17}
$$

Using the symmetric nature of the QAM constellation, we observe that $E[|a_R(m)|^p] = E[|a_I(m)|^p]$, which simplifies (17) to

$$
\begin{aligned}
&E\left[\left|\delta w_i(n)\right|^2\right] \\
&= \mu^2 \left|h_{\Delta-i}\right|^2 \\
&\qquad \cdot E\Big[\left\{a_R^2(m)+a_I^2(m)\right\} \\
&\qquad\quad \cdot \left\{a_R^2(m)(a_R^2(m)-R_R)^2 + a_I^2(m)(a_I^2(m)-R_I)^2\right\}\Big] \\
&\quad + 4\mu^2 E\left[a_R^2(m)\right] E\left[a_R^2(m)(R_R-a_R^2(m))^2\right] \sum_{k\neq\Delta-i} \left|h_k\right|^2 \\
&= \mu^2 \gamma_1 \left|h_{\Delta-i}\right|^2 + \mu^2 \gamma_2 \sum_{k\neq\Delta-i} \left|h_k\right|^2 .
\end{aligned} \tag{18}
$$

The terms, γ_1 and γ_2, in (18) indicate the mismatch and are expressed as

$$
\begin{aligned}
\gamma_1 &= 2E\left[\left|a(m)\right|^2 \cdot a_R^2(m)(a_R^2(m)-R_R)^2\right], \\
\gamma_2 &= 4E[a_R^2(m)] \cdot E[a_R^2(m)(a_R^2(m)-R_R)^2].
\end{aligned} \tag{19}
$$

Examining (18) reveals that the only way to reduce the jitter is to either decrease "μ" or "γ_1 and γ_2". However, any decrement in μ decreases the speed of convergence of the equalizer. In addition, finite precision effects become a factor in a practical implementation if μ is too small [16]. Rather than decreasing the step size μ for large constellations, we propose to keep the misadjustment metrics small, by modifying the cost function of MMA, such that the values of γ_1 and γ_2 are decreased.

VI. Sliced Multi-modulus Algorithm

In this section, we provide a new algorithm, similar to the MMA, with reduced metrics γ_1 and γ_2. In order to decrease the values of metrics γ_1 and γ_2, we propose to assign a separate MMA cost function and (thus a separate) modulus $R_{R;I}$ to a group of symbols having the same values of $|\hat{a}_R(n)|$ or $|\hat{a}_I(n)|$. It can easily be achieved by weighting the dispersion constants R_R and R_I with absolute values of $\hat{a}_R(n)$ and $\hat{a}_I(n)$, respectively. In this way, both the real and imaginary parts of $y(n)$ are forced to belong to the contour weighted with the absolute value of the real and imaginary parts of the closest symbol, respectively.

The proposed algorithm is thus devised by embedding the *sliced* symbols in the dispersion constants; it is named the *sliced multi-modulus algorithm* (S-MMA). The S-MMA cost function is

$$
J = E\left[\left(y_R^2(n) - \left|\hat{a}_R(n)\right|^c R_R\right)^2 + \left(y_I^2(n) - \left|\hat{a}_I(n)\right|^c R_I\right)^2\right].
$$

The corresponding S-MMA tap updating algorithm is

$$
\begin{aligned}
&\mathbf{w}(n+1) \\
&= \mathbf{w}(n) + \mu\Big[y_R(n)\left(\left|\hat{a}_R(n)\right|^c R_R - y_R^2(n)\right) \\
&\qquad + j.y_I(n)\left(\left|\hat{a}_I(n)\right|^c R_I - y_I^2(n)\right)\Big] \\
&\qquad \cdot \mathbf{x}^*(n),
\end{aligned} \tag{20}
$$

where c is a positive constant (possibly $c \leq 1$). A closer look at (20) reveals that the S-MMA has an update rule very similar to the MMA (as specified in (8)). Especially, for $c = 0$, the S-MMA reduces to the MMA. Observe that, due to using both equalizer and slicer outputs, the update rule (20) forces $y_R(n)$ and $y_I(n)$ to lie on point contours of values $\text{sign}[y_R(n)]\sqrt{|\hat{a}_R(n)|^c R_R}$ and $\text{sign}[y_I(n)]\sqrt{|\hat{a}_I(n)|^c R_I}$, respectively, where sign[·] is a standard signum function. In this way, the S-MMA update mechanism is aware of the dispersion of $y(n)$ away from the closest symbol $\hat{a}(n)$ in some statistical sense.

1. Misadjustment of S-MMA

Our main intention in the proposal of the S-MMA has been to propose a new algorithm, similar to the MMA, such that the steady-state misadjustment is minimized. Based on the procedure discussed in section V, the values of the misadjustment metrics γ_1 and γ_2, obtained for the S-MMA, are as follows (c = 1):

$$\gamma_1 = 2E\left[|a(m)|^2 \cdot a_R^2(m)\left(a_R^2(m) - |a_R(m)|R_R\right)^2\right],$$
$$\gamma_2 = 4E[a_R^2(m)] \cdot E\left[a_R^2(m)\left(a_R^2(m) - |a_R(m)|R_R\right)^2\right]. \quad (21)$$

The values of γ_1 and γ_2, as specified in (21), are much smaller than those in (19). To corroborate this claim, the ratios of γ_1's and γ_2's, exhibited by the MMA and the S-MMA are depicted in Fig. 2 for different QAM constellation sizes. Observe that the misadjustment metrics exhibited by the S-MMA are at least five times smaller than those of the MMA.

To gain further insight, we compare the error-functions of the MMA and S-MMA The error-functions of the MMA and S-MMA are depicted in Fig. 3 for 64-QAM. Observe that the error-function of the MMA is continuous while the S-MMA error-function is discontinuous; or in other words, the S-MMA exhibits a piecewise combination of several continuous error-functions. Due to a continuous error-function, the MMA forces all symbols to lie on a single contour; while due to a discontinuous error function, the S-MMA forces the equalizer output to lie on one of the multiple contours, the one statistically closest to it. Because of these increased contours in the S-MMA, the average distance between the constellation symbols and their respective contours is decreased, and as a result it yields a smaller steady state misadjustment.

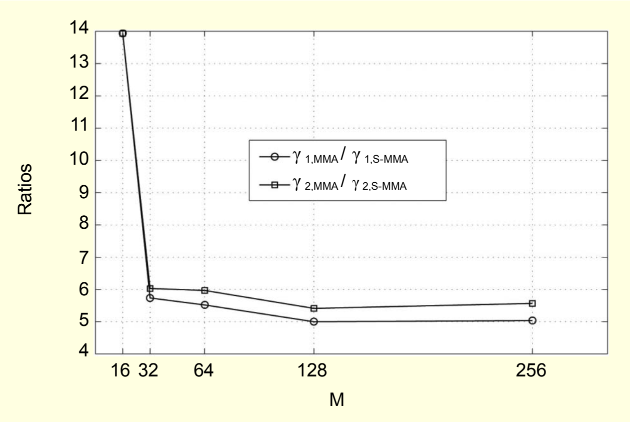

Fig. 2. Ratios of γ_1's and γ_2's for MMA and S-MMA.

2. Dispersion Constants of S-MMA

Dispersion constants in a blind equalization algorithm are pre-computed positive constants which play a vital role in adjusting the *gain* of the equalizer such that the statistics of the equalizer output $y(n)$ (upon convergence) get matched with the statistics of the transmitted signal $a(n)$. So, they contain the information about the *size* (number of symbols), *shape* (whether square, cross, or something else), and *energy* (which reflects the average symbol-to-symbol distance) of the transmitted signal.

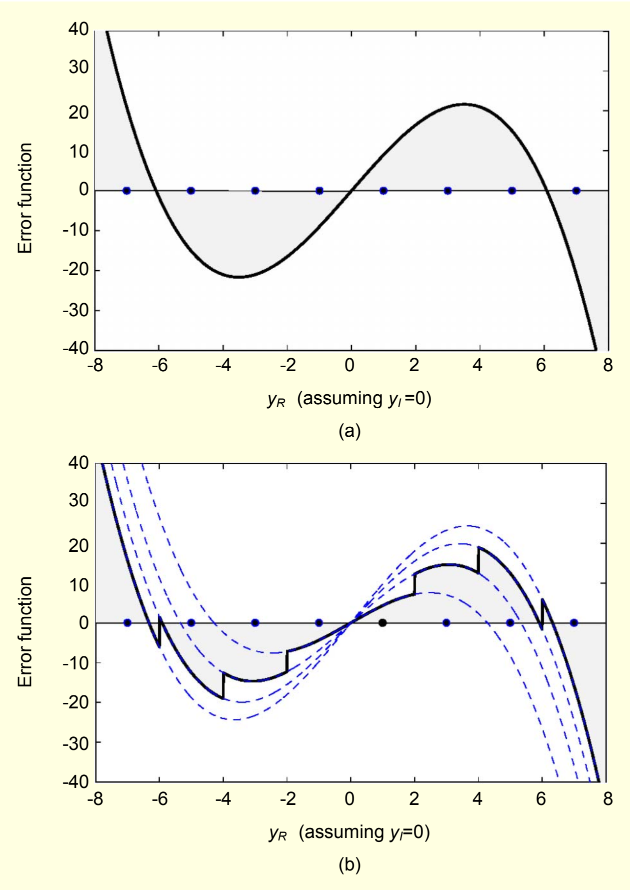

Fig. 3. (a) MMA and (b) S-MMA (c = 0.4) error-functions for 64-QAM.

Assuming a perfect convergence condition, that is $y(n) = a(n)$, the dispersion constants R_R and R_I in the S-MMA are computed by solving the equations,

$$E\left[a^*(n)a_R(n)\left(|a_R(n)|^c R_R - a_R^2(n)\right)\right] = 0,$$
$$E\left[a^*(n)a_I(n)\left(|a_I(n)|^c R_I - a_I^2(n)\right)\right] = 0,$$

which gives

$$R_R = E[a_R^4] / E\left[|a_R|^{2+c}\right], \quad (22)$$

$$R_I = E[a_I^4] / E\left[|a_I|^{2+c}\right]. \quad (23)$$

Note that the incorporation of c in R_R and R_I reflects the weight of the information contributed by $|\hat{a}_R(n)|$ and $|\hat{a}_I(n)|$, respectively.

Consider the case of the very first attempt, made by Benveniste et al. [22], in which the misadjustment was reduced by using the weighted sum of blind and DD errors. However, due to the nonlinearity of the DD error, it was impossible to incorporate the contribution of $\hat{a}(n)$ into the dispersion constants. Later, other researchers proposed dual-mode solutions (for example [7]), where the blind equalizer made the switching between blind and DD modes of adaptation. Since dual-mode is actually a special case of Benveniste's idea (in which the weights are simply replaced with binary flags), the dispersion constants were again blind to the contribution made by $\hat{a}(n)$.

3. Dimensions of the S-MMA

Assume that the equalizer has successfully converged and the equalizer output $y(n)$ is in a close vicinity to the constellation symbol $\hat{a}(n)$ within an angle θ, such that $y(n)=\hat{a}(n)e^{-j\theta}$ (probably due to a residual phase-offset). The real and imaginary parts of $\hat{a}(n)$ can be expressed in terms of $y(n)=y_R(n)+j.y_I(n)$ and θ as follows:

$$\hat{a}_R(n)=y_R(n)\cos(\theta)-y_I(n)\sin(\theta), \tag{24}$$

$$\hat{a}_I(n)=y_I(n)\cos(\theta)+y_R(n)\sin(\theta), \tag{25}$$

where

$$\theta=\arctan\left(\frac{\hat{a}_I(n)}{\hat{a}_R(n)}\right)-\arctan\left(\frac{y_I(n)}{y_R(n)}\right). \tag{26}$$

For the sake of simplicity, we assume that both $y(n)$ and $\hat{a}(n)$ lie in the first quadrant. Under this assumption, $|\hat{a}_R(n)|=\hat{a}_R(n)$ and $|\hat{a}_I(n)|=\hat{a}_I(n)$. For c = 1, we obtain the real part of the error-function in (20) as follows:

$$\text{S-MMA}: \quad \psi_R[y(n)]=-y_R(n)\left[\{y_R(n)\cos(\theta)-y_I(n)\sin(\theta)\}R_R-y_R^2(n)\right]. \tag{27}$$

On the other hand, the real part of the error-function in the CMA update rule (6) is

$$\text{CMA}: \quad \psi_R[y(n)]=-y_R(n)[R-y_R^2(n)-y_I^2(n)]. \tag{28}$$

Similarly, the real part of the error-function in the MMA update rule (8) is

$$\text{MMA}: \quad \psi_R[y(n)]=-y_R(n)[R_R-y_R^2(n)]. \tag{29}$$

Comparing (29), (28) and (27), it reveals that the error functions in the MMA, CMA and S-MMA are one-, two- and pseudo-two-dimensional, respectively. The S-MMA is pseudo-two-dimensional because it contains both $y_R(n)$ and $y_I(n)$ only when θ is not equal to zero ($\theta \neq 0$). Also, by considering the angle θ, it is apparent that the S-MMA is utilizing more information (or higher statistics in a loose sense) than both the CMA and MMA.

4. Complexity of the S-MMA

The computational complexity of the S-MMA is almost the same as that of the MMA. In fact, the dispersion constants R_R and R_I in the MMA are replaced with $|\hat{a}_R(n)|^c R_R$ and $|\hat{a}_I(n)|^c R_I$, respectively. However, these constants need not be computed at each iteration. For example, for 16-QAM, only two values R_R and 3^cR_R (= R_I and 3^cR_I, resp.) are needed to be precomputed and stored in registers. The binary representation for $|\hat{a}_{R;I}|$ can be used to address these registers. Thus, the S-MMA needs some extra storage space compared to the MMA.

VII. Simulation Results

In simulations, a complex-valued seven-tap transversal equalizer was used and initialized so that the center tap was set to one and the other taps were set to zero. The channel is a complex-valued representation of a typical voice band telephone channel, taken from [23] with additive white Gaussian noise. The residual ISI and symbol-error rate (SER) are measured and compared as performance parameters. Each of the ISI traces is obtained from the ensemble average of 200 independent Monte Carlo experiments. The values of dispersion constants and step-sizes used in these experiments are mentioned in the labels of the figures.

Figures 4, 5 and 6 depict residual ISI plots for 16-, 64- and 256-QAM constellations, respectively, for the MMA and S-MMA (c = 1). It can be observed that the steady-state ISI in the S-MMA is lower by at least 5dB compared to those obtained from the MMA. It is evident from the ISI curves that the S-MMA improves the equalizer performance by offering better removal of multipath effects at a steady-state without compromising over the convergence rate.

A lower residual ISI floor directly implies a lesser

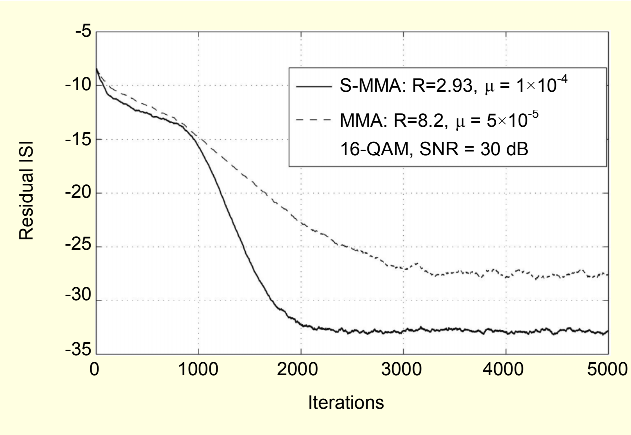

Fig. 4. Residual ISI for 16-QAM signaling.

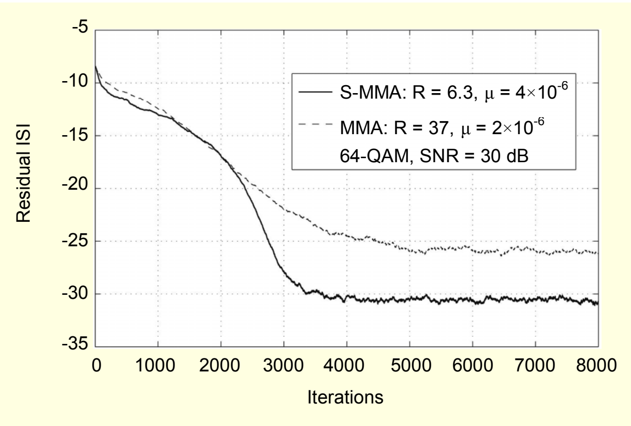

Fig. 5. Residual ISI for 64-QAM signaling.

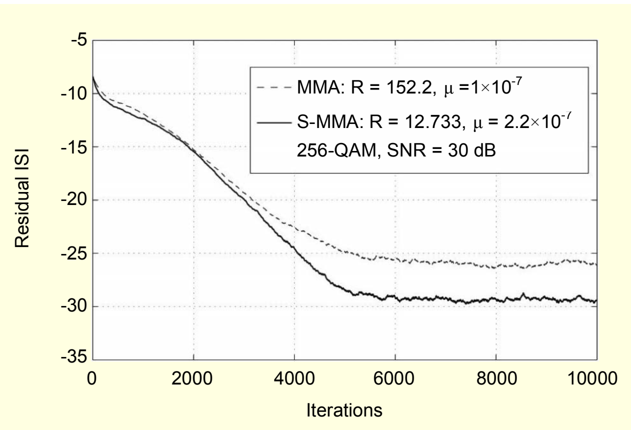

Fig. 6. Residual ISI for 256-QAM signaling.

misadjustment. It is also evident from this set of experiments that the S-MMA is capable of giving better performance regardless of the size of the QAM constellation.

Next, we compare the performance of the MMA and S-MMA algorithms by measuring the SER versus SNR. Figures 7(a), 7(b) and 7(c) depict SER plots for 16-, 64-, and 256-QAM constellations, respectively. Each of the SER plots is computed when convergence has been achieved at the same rate for both the S-MMA and MMA (the first 15000 symbols were not used in the computation of the SER). The values of step-sizes are similar to those which are used in the results shown in the legends of Figs. 4 through 6. Remember, the SER of 10^{-2} is usually considered acceptable to guarantee a safe switch between the blind equalization and DD tap-updating algorithms [16]. Observe that the S-MMA is yielding a similar SER as those of the MMA at a low SNR; however, for a higher SNR, the S-MMA is outperforming the MMA. Especially, for 256-QAM, the SER of the S-MMA is approaching 10^{-3} while the SER of the MMA is far above 10^{-2}. It can be concluded that the performance of the S-MMA becomes far better than the MMA at higher constellation sizes. Further improvement in SER can be gained by using coding techniques in a transmitted data sequence; a detailed analysis of the performance of different coding schemes in a blind receiver has been explored in [24].

Blind equalizers are usually switched to the DD equalization mode once the error level is reasonably low. Considerable attention has to be paid in determining the point at which this switch-over is made in order to avoid the error-propagation effects associated with the DD mode. Non-convergence of the algorithm or large steady state errors may result if the algorithm is switched to the DD mode too early. On the other hand, too late a switch to the DD mode may result in a long delay in the convergence process. The switchover process is thus not trivial and demands extra computation to determine the error level [25].

Under the above mentioned scenario, observe that the S-MMA is not only capable of providing a reliable initial convergence but also minimizes the error level (by decreasing the misadjustment at almost no additional cost), so that a reliable switch-over to DD mode is possible. It can be argued that the S-MMA is using $\hat{a}(n)$ from the start-up, when $\hat{a}(n)$ cannot be considered reliable. This problem can be solved by selecting a small value of c at start-up. However, on the basis of computer simulations on many different channels, it is observed that for *square* constellations, c can reliably be selected as high as 1 from the start-up phase. Moreover, for *non-square* constellations, the reliable convergence along with a smaller misadjustment were observed to be achieved with values of c much smaller than 1 (these results are omitted to limit the size of this paper).

Finally, the idea of slicing the error-function can equally be applied to the CMA and/or other error-function-based algorithms to decrease their misadjustment.

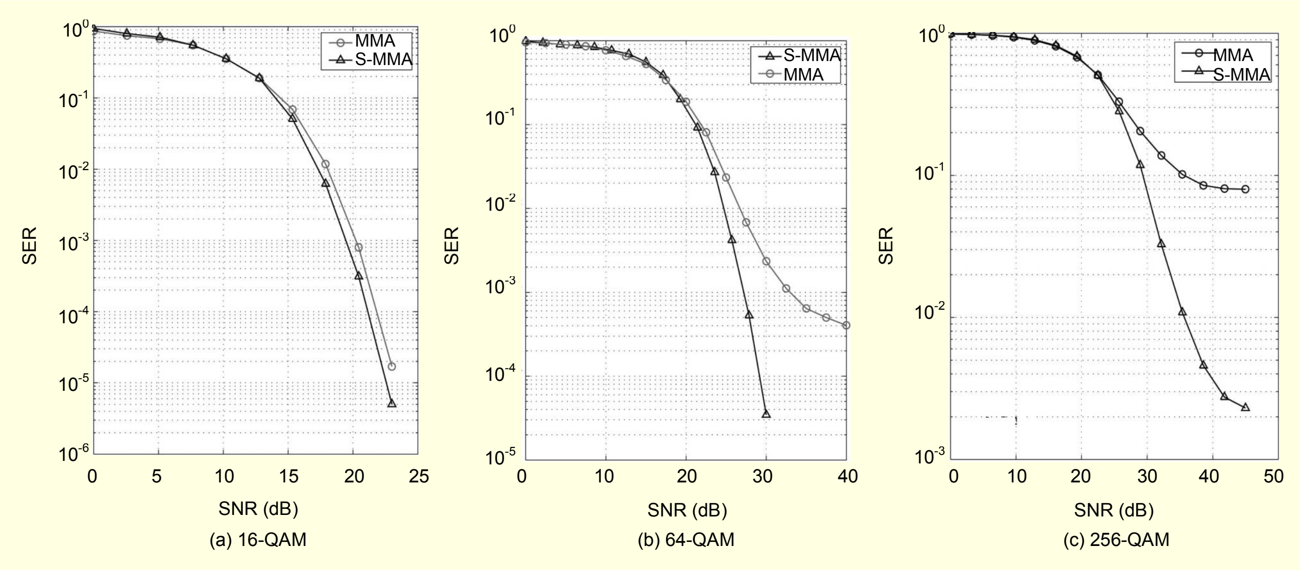

Fig. 7. SER results for (a) 16-, (b) 64- and (c) 256-QAM.

VIII. Conclusion

In this work, we have introduced an adaptive equalizer for the blind equalization of QAM signals that minimizes a cost function composed of equalized and sliced symbols. The contribution lies in the technique to incorporate the sliced symbols in the multi-modulus type weight adaptation process. The proposed implementation is referred to as the sliced multi-modulus algorithm (S-MMA). The steady-state misadjustment analysis of an existing technique and the proposed one is carried out. Both analysis and simulations demonstrate the advantage of using the proposed cost function over the traditional multi-modulus cost function associated with the conventional MMA. The performance of the equalizers implementing the MMA and S-MMA are compared. The simulation-based experiments show that the S-MMA exhibits a superior performance compared to the MMA yielding a better residual ISI and SER, without compromising the convergence rate.

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References

[1] J.R. Treichler and B.G. Agee, "A New Approach to Multipath Correction of Constant Modulus Signals," *IEEE Trans. Acoust. Speech Signal Proc.*, vol. ASSP-31, 1983, pp. 459–471.

[2] D.N. Godard, "Self-Recovering Equalization and Carrier Tracking in Two-Dimensional Data Communications Systems," *IEEE Trans. Commun.*, vol. COM-28, Nov. 1980, pp. 1867–1875.

[3] W.A. Sethares, G.A. Rey, and Jr. C.R. Johnson, "Approach to Blind Equalization of Signal with Multiple Modulus," *IEEE Proc. ICASSP*, Apr. 1989, pp. 972–975.

[4] R.A. Axford Jr., L.B. Milstein, and J.R. Zeidler, "The Transversal and Lattice Multiple Modulus Algorithms for Blind Equalization Of QAM Signals," *Proc. IEEE MILCOM '95*, vol. 2, Nov. 1995, pp. 586–591.

[5] M.J. Ready and R.P. Gooch, "Blind Equalization Based on Radius Directed Adaptation," *Proc. IEEE ICASSP*, Apr. 1990, pp. 1699–1702.

[6] Roy A. Axford Jr., *Refined Techniques for Blind Equalization of Phase Shift Keyed (PSK) and Quadrature Amplitude Modulated (QAM) Digital Communications Signals,* PhD thesis, University of California, San Diego, 1995.

[7] V. Weerackody and S.A. Kassam, "Dual-Mode Type Algorithms for Blind Equalization," *IEEE Trans. Commun.*, vol. 42, no. 1, Jan. 1994, pp. 22–28.

[8] Ching-Hsiang Tseng and Cheng-Bin Lin, "A Stop-and-Go Dual Mode Algorithm for Blind Equalization," *Proc. GLOBECOM '96*, vol. 2, pp. 1427–1431, 1996.

[9] Jai-Chul Song, "The Performance Evaluation Of Stop-And-Go Dual-Mode Algorithm For Blind Equalization," *Proc. IEEE ICICS*, vol. 1, Sept. 1997, pp. 336–339.

[10] R.A. Axford Jr., L.B. Milstein, and J.R. Zeidler, "A Dual-Mode Algorithm for Blind Equalization of QAM Signals: CADAMA," *Proc. Asilomar-29*, vol. 1, 1996, pp. 172–176.

[11] M. Shahmohammadi and M.H. Kahaei, "A New Dual-Mode Approach to Blind Equalization Of QAM Signals," *Proc. IEEE 8th Int'l Symp. Computers & Commun.*, 2003, pp. 277–281.

[12] K. Wesolowski, "Self-Recovering Adaptive Equalization

Algorithms for Digital Radio and Voiceband Data Modems," *Proc. European Conf. Circuit Theory and Design*, 1987, pp.19–24.

[13] K.N. Oh and Y.O. Chin, "Modified Constant Modulus Algorithm: Blind Equalization and Carrier Phase Recovery Algorithm," *Proc. IEEE Int'l Conf. Commun.*, vol. 1, June 1995, pp. 498–502.

[14] J. Yang, J.-J. Werner, and G.A. Dumont, "The Multimodulus Blind Equalization Algorithm," *Proc. IEEE Int'l Conf. DSP*, vol. 1, 1997, pp. 127–130.

[15] L.M. Garth, J. Yang, and J.-J. Werner, "Blind Equalization Algorithms for Dual-Mode CAP-QAM Reception," *IEEE Trans. Commun.*, vol. 49, no. 3, Mar. 2001, pp. 455–466.

[16] J. Yang, J.-J. Werner, and G.A. Dumont, "The Multimodulus Blind Equalization and its Generalized Algorithms," *IEEE J. Selected Areas Commun.,* vol. 20, no. 5, June 2002, pp. 997–1015.

[17] J. Yang, J.-J. Werner, D.D. Harman, and G.A. Dumont, "Blind Equalization For Broadband Access," *IEEE Commun. Magazine,* vol. 37, no. 4, Apr. 1999, pp. 87–93.

[18] Kun-Da Tsai and Jenq-Tay Yuan, "A Modified Constant Modulus Algorithm (MCMA) for Joint Blind Equalization and Carrier Recovery in Two-Dimensional Digital Communication Systems," *Proc. 7th Int'l Symp. Signal Processing & Applications*, vol. 2, July 2003, pp. 563–566.

[19] K. Wesolowski, "Analysis and Properties of the Modified Constant Modulus Algorithm for Blind Equalization," *European Trans. Telecommunications*, vol. 3, no. 3, May-June 1992, pp. 225–230.

[20] N. K. Jablon, "Joint Blind Equalization, Carrier Recovery and Timing Recovery for High-Order QAM Constellations," *IEEE Trans. Signal Proc.*, vol.40, pp. 1383–1398, 1992.

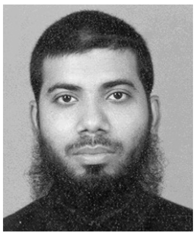
[21] A. Benveniste, M. Goursat, and G. Ruget, "Robust Identification of a Nonminimum Phase System: Blind Adjustment of a Linear Equalizer in Data Communications," *IEEE Trans. Automatic Control*, vol. AC-25, no. 3, June 1980, pp. 385–389.

[22] A. Benveniste and M. Goursat, "Blind Equalizers," *IEEE Trans. Commun.*, vol. COM-32, 1984, pp. 871–883.

[23] G. Picchi and G. Prati, "Blind Equalization and Carrier Recovery using a Stop-and-Go Decision-Directed Algorithm," *IEEE Trans. Commun.*, vol. COM-35, 1987, pp. 877–887.

[24] S. Vlahoyiannatos, C.-S. Lee, and L. Hanzo, "Satellite Based Turbo-Coded, Blind-Equalized 4-QAM and 16-QAM Digital Video Broadcasting," *IEEE Trans. Broadcasting*, vol. 46, no. 1, Mar. 2000, pp. 23–33.

[25] K. Hilal and P. Duhamel, "A Blind Equalizer Allowing Soft Transition between the Constant Modulus and the Decision-Directed Algorithm for PSK Modulated Signals," *Proc. IEEE Int'l Conf. Commun., ICC'93*, vol. 2, 1993, pp. 1144–1148.



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