Realization of *n*-th Order Voltage Transfer Function Using a Single Operational Transresistance Amplifier

Selçuk Kilinç and Ugur Çam

ABSTRACT—A new configuration to realize the most general n-th order voltage transfer function is proposed. It employs only one operational transresistance amplifier (OTRA) as the active element. In the synthesis of the transfer function, the RC:–RC decomposition technique is used. To the best of the author's knowledge, this is the first topology to be used in the realization of an n-th order transfer function employing a single OTRA.

Keywords—High-order filters, operational transresistance amplifier, transfer function synthesis.

I. Introduction

The operational transresistance amplifier (OTRA), known also as a current differencing amplifier or Norton amplifier, is an important active element in analog integrated circuits and systems. Both input and output terminals of an OTRA are characterized by low impedance, thereby eliminating response limitations incurred by capacitive time constants. The input terminals are internally grounded, leading to circuits that are insensitive to stray capacitances. Thus, it is possible to obtain very accurate transfer functions by using an OTRA in contrast to its unity-gain active device counterparts. Furthermore, it has been shown that the differential current input nature of this device considerably simplifies the implementation of a MOS-C analog integrated circuit [1]. OTRA has the advantages of a high slew rate and wide bandwidth due to the fact that it benefits from the current processing capabilities at the input terminals. On the other hand, since its output terminal is characterized as having low impedance, an OTRA is suitable for voltage mode operations, keeping compatibility with existing signal processing circuits [1].

Some filter applications of an OTRA are present in the literature [1]-[8]. Also, realizations of *n*-th order transfer functions using OTRAs were reported a long time ago [9]-[12]. To synthesize an *n*-th order transfer function, [9] and [10] need *n*+1 active elements, while [11] and [12] require *n* OTRAs. In this paper, we present a configuration that is suitable for a high-order filter response, involving a single OTRA and the RC:–RC decomposition technique. This is a significant reduction in comparison with the previously reported configurations.

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II. Proposed Configuration

The general configuration to be used in the realization of an *n*-th order transfer function is shown in Fig. 1. With the following defining equations of an OTRA,

$$\begin{bmatrix} V_p \\ V_n \\ V_z \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ R_m & -R_m & 0 \end{bmatrix} \begin{bmatrix} I_p \\ I_n \\ I_z \end{bmatrix}, \qquad (1)$$

where R_m is the transresistance gain and ideally approaches to infinity forcing the input currents to be equal, the voltage transfer function of the network in Fig. 1 is found as

$$T(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{Y_a - Y_b}{Y_c - Y_d}, \qquad (2)$$

where Y_a, Y_b, Y_c and Y_d are positive real admittance functions of

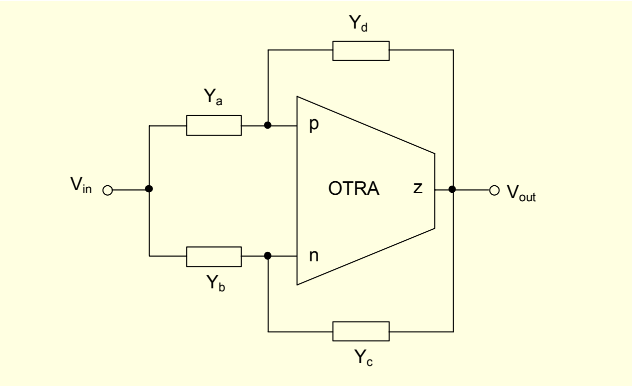

Fig. 1. Proposed configuration.

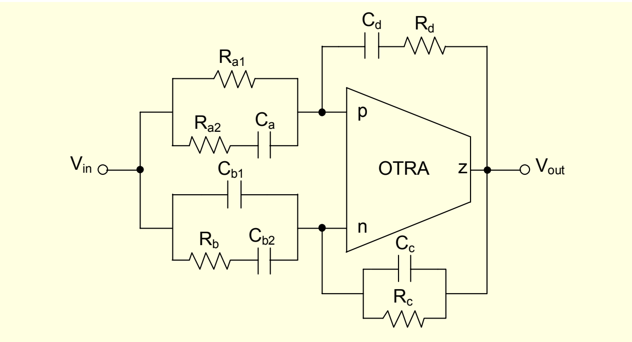

Fig. 2. Third-order all-pass filter.

passive two-terminal elements. One of their terminals is internally grounded due to input properties of an OTRA. In the literature, a current-differencing buffered-amplifier-based n-th order current transfer function is realized using the same transfer function as (2) [13].

The form of $T(s)$ in (2) and the RC:–RC decomposition technique prove that the proposed configuration can realize any voltage transfer function of the form

$$T(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{a_m s^m + a_{m-1}s^{m-1} + \cdots + a_1 s + a_0}{b_n s^n + b_{n-1}s^{n-1} + \cdots + b_1 s + b_0}, \quad (3)$$

where $m \le n$ and a_i's and b_i's are real constants indicating coefficients of numerator and denominator polynomials, respectively. Note that to realize the prescribed transfer function $T(s)$ of (3), we write

$$T(s) = \frac{A(s)}{B(s)} = \frac{A(s)/D(s)}{B(s)/D(s)}, \quad (4)$$

where $D(s)$ is an arbitrary polynomial of degree n_D having only a simple negative real root, and where $n_D \ge \max(m,n) - 1$. Note that m and n are the degrees of the numerator and denominator polynomials, respectively [13].

III. Third-Order All-Pass and Low-Pass Filter Examples

As an example, a network for a third-order normalized all-pass function $T(s) = (-s^3+2s^2-2s+1) \; / \; (s^3+2s^2+2s+1)$ is obtained and simulated using the configuration in Fig. 1. In this design, the RC:–RC decomposition technique is used by choosing the arbitrary polynomial as D(s) = (s+1) (s+2). From (4), $T(s)$ can be written as

$$T(s) = \frac{(-s^3 + 2s^2 - 2s + 1)/[(s+1)\,(s+2)]}{(s^3 + 2s^2 + 2s + 1)/[(s+1)\,(s+2)]}. \quad (5)$$

Considering the transfer function of the proposed configuration given in (2) and equating its numerator to the numerator of (5) yields

$$\begin{aligned} Y_a - Y_b &= \frac{-s^3 + 2s^2 - 2s + 1}{(s+1)(s+2)} \\ &= -s + 5 + \frac{6}{s+1} - \frac{21}{s+2} \\ &= -s + \frac{1}{2} + \left(-6 + \frac{6}{s+1}\right) + \left(\frac{21}{2} - \frac{21}{s+2}\right) \\ &= -s + \frac{1}{2} - \frac{6s}{s+1} + \frac{21s}{2s+4} \\ &= \left(\frac{1}{2} + \frac{21s}{2s+4}\right) - \left(s + \frac{6s}{s+1}\right). \end{aligned} \quad (6)$$

From (6), the driving-point RC admittance functions are found as $Y_a = 1/2 + 21s/(2s+4)$ and $Y_b = s + 6s/(s+1)$. If the same procedure is applied for the denominators of (2) and (5), it is found that $Y_c = s + 1/2$ and $Y_d = 3s/(2s+4)$. The resulting third-order all-pass filter is shown in Fig. 2. Normalized values of passive components comprising the admittances are found as R_{a1} = 2 Ω, R_{a2} = 2/21 Ω, C_a = 21/4 F, R_b = 1/6 Ω, C_{b1} = 1 F, C_{b2} = 6 F, R_c = 2 Ω, C_c = 1 F, R_d = 2/3 Ω, and C_d = 3/4 F. If we choose the impedance scaling factor as 80 × 10^3 and the frequency scaling factor as 2π × 100 × 10^3, the element values of the filter are calculated as R_{a1} = 160 kΩ, R_{a2} = 7.619 kΩ, C_a = 104.445 pF, R_b = 13.333 kΩ, C_{b1} = 19.894 pF, C_{b2} = 119.366 pF, R_c = 160 kΩ, C_c = 19.894 pF, R_d = 53.333 kΩ, and C_d = 14.921 pF. This choice leads to a resonant frequency of f_0=100 kHz.

The filter is simulated using the SPICE program. The CMOS configuration proposed in [1] is used for the realization of the OTRA, with the MIETEC 0.5 μ CMOS process parameters. In this configuration, the supply voltages are chosen as VDD = 2.5 V and VSS = –2.5 V. As can be seen from the gain and phase responses of Fig. 3, the simulated results agree quite well with the theoretical ones.

The large signal behavior of the filter is also tested with

SPICE simulations for a sinusoidal input voltage. Figure 4 shows the simulated transient response of the third-order all-pass filter. It can be seen from this figure that the dynamic range of the circuit extends up to an amplitude of 4 V peak-to-peak. The dependence of the output harmonic distortion on the input signal amplitude is illustrated in Fig. 5.

As another example, a network for a third-order normalized low-pass function $T(s) = 1/(s^3+2s^2+2s+1)$ is obtained. In this

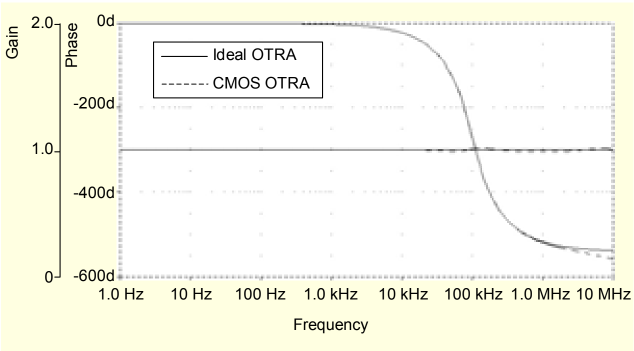

Fig. 3. Simulated frequency response for the third-order all-pass filter.

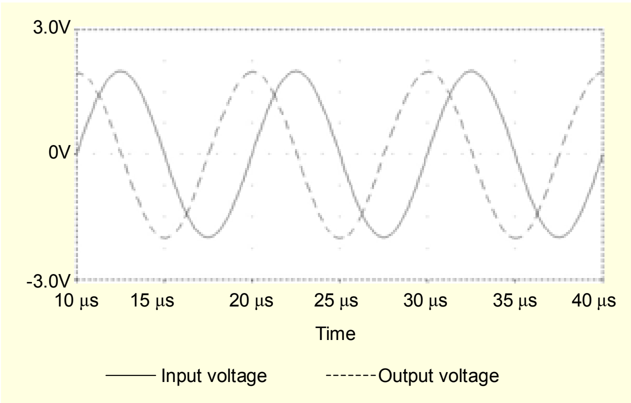

Fig. 4. Simulated transient response for the third-order all-pass filter.

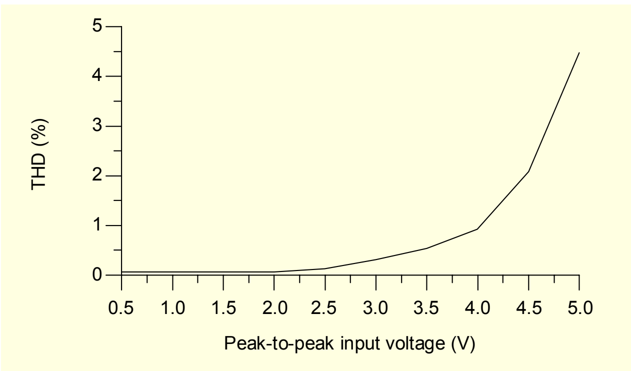

Fig. 5. Dependence of the output harmonic distortion on the input voltage amplitude for the third-order all-pass filter.

design, using the RC:–RC decomposition technique again, the driving-point RC admittance functions are found as $Y_a = 1/2 + s/(2s+4)$, $Y_b = s/(s+1)$, and $Y_c = s + 1/2$, $Y_d = 3s/(2s+4)$ with the same arbitrary polynomial $D(s) = (s+1)(s+2)$. The resulting circuit is shown in Fig. 6. The element values of this filter are $R_{a1} = 160$ kΩ, $R_{a2} = 160$ kΩ, $C_a = 4.974$ pF, $R_b = 80$ kΩ, $C_b = 19.894$ pF, $R_c = 160$ kΩ, $C_c = 19.894$ pF, $R_d = 53.333$ kΩ, and $C_d = 14.921$ pF. This choice also leads to a resonant frequency of $f_0 = 100$ kHz.

Figure 7 shows the simulated frequency response for the third-order low-pass filter. The simulations give similar results for the transient response and harmonic distortion of a low-pass filter as those of an all-pass filter.

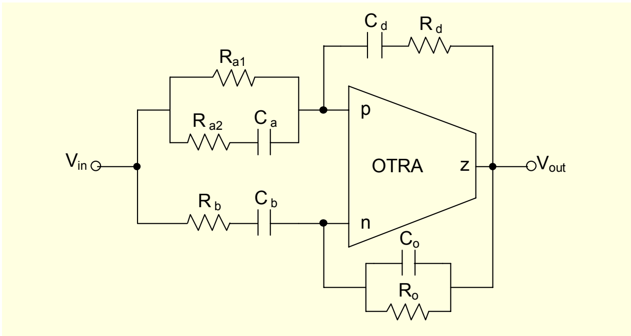

Fig. 6. Third-order low-pass filter.

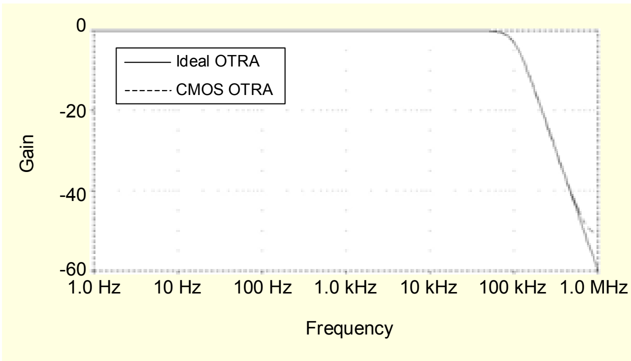

Fig. 7. Simulated frequency response for the third-order low-pass filter.

IV. Non-ideality Analysis

Considering a single-pole model for the transresistance gain, R_m can approximately be expressed at high frequencies as

$$R_m(s) \approx \frac{1}{sC_p}, \tag{7}$$

where $C_p = 1/(R_0\omega_0)$, R_0 is the DC transresistance gain, and ω_0 is the pole frequency. Taking into account the relation in (7), the voltage transfer function of the configuration in Fig. 1 becomes

$$T(s)=\frac{V_{out}(s)}{V_{in}(s)}=\frac{Y_a-Y_b}{Y_c-Y_d+sC_p}. \quad (8)$$

If the admittances Y_c and/or Y_d contain a parallel capacitor, we will have a complete self compensation [2]. In both of our example circuits, Y_c contains a parallel capacitor branch (Y_c=1/R_c+sC_c); hence the filters can be designed taking the magnitude of C_p into consideration and by subtracting its magnitude from C_c. In this way, the effect of C_p can be absorbed in capacitance C_c without using additional elements and achieving complete self compensation [2].

V. Discussion

The approach presented in this study requires determining the transfer function to be realized at the beginning of the synthesis procedure. Therefore, the resulting circuits will not have any right-hand side pole, and hence there will be no stability problem if an appropriate transfer function is selected at the beginning. This situation is valid for the example circuits of third-order all-pass and low-pass filters.

From a practical point of view, it can be said that the presented circuits would be less sensitive to stray capacitances because of the internally grounded input terminals of an OTRA. On the other hand, the low output impedance of the presented circuits allows for driving the loads without the addition of a buffer. Furthermore, the resistors connected to the input terminals of an OTRA can be realized with MOS transistors leading to fully integrated circuits. By this way, the filter parameters can also be adjusted electronically.

Since the proposed configuration uses only one active element to realize an n-th order transfer function, it is possible to implement high-order filters with lower power consumption than those in the literature [9]-[12]. Therefore, the presented approach could be preferred to the others if power consumption is an important design criterion. On the other hand, the resulting circuits obtained from the proposed configuration have no canonical structure. This results in the occupation of larger areas on the integrated circuits for on-chip applications.

VI. Conclusion

A single OTRA-based configuration for the synthesis of an n-th order voltage transfer function is presented. In the realization, the RC:–RC decomposition technique is employed. As examples of the general configuration, third-order all-pass and low-pass filters are obtained and simulated using the SPICE program. The results verify the theoretical analysis. The proposed configuration saves a considerable number of OTRAs in comparison with the previously reported ones [9]-[12].

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