KYUNGPOOK Math. J. 45(2005), 231-240

Hypersurfaces of the Contact Metric Manifold with a Nullity Condition and φ-constant Sectional Curvature

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Abstract. The paper shows that a hypersurface with constant curvature with the condition $hD \subset D$, of a contact metric manifold with a nullity condition and φ-constant sectional curvature, has the curvature equals to k and $\mu = 0$.

1. Introduction

The theory of CR-submanifold has been introduced by A. Bejancu in [1] and was developed in many papers. In [1] is given the first and the most important example of CR-submanifold, which is the hypersurface.

Let $\widetilde{M}(\varphi,\ \xi,\ \eta,\ \widetilde{g})$ be a $(2n+1)$-dimensional contact metric manifold with the contact structure given by the Riemannian metric $\widetilde{g}$, structure vector field ξ, 1-form η and the $(1,1)$-*tensor* field φ. A submanifold M of $\widetilde{M}$ is called a *CR-submanifold* (*semi-invariant,* [2]) if there exist two differentiable distributions D and $D^{\perp}$ on M which satisfying

(a) $TM = D \oplus D^{\perp} \oplus \{\xi\}$, where $D \oplus D^{\perp}$ and $\{\xi\}$ are mutually orthogonal to each other

(b) the distribution D is invariant by φ, that $\varphi(D_x) = D_x$ for each $x \in M$

(c) the distribution $D^{\perp}$ is anti-invariant by φ, that $\varphi(D_x^{\perp}) = T_xM^{\perp}$ for each $x \in M$.

In this way we obtain in $TM^{\perp}$ a vector subbundle $\nu^{\perp} = \varphi(D^{\perp})$. The complementary orthogonal subbundle to $\nu^{\perp}$ in $TM^{\perp}$ well denote by ν, so that we have the decomposition
$$TM^{\perp} = \nu \oplus \nu^{\perp},$$
and ν is invariant to φ, i.e., $\varphi\nu \subseteq \nu$.

The goal of this paper is to study the hypersurfaces of contact metric manifolds $\widetilde{M}(c)$ with a nullity condition and having the φ-*sectional* curvature as a constant number denoted by c.

Received March 3, 2004.
2000 Mathematics Subject Classification: 53C25, 53D10, 53D15.
Key words and phrases: contact metric manifolds, hypersurfaces, submanifolds.

Let $\widetilde{M}$ be a Riemannian manifold. It is known that the tangent sphere bundle $T_1\widetilde{M}$ admits a contact Riemannian structure $(\varphi,\ \xi,\ \eta,\ g)$. $T_1\widetilde{M}$ together with this structure is a contact Riemannian manifold. If $\widetilde{M}$ is of constant sectional curvature $c = 1$, then $T_1\widetilde{M}$ is a Sasakian manifold, i.e., its curvature tensor $\widetilde{R}$ satisfies $\widetilde{R}(X,Y)\xi = \eta(Y)X - \eta(X)Y$ for all vector fields X, Y. If $c = 0$, then the curvature tensor of $T_1\widetilde{M}$ satisfies the condition $\widetilde{R}(X,Y)\xi = 0$. Applying a D-homothetic deformation on a contact Riemannian manifold satisfying $\widetilde{R}(X,Y)\xi = 0$, we get a contact Riemannian manifold such that $\widetilde{R}(X,Y)\xi = k[\eta(Y)X - \eta(X)Y] + \mu[\eta(Y)hX - \eta(X)hY]$, where k, μ are real constants and $2h$ is the Lie differentiation of φ in the direction of ξ. We call (see [3]) this kind of manifold, $(k,\mu)-$*contact Riemannian manifold*, or *contact metric manifold with nullity condition.*

The tensor field $\widetilde{K}(X,\varphi X) = g(\widetilde{R}(X,\varphi X)\varphi X, X)$ is called the φ-sectional curvature of $\widetilde{M}$. In [6] the author gave an expression of the curvature tensor when the ambient manifold $\widetilde{M}$ has a constant φ-sectional curvature c, denoted by $\widetilde{M}(c)$.

Theorem 4 will give as the form of the φ-sectional curvature and also some information about the Weigarten operator A on hypersurfaces.

2. Contact metric manifolds with nullity condition

A differential 1-$form$ η on a differentiable $(2n+1)$-dimensional manifold $\widetilde{M}$ is called a *contact form* if it satisfies $\eta \wedge (d\eta)^n \neq 0$ everywhere on $\widetilde{M}$. By a *contact manifold* $(\widetilde{M}, \eta)$ we mean a manifold $\widetilde{M}$ together with a contact form η. Since $d\eta$ is of rank $2n$, there exists a global vector field ξ, called the *structure vector field*, such that

(i) $\eta(\xi) = 1$ and $L_\xi \eta = 0$, where L_ξ denotes the Lie differentiation by ξ. Moreover it is well known that there exist a Riemannian metric $\widetilde{g}$ and a $(1,1)$-*tensor* field φ satisfying

(ii) $\varphi^2 = -I + \eta \oplus \xi$

(iii) $\widetilde{g}(\varphi X, \varphi Y) = \widetilde{g}(X,Y) - \eta(X)\eta(Y)$

(iv) $\varphi\xi = 0$

(v) $\eta \circ \varphi = 0$, for all vector fields $X,\ Y$ on $\widetilde{M}$.

The structure $(\varphi,\ \xi,\ \eta,\ \widetilde{g})$ is called a *contact Riemannian structure* and the manifold $\widetilde{M}$ carrying such structure is said to be a *contact metric manifold*, and we denote it by $\widetilde{M}(\varphi,\ \xi,\ \eta,\ \widetilde{g})$.

Following [3] we define and we define the $(1,1)$-*type* field h by:

$$2hX = (L_\xi \varphi)X,\ \forall X \in \Gamma(T\widetilde{M})$$

which satisfies the relations:

(vi) (a) $h\varphi = -\varphi h$ and

(vi) (b) $h\xi = 0$

Using (vi)(a), if X is an eigen vector field for h with respect the eigen value λ, then φX is also an eigen vector of h, but with respect the eigen value-λ.

The sectional curvature $\widetilde{K}(X, \varphi X)$ of a plane section spanned by a vector X orthogonal to ξ is called a φ-sectional *curvature*.

It is known that the tangent sphere bundle $T_1\widetilde{M}$ of a Riemannian manifold $\widetilde{M}$ admits a contact Riemannian structure $(\varphi,\ \xi,\ \eta,\ g)$, known as the *standard contact metric structure*.

For real constants $k,\ \mu$, the $(k,\ \mu)$-*nullity* distribution of a contact metric manifold $\widetilde{M}(\varphi,\ \xi,\ \eta,\ \widetilde{g})$ is a distribution

$$\begin{aligned} N(k,\mu) : P \to \quad & N_P(k,\mu) \\ = \quad & \Big\{ Z \in T_P\widetilde{M} \mid \widetilde{R}(X,Y)Z = k\big[g(Y,Z)X - g(X,Z)Y\big] \\ & +\mu\big[g(Y,Z)hX - g(X,Z)hY\big]\Big\}. \end{aligned}$$

So, if the structure vector field ξ belongs to the (k,μ)-*distribution* we have

$$\widetilde{R}(X,Y)\xi = k\big[\eta(Y)X - \eta(X)Y\big] + \mu\big[\eta(Y)hX - \eta(X)hY\big]. \tag{1}$$

We'll call $\widetilde{M}$ to be a $(k,\mu)-$*contact manifold*, or *contact metric manifold with nullity condition* (1), where k and μ are real contents, and $2h$ is the Lie differentiantion of φ in direction of ξ (conf. [3]).

The above construction is given in [5] and it study the $(k,\mu)-$contact manifold introduced by the authors in [3].

It's important to observe that if $h = 0$, then the contact metric manifold with nullity condition (1) is a Sasakian manifold.

The following are true ([3]);

(vii) $\widetilde{\nabla}_X\xi = -\varphi X - \varphi hX$

(viii) $\widetilde{\nabla}_\xi\varphi = 0$.

For any vector fields $X, Y \in \Gamma(T\widetilde{M})$ orthogonal to the structure vector filed ξ, $\widetilde{K}(X,Y) = g(\widetilde{R}(X,Y)Y, X)$ is called *sectional curvature* of the manifold $\widetilde{M}$.

We remind the following results:

Lemma 1 ([3]). *Let $\widetilde{M}^{2n+1}(\varphi,\ \xi,\ \eta,\ \widetilde{g})$ be a contact metric manifold with ξ belonging to the $(k,\ \mu)$-nullity distribution. Then*

(ix) $(\widetilde{\nabla}_X\varphi)Y = \widetilde{g}(X + hX, Y)\xi - \eta(Y)(X + hX)$

(x) $\widetilde{R}(\xi, X)Y = k[\widetilde{g}(X,Y)\xi - \eta(Y)X] + \mu[\widetilde{g}(hX,Y)\xi - \eta(Y)hX]$

(xi) $h^2 = (k-1)\varphi^2$, *for any $X, Y \in \Gamma(T\widetilde{M})$.*

We recall that for any vector fields $X,\ Y$ mutual orthogonal and orthogonal to the structure vector field ξ, the tensor field $\widetilde{K}(X,Y) = g(\widetilde{R}(X,Y)Y,X)$ is called the *sectional curvature* of $\widetilde{M}$ given by the sectional plane $\{X,Y\}$. The tensor field $\widetilde{K}(X,\varphi X) = g(\widetilde{R}(X,\varphi X)\varphi X,X)$ is called the φ-*sectional curvature* of $\widetilde{M}$. If the manifold $\widetilde{M}$ has a constant φ-*sectional* curvature c for any sectional plane $\{X,\varphi X\}$, we denote it by $\widetilde{M}(c)$. The sectional curvature $\widetilde{K}(X,\xi)$ of a sectional plane spanned by ξ and another vector field X orthogonal to ξ is called the ξ-*sectional curvature*.

Theorem 1 ([3]). *Let $\widetilde{M}^{2n+1}(\varphi,\ \xi,\ \eta,\ \widetilde{g})$ be a contact metric manifold, with the nullity condition* (1). *Then $k \leqslant 1$. If $k=1$, then $h=0$ and $\widetilde{M}$ is a Sasakian manifold. $k<1$, then $\widetilde{M}$ determined by the eigenspaces of h, where $\lambda=\sqrt{1-k}$.*

Theorem 2 ([3]). *Let $\widetilde{M}^{2n+1}(\varphi,\ \xi,\ \eta,\ \widetilde{g})$ be a contact metric manifold, with the nullity condition* (1)*. If $k<1$, then for any X orthogonal to ξ :*

(a) *the ξ-sectional curvature $\widetilde{K}(X,\xi)$ is given by:*

$$\widetilde{K}(X,\xi) = k + \mu\widetilde{g}(hX,X) = \begin{cases} k+\lambda\mu, & \text{if } X \in D(\lambda) \\ k-\lambda\mu, & \text{if } X \in D(-\lambda) \end{cases} \tag{2}$$

(b) *the sectional curvature of a sectional plane $\{X,\ Y\}$ normal to ξ is given by:*

$$\widetilde{K}(X,Y) = \begin{cases} 2(1+\lambda)-\mu, & X,Y \in D(\lambda) \\ -(k+\mu)(g(X,\varphi Y))^2, & X \in D(\lambda) \text{ and } Y \in D(-\lambda) \\ 2(1-\lambda)-\mu, & X,Y \in D(-\lambda) \end{cases} \tag{3}$$

(c) *$\widetilde{M}$ has constant scalar curvature, given by*

$$S = 2n[2(n-1)+k-n\mu]. \tag{4}$$

In [6] the author obtained the form for the curvature tensor field of the manifold $\widetilde{M}(c)$ with the nullity condition (1) and constant φ-sectional curvature c.

Theorem 3 ([6]). *Let $\widetilde{M}^{2n+1}(\varphi,\ \xi,\ \eta,\ \widetilde{g})$ be a contact metric manifold $(n>1)$ with ξ belonging to the $(k,\mu)-$nullity distribution. If the φ-sectional curvature of any point of $\widetilde{M}$ is independent of the choice of φ-section at the point, then it is constant on $\widetilde{M}$ and the curvature tensor is given by*

$$\begin{aligned}
(5)\ 4\widetilde{R}(X,Y)Z \;=\; & (c+3)\{\widetilde{g}(Y,Z)X-\widetilde{g}(X,Z)Y\} \\
& +(c+3-4k)\{\eta(X)\eta(Z)Y-\eta(Y)\eta(Z)X+\widetilde{g}(X,Z)\eta(Y)\xi \\
& -\widetilde{g}(Y,Z)\eta(X)\xi\} \\
& +(c-1)\{2\widetilde{g}(X,\varphi Y)\varphi Z+\widetilde{g}(X,\varphi Z)\varphi Y-\widetilde{g}(Y,\varphi Z)\varphi X\} \\
& -2\{\widetilde{g}(hX,Z)hY-\widetilde{g}(hY,Z)hX+2\widetilde{g}(X,Z)hY-2\widetilde{g}(Y,Z)hX \\
& -2\eta(X)\eta(Z)hY+2\eta(Y)\eta(Z)hX+2\widetilde{g}(hX,Z)Y-2\widetilde{g}(hY,Z)X \\
& +2\widetilde{g}(hY,Z)\eta(X)\xi-2\widetilde{g}(hX,Z)\eta(Y)\xi
\end{aligned}$$

$$-\widetilde{g}(\varphi hX, Z)\varphi hY + \widetilde{g}(\varphi hY, Z)\varphi hX\}$$
$$+4\mu\{\eta(Y)\eta(Z)hX - \eta(X)\eta(Z)hY + \widetilde{g}(hY, Z)\eta(X)\xi$$
$$-\widetilde{g}(hX, Z)\eta(Y)\xi\},$$

for any $X, Y \in \Gamma(T\widetilde{M})$, where c is the constant φ-sectional curvature. Moreover, if $k \neq 1$, then $\mu = k + 1$ and $c = -2k - 1$.

3. Basic results

Let $\widetilde{M}(\varphi,\ \xi,\ \eta,\ \widetilde{g})$ be a $(2n+1)$-dimensional contact metric manifold with the contact structure given by the Riemannian metric $\widetilde{g}$, structure vector field ξ, 1-form η and the $(1,1)$-*tensor* field φ. We denote by $\widetilde{\nabla}$ the Levi-Civita connection determined by $\widetilde{g}$ and the curvature tensor field $\widetilde{R}$. We consider M be a submanifold of $\widetilde{M}$, with the induced connection denoted by ∇, curvature tensor field R and induced metric tensor denoted also by g.

A submanifold M of a contact metric manifold $\widetilde{M}$ is called a *CR-submanifold* (*semi-invariant*, [1]) if there exist two differentiable distributions D and $D^\perp$ on M which satisfying

(a) $TM = D \oplus D^\perp \oplus \{\xi\}$, where $D \oplus D^\perp$ and $\{\xi\}$ are mutually orthogonal to each other

(b) the distribution D is invariant by φ, that $\varphi(D_x) = D_x$ for each $x \in M$

(c) the distribution $D^\perp$ is anti-invariant by φ, that $\varphi(D_x^\perp) \subseteq T_xM^\perp$ for each $x \in M$.

We denote by $2p$ and q the real dimensions of D and $D^\perp$ respectively, $x \in M$. If $p \cdot q \neq 0$ then M is called a *proper CR-submanifold.*

Any hypersurface is a CR-submanifold of $\widetilde{M}$.

This paper's purpose is to study the hypersurfaces into contact metric manifolds with nullity condition (1).

We recall some properties of hypersurfaces here.

Let $M \subset \widetilde{M}$ be an orientable hypersurface. We denote by V the normal vector field to M, into $\widetilde{M}$. Then $T\widetilde{M} = TM \oplus TM^\perp$, where $TM^\perp = span\{V\}$.

We suppose that the structure vector field ξ is tangent to M, so that we have $\widetilde{g}(V, \xi) = 0$. Let denote $U = \varphi V$. We have $\widetilde{g}(U, V) = 0$, so that the unit vector field U is tangent to M. Let $D^\perp = span\{U\}$, so that $\dim D_x^\perp = 1$, for any point $x \in M$ and also $\varphi(D^\perp) = TM^\perp$. The distribution generated by ξ will be denoted by $\{\xi\}$, and the complementary orthogonal distribution on TM of the distribution $D^\perp \oplus \{\xi\}$ will be denoted by D. So, we get the following decomposition

$$TM = D \oplus D^\perp \oplus \{\xi\}. \tag{6}$$

From the last above relation, for any $X \in \Gamma(TM)$ we can write the decomposition

$$X = PX + \omega(X)U + \eta(X)\xi, \tag{7}$$

where P is the projection operator of TM on D, and ω is a 1-form defined by

$$\omega(X) = g(X, U), \forall X \in \Gamma(TM). \tag{8}$$

From (8), applying φ, we get

$$\varphi X = \varphi PX - \omega(X)V. \tag{9}$$

If we denote $f = \varphi P$, then f is a tensor filed of type (1,1) over M and we have

$$\varphi X = fX - \omega(X)V. \tag{10}$$

If we apply the operator φ to the relation (10) and taking account on (9), we have

$$f^2 = -I + \eta \otimes \xi + \omega \otimes U. \tag{11}$$

Also,

$$\omega \circ f = 0,\ \omega(\xi) = 0,\ f\xi = 0,\ fU = 0,\ \eta \circ f = 0,\ \eta(\xi) = 1,\ \text{and } \omega(U) = 1. \tag{12}$$

From (iii) and (10) we have

Proposition 1. *Let M be a hypersurface of $\widetilde{M}$. Then*

$$g(fX, fY) = g(X, Y) - \eta(X)\eta(Y) - \omega(X)\omega(Y), \tag{13}$$

for any $X, Y \in \Gamma(TM)$.

Corollary 1. *The tensor field f is skew-symmetric, i.e., $g(fX, Y) = -g(X, fY)$.*

We recall the Gauss' and Weingarten's equations for a hypersurface

$$\widetilde{\nabla}_X Y = \nabla_X Y + g(AX, Y)V, \tag{14}$$

$$\widetilde{\nabla}_X V = -AX, \tag{15}$$

for any $X, Y \in \Gamma(TM)$, where A is the Weingarten's operator.

The Gauss equation for hypersurfaces is

$$\begin{aligned} g(\widetilde{R}(X, Y)Z, W) &= g(R(X, Y)Z, W) \\ &\quad + g(AY, W)g(AX, Z) - g(AX, W)g(AY, Z), \end{aligned} \tag{16}$$

for any $X, Y, Z, W \in \Gamma(TM)$.

In the above situations, we can write

$$hX = TX + \delta(X)V, \tag{17}$$

for any vector field $X \in \Gamma(TM)$, where δ is a 1-form defined by

$$\delta(X) = g(hX, V). \tag{18}$$

4. Main results

In this paragraph we'll suppose that M is a hypersurface of the $(2n+1)$-dimensional contact metric manifold $\widetilde{M}$, with the nullity condition (1).

Lemma 2. *We have*

$$\nabla_X \xi = -fX - fTX - \delta(X)U, \tag{19}$$

$$g(AX,\ \xi) = \omega(X) + \omega(TX), \tag{20}$$

for any $X \in \Gamma(TM)$.

Proof. Let be $X \in \Gamma(TM)$. From (vii), using the Gauss formula, we have

$$\widetilde{\nabla}_X \xi = -\varphi X - \varphi hX \tag{21}$$

i.e.,

$$\begin{aligned} \nabla_X \xi + g(AX, \xi)V &= -fX + \omega(X)V - \varphi(TX) - \delta(X)U \\ &= -fX + \omega(X)V - fTX + \omega(TX)V - \delta(X)U. \end{aligned} \tag{22}$$

Comparing by tangent and normal components, we get the results. □

From (2), (5) and Gauss equation we obtain the following:

Proposition 2. *Let* M *be a hypersurface of* $\widetilde{M}^{2n+1}(c)$. *Then the sectional curvatures of* M *are given by*:

$$K(U,\ \xi) = k + \mu g(U, hU) - [1 + g(U, hU)]^2, \tag{23}$$

$$K(X,\ \xi) = k + \mu g(X, hX) - g(hX, U)^2, \tag{24}$$

$$\begin{aligned} K(X, U) = {}& \frac{1}{4}(c+3) - 2\{g(hX, U)g(hU, X) \\ & - g(hU, U)g(hX, X) - 2g(hX, X) - 2g(hU, U) \\ & + g(hX, \varphi U)g(\varphi hU, X) - g(hU, \varphi U)g(\varphi hX, X)\} \\ & - g(AX, U)^2 + g(AX, X)g(AU, U), \end{aligned} \tag{25}$$

for any unitary vector field $X \in \Gamma(D)$.

Corollary 2. *If* M *is a hypersurface in* $\widetilde{M}^{2n+1}$ *so that* $g(hU, U) = 0$, *then* $K(U, \xi) = k - 1$.

Corollary 3. *If* M *is a hypersurface in* $\widetilde{M}^{2n+1}$ *so that*

(a) $g(hX,X)=0$, *and*

(b) $g(hX,U)=0$, *for any* $X\in\Gamma(D)$,

then $K(X,\xi)=\widetilde{K}(X,\xi)$, *for any* $X\in\Gamma(D)$.

Theorem 4. *Let M be a hypersurface, with $hD\subset D$ and constant curvature C, of $\widetilde{M}^{2n+1}$. We have $C=k$, $\mu=0$ and $A/_D=0$.*

Proof. If M is a hypersurface with constant curvature C. We have:

$$R(X,\xi)Y=C[\eta(Y)X-g(X,Y)\xi], \tag{26}$$

for any vector fields $X,Y\in\Gamma(TM)$.

On the other hand, using the Gauss equation and (20), from (x) we have:

$$\begin{aligned} R(X,\xi)Y &= k[\eta(Y)X-g(X,Y)\xi]+\mu[\eta(Y)hX-g(hX,Y)\xi] \\ &\quad +g(Y+hY,U)AX-g(AX,Y)A\xi, \end{aligned}$$

for any vector fields $X,Y\in\Gamma(TM)$.

From (26) and (27) we get

$$\begin{aligned} &(k-C)[\eta(Y)X-g(X,Y)\xi]+\mu[\eta(Y)hX-g(hX,Y)\xi] \\ &+g(Y+hY,U)AX-g(AX,Y)A\xi \;=\; 0, \end{aligned} \tag{27}$$

for any $X,Y\in\Gamma(TM)$.

Now, we suppose that $Y\in\Gamma(D)$. Taking account on hypothesis that $hD\subset D$, from (28) we derive that:

$$(C-k)g(X,Y)\xi-\mu g(hX,Y)\xi-g(AX,Y)A\xi=0. \tag{28}$$

We recall (20), where we have that $A\xi$ and ξ are linear independent and, also, $g(AU,\xi)=1+g(hU,U)$. So, from (29), we obtain:

$$(C-k)g(X,Y)-\mu g(hX,Y)=0 \tag{29}$$

and

$$g(AX,Y)=0, \tag{30}$$

for any $X\in\Gamma(TM)$.

From (31) we obtain that $A/_D=0$.

In (30) we chose $X=Y$ to be an unit vector field, and we obtain

$$C-k=\mu g(hY,Y), \tag{31}$$

for any $Y\in\Gamma(D)$.

Now, if we replace Y by φY, which is also unitary vector field, and taking account of (vi)(a) we have

$$C - k = \mu g(h\varphi Y, \varphi Y) = -\mu g(hY, Y). \tag{32}$$

From the last two relations, (32) and (33), we have the rest of the proof. □

On the final of this paper we'll suppose that $\widetilde{M}(\varphi,\ \xi,\ \eta,\ g)$ is a contact metric manifold with the nullity condition which has the dimension 3, and M be a surface, i.e., $\dim M = 2$.

Because $\dim M = 2$ and $TM = span\{U, \xi\}$, we have that $A\xi = \alpha U$ and from (17) we get

$$\begin{aligned} g(\widetilde{R}(U,\xi)\xi, U) &= g(R(U,\xi)\xi, U) - g(AU, U)g(A\xi, \xi) \\ &\quad + g(A\xi, U)g(AU, \xi), \end{aligned} \tag{33}$$

i.e.,

$$\widetilde{K}(U,\xi) = K(U,\xi) + \alpha^2, \tag{34}$$

where $\widetilde{K}(U,\xi)$ and $K(U,\ \xi)$ are the sectional curvatures of $\widetilde{M}$ and M, respectively, given by the sectional plane $\{U,\ \xi\}$.

Proposition 3. *If M is a surface of the contact metric manifold $\widetilde{M}^{2n+1}$ with the nullity condition, then the curvature of M is $k - \mu + \alpha(\mu - \alpha)$ and $A\xi = U + g(hU, U)U$.*

Proof. From (3) and (36) we have

$$K(U,\ \xi) = k + \mu g(hU, U) - \alpha^2. \tag{35}$$

On the other hand, from (21) we have

$$g(hU,\ U) = \alpha - 1, \tag{36}$$

so we get the first relation of the Theorem.

The second relation is deriving from the dimension of M and from (21). □

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