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ON RELATIVE-INVARIANT CIRCULAR UNITS IN FUNCTION FIELDS

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Abstract. Let K be an absolutely real abelian number field with $G = Gal(K/\mathbb{Q})$. Let E be a subfield of K and $\Delta = Gal(K/E)$. Let C_K and C_E be the group of circular units of K and E respectively. In [G], Greither has shown that if G is cyclic then $C_K^\Delta = C_E$. In this paper we show that the same result holds in function field case.

1. Introduction

For an absolutely abelian number field K with $G = Gal(K/\mathbb{Q})$, let C_K be the group of circular units of K defined by Sinnott in [S]. Let E be a subfield of K and $\Delta = Gal(K/E)$. Let C_K^Δ be the subgroup of C_K consisting of all circular units of K which are fixed under Δ. It holds that $C_E \subset C_K$ and $N_{K/E}C_K \subset C_K^\Delta \subset C_E$. In [G], Greither has asked the following question: "*Does* $C_K^\Delta = C_E$ *?* ", and has proven that if G is cyclic, then $C_K^\Delta = C_E$. When both K and E are cyclotomic fields, Gold and Kim in [GK] have shown that the question holds true.

In this paper we treat the same question in function fields case. Let $\mathbb{A} = \mathbb{F}_q[T]$ be the ring of polynomials over the finite field $\mathbb{F}_q$ and $k = \mathbb{F}_q(T)$. Let ∞ be the place of k associated to $(1/T)$. For each $N \in \mathbb{A}$, one uses the Carlitz module to construct a field extension $k(\Lambda_N)$,

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called the N-th cyclotomic function field, and its maximal real subfield $k(\Lambda_N)^+$. For the theory of cyclotomic function fields we refer to the Rosen's book [R, Chap.12,16]. Let K be a finite abelian extension of k which is contained in some cyclotomic function field with $G = Gal(K/k)$. Let C_K be the group of circular units of K defined by Harrop in [H]. Let E be a subfield of K and $\Delta = Gal(K/E)$. In [BJ], Bae and Jung have shown that if both E and K are cyclotomic function fields, then $C_K^\Delta = C_E$. The aim of this paper is to show that if K is a real cyclic extension over k then $C_K^\Delta = C_E$ holds (Theorem 3.1).

2. Preparations

We keep the notations in the preceding section. In this section, we give some basic facts of the cyclotomic function fields and circular units in function fields. Let k^{ac} be a fixed algebraic closure of k. Then k^{ac} becomes an $\mathbb{A}$-module under the following action (called the Carlitz module): for $u \in k^{ac}$ and $N \in A$, define $u^N = N(\varphi + \mu_T)(u)$, where the map φ is defined as $\varphi(u) = u^q$ and μ_T is defined as $\mu_T(u) = Tu$. It is well known that the set Λ_N of roots of $u^N = 0$ generates a finite abelian extension $k(\Lambda_N)$, called the N-th cyclotomic function field. Let $k(\Lambda_N)^+$ be the maximal real subfield of $k(\Lambda_N)$, i.e., $k(\Lambda_N)^+$ is the largest subfield of $k(\Lambda_N)$ in which the infinite place ∞ of k splits completely. Let e_C be the Carlitz exponential function and $\tilde{\pi}$ be a fixed generator of the lattice associated to Carlitz module. Then Λ_N is cyclic as an $\mathbb{A}$-module via the Carlitz module with a generator $\lambda_N = e_C(\tilde{\pi}/N)$. The Carlitz $\mathbb{A}$-action on λ_N is given as follows: $\lambda_N^A = e_C(\frac{\tilde{\pi}A}{N})$ for any $A \in \mathbb{A}$. There is a canonical isomorphism $\tau : (\mathbb{A}/N\mathbb{A})^* \simeq Gal(k(\Lambda_N)/k)$, $A + N\mathbb{A} \mapsto \tau_A$; where $\tau_A(\lambda_N) = \lambda_N^A$. Let us denote by $\mathbb{A}_+$ the set of all monic polynomials.

Let K be a finite abelian extension of k with $G = Gal(K/k)$. We always assume that K is contained in some cyclotomic function field. Let

$F \in \mathbb{A}_+$ be the conductor of K, i.e., $k(\Lambda_F)$ is the smallest cyclotmic function field containing K. When the infinite place ∞ of k splits completely in K, we call K a real extension of k. Let $\mathcal{O}_K$ be the integral closure of $\mathbb{A}$ and U_K its unit group. For any $N \in \mathbb{A}_+$, let $K_N = K \cap k(\Lambda_N)$. Let D_K be the subgroup of K^* generated by $\mathbb{F}_q^*$ and $\{g'_N(A) : A, N \in \mathbb{A}_+ \text{ with } \deg A < \deg N\}$, where $g'_N(A) = N_{k(\Lambda_N)/K_N}(\lambda_N^A)$. The group $C_K = D_K \cap U_K$ is called the group of circular units of K.

Lemma 2.1. *D_K is a $\mathbb{Z}[G]$-submodule of K^* and it is generated by $\mathbb{F}_q^* \cup \{g'_N(A) : N, A \in \mathbb{A}_+$ with $A|N\}$ as $\mathbb{Z}[G]$-module.*

Proof. To show that D_K is a $\mathbb{Z}[G]$-module, it suffices to show that $\sigma(g'_N(A)) \in D_K$ for any $\sigma \in G$ and $N, A \in \mathbb{A}_+$ with $\deg A < \deg N$. Let $\tau_B \in Gal(k(\Lambda_N)/k)$ be an extension of $\sigma|_{K_N} \in Gal(K_N/k)$. Then one has

$$\begin{aligned}\sigma(g'_N(A)) &= N_{k(\Lambda_N)/K_N}(\tau_B(\lambda_N^A)) = N_{k(\Lambda_N)/K_N}(\lambda_N^{AB}) \\ &= N_{k(\Lambda_N)/K_N}(\lambda_N^C) = g'_N(C),\end{aligned}$$

where $C \equiv AB \mod N$ with $\deg(C) < \deg(N)$. Thus D_K is a $\mathbb{Z}[G]$-submodule of K^*. Let $D = \gcd(N, A)$ and $N = DN', A = DA'$. Then one has

$$\lambda_N^A = e_C\left(\frac{\tilde{\pi}A}{N}\right) = e_C\left(\frac{\tilde{\pi}A'}{N'}\right) = \tau_{A'}(\lambda_{N'}) = \tau_{A'}(\lambda_N^D),$$

and so $g'_N(A) = N_{k(\Lambda_N)/K_N}(\tau_{A'}(\lambda_N^D)) = \sigma(N_{k(\Lambda_N)/K_N}(\lambda_N^D)) = \sigma(g'_N(D))$ for some $\sigma \in G$. Hence D_K is generated by $\mathbb{F}_q^* \cup \{g'_N(A) : N, A \in \mathbb{A}_+$ with $A|N\}$ as $\mathbb{Z}[G]$-module. □

For any subset S of K^*, we denote by $\langle S \rangle_{\mathbb{Z}[G]}$ the $\mathbb{Z}[G]$-submodule of K^* generated by S. Any monic irreducible polynomial of A will be called a prime element of $\mathbb{A}$.

Lemma 2.2. *Let V be the subgroup of k^* generated by $\mathbb{F}_q^*$ and $\{P : P$ is a prime with $P \nmid F\}$. Then one has*

$$D_K = V \cdot \big\langle \{g'_D(1) : D \in A_+ \text{ with } D|F\} \big\rangle_{\mathbb{Z}[G]}.$$

Proof. For any prime P with $P \nmid F$, one has $K \cap k(\Lambda_P) = k$ and $N_{k(\Lambda_P)/k}(\lambda_P) = P$. Thus ($\supseteq$) holds. For the converse we only need to consider $g'_N(A)$ with $N, A \in \mathbb{A}_+$ and $A|N$ (by Lemma 2.1). Let $M = N/A$. Since $\lambda_N^A = \lambda_M$, one has

$$\begin{aligned} g'_N(A) &= N_{k(\Lambda_N)/K_N}(\lambda_M) \\ &= N_{k(\Lambda_M)/K_M}(\lambda_M)^{[k(\Lambda_N):k(\Lambda_M)K_N]} = g'_M(1)^{[k(\Lambda_N):k(\Lambda_M)K_N]}. \end{aligned}$$

Thus it suffices to consider $g'_N(1)$ with $N \in \mathbb{A}_+$. Let $D = \gcd(F, N)$. Note that $K_N = k(\Lambda_N) \cap K = k(\Lambda_D) \cap K = K_D$. If $D = 1$, then $K_N = k$ and $g'_N(1) = N_{k(\Lambda_N)/k}(\lambda_N) \in V$. If $D \neq 1$, then $g'_N(1) = N_{k(\Lambda_D)/K_D}(N_{k(\Lambda_N)/k(\Lambda_D)}(\lambda_N)) \in \langle g'_D(1)\rangle_{\mathbb{Z}[G]}$ by [AJ, Lemma 2.3]. Thus the proof is complete. □

Corollary 2.3.

$$\begin{aligned} C_K &= \mathbb{F}_q^* \cdot \prod_{D|F} \left(U_K \cap \langle g'_D(1)\rangle_{\mathbb{Z}[G]}\right) \\ &= \mathbb{F}_q^* \cdot \prod_{\substack{D|F \\ D:\text{ not prime power}}} \langle g'_D(1)\rangle_{\mathbb{Z}[G]} \cdot \prod_{\substack{P|F \\ P:\text{ prime}}} \langle g'_{P^{\varepsilon_P}}(1)\rangle_{I_G}. \end{aligned}$$

Proof. Let $D \in \mathbb{A}_+$ be a divisor of F. If D is not a prime power, then $g'_D(1) \in U_K$. If D is a power of prime P, set $\varepsilon_P = \mathrm{ord}_P(F)$, then for any $e < \varepsilon_P$, we have $\lambda_{P^e} = N_{k(\Lambda_{P^{\varepsilon_P}})/k(\Lambda_{P^e})}(\lambda_{P^{\varepsilon_P}})$ and so $g'_{P^e}(1) = g'_{P^{\varepsilon_P}}(1)^a$ for some $a \in \mathbb{Z}[G]$. Thus any element of $\big\langle\{g'_D(1) : D \in A_+ \text{ with } D|F\}\big\rangle_{\mathbb{Z}[G]}$ is of the form

$$\prod_{\substack{D|F \\ D:\text{ not prime power}}} g'_D(1)^{a_D} \times \prod_{\substack{P|F \\ P:\text{ prime}}} g'_{P^{\varepsilon_P}}(1)^{a_P}$$

for some $a_D, a_P \in \mathbb{Z}[G]$. Let I_G be the augmentation ideal of $\mathbb{Z}[G]$. Note that $g'_{P^{\varepsilon_P}}(1)^{a_P} \in U_K$ if and only if $a_P \in I_G$. Thus we get the result. □

3. Circular units for absolutely cyclic extension K

In this section we assume that K is a real extension of k. For any $D \in \mathbb{A}_+$ with $D|F$, we set $g_D^K := g'_D(1)$. Our main result is

Theorem 3.1. *If K/k is a real cyclic extension, and $E \subset K$ is any subfield with $\Delta = Gal(K/E)$, then $C_K^\Delta = C_E$.*

Since $C_E \subset C_K$, one has $C_E \subset C_K^\Delta$. For any $D \in \mathbb{A}_+$ with $D|F$, let $Y_{D,K} = \langle g_D^K \rangle_{\mathbb{Z}[G]}$ and $Y_{D,E} = \langle g_D^E \rangle_{\mathbb{Z}[G/\Delta]}$. Furthermore we set $Z_{D,K} = U_K \cap Y_{D,K}$ and $Z_{D,E} = U_E \cap Y_{D,E}$. Then Corollary 2.3 implies that

$$C_K = \mathbb{F}_q^* \cdot \prod_{D|F} Z_{D,K}, \text{ and } C_E = \mathbb{F}_q^* \cdot \prod_{D|F} Z_{D,E}.$$

Lemma 3.2. *For any $D \in \mathbb{A}_+$ with $D|F$, one has $Y_{D,E} = N_{K_D/E_D} Y_{D,K}$ and $Z_{D,E} = N_{K_D/E_D} Z_{D,K}$.*

Proof. Since $g_D^E = N_{K_D/E_D}(g_D^K)$, one has $Y_{D,E} = N_{K_D/E_D} Y_{D,K}$. If D is not a power of prime, there is nothing to prove because $Z_{D,K} = Y_{D,K}$ and $Z_{D,E} = Y_{D,E}$. If D is a power of prime P, $N_{K_D/E_D} Z_{D,K} \subseteq U_E \cap Y_{D,E} = Z_{D,E}$. Conversely if $z \in Z_{D,E} = U_E \cap Y_{D,E} \subseteq Y_{D,E} = N_{K_D/E_D} Y_{D,K}$, then $z = N_{K_D/E_D}(y)$ for some $y \in Y_{D,K}$. Then y is already unit in K, so $y \in Z_{D,K}$. Thus $Z_{D,E} = N_{K_D/E_D} Z_{D,K}$. □

For any subset M of G, we denote by $s(M)$ the element $\sum_{\sigma \in M} \sigma$ of $\mathbb{Z}[G]$.

Corollary 3.3. *If $[K : E] = \ell^e$ (ℓ : prime, $e \geq 1$) and E is the maximal proper subfield of K with $\Delta = Gal(K/E)$, then*

$$Z_{F,E} = s(\Delta) \cdot Z_{F,K} \text{ (additive notation in } \mathbb{Z}[G]\text{-module } Z_{F,K}),$$

$$Z_{D,E} = Z_{D,K} \text{ for all } D|F, D \neq F.$$

Proof. The first statement follows immediately from Lemma 3.2, because $k(\Lambda_F) \cap K = K$ and $k(\Lambda_F) \cap E = E$. Suppose that D is a proper divisor of F. By definition of conductor K is not in $k(\Lambda_D)$, and so $k(\Lambda_D) \cap K = k(\Lambda_D) \cap E$. Thus one has $Z_{D,E} = Z_{D,K}$. □

Lemma 3.4. *Let K/k be a finite abelian extension with $G = Gal(K/k)$. Let E/k be a subextension of K with $\Delta = Gal(K/E)$. Then the $\mathbb{Z}[G]$-modules C_K/C_E and $\mathbb{Z}[G]/s(\Delta)\mathbb{Z}[G]$ have the same $\mathbb{Z}$-rank $[K:k]-[E:k]$.*

Proof. Since C_K and U_K (resp. C_E and U_E) have the same $\mathbb{Z}$-rank, the first statement follows directly from the Dirichlet unit theorem in function field. The second statement follows from the isomorphism $s(\Delta)\mathbb{Z}[G] \simeq \mathbb{Z}[G/\Delta]$. □

Proposition 3.5. *Let K, E, G and Δ be as in Lemma 3.4. Suppose that $|\Delta| = \ell$ is a prime and C_K/C_E is a cyclic over $\mathbb{Z}[G]$. Then one has*

(i) $C_K/C_E \simeq \mathbb{Z}[G]/s(\Delta)\mathbb{Z}[G]$.

(ii) $(C_K/C_E)^\Delta = 0$.

(iii) $C_K^\Delta \subseteq C_E$.

Proof. (i) Since $s(\Delta)C_K = N_{K/E}C_K \subseteq C_E$, $M = C_K/C_E$ is annihilated by $s(\Delta)$. Thus M is a $\mathbb{Z}[G]/s(\Delta)\mathbb{Z}[G]$-module. Since M is cyclic over $\mathbb{Z}[G]$, there exists a surjective homomorphism $\gamma : \mathbb{Z}[G]/s(\Delta)\mathbb{Z}[G] \to M$. Note that M and $\mathbb{Z}[G]/s(\Delta)\mathbb{Z}[G]$ have the same $\mathbb{Z}$-rank (Lemma 3.4). Thus the kernel of γ must be a finite torsion $\mathbb{Z}$-submodule of $\mathbb{Z}[G]/s(\Delta)\mathbb{Z}[G]$. But $\mathbb{Z}[G]/s(\Delta)\mathbb{Z}[G]$ is $\mathbb{Z}$-torsion free. Hence γ must be an isomorphism.

(ii) Write $G = G' \times H$, where $|G'|$ is a power of ℓ and $\gcd(\ell, |H|) = 1$. Then $\Delta \subseteq G'$ and $\mathbb{Z}[G']/s(\Delta)\mathbb{Z}[G'] \simeq \mathbb{Z}[G]/s(\Delta)\mathbb{Z}[G]$ as $\mathbb{Z}[\Delta]$-modules. On the other hand Δ is the subgroup of order ℓ in the cyclic ℓ-group G', thus one has

$$\mathbb{Z}[G']/s(\Delta)\mathbb{Z}[G'] \simeq \mathbb{Z}[\zeta_{\ell^e}]$$

with $G' \simeq \langle \zeta_{\ell^e} \rangle, \Delta \simeq \langle \zeta_\ell \rangle$. Here ζ_n denotes a primitive n-th root of unity in $\mathbb{C}$. Therefore $(\mathbb{Z}[G']/s(\Delta)\mathbb{Z}[G'])^\Delta$ is isomorphic to the annihilator of $1 - \zeta_\ell$ in $\mathbb{Z}[\zeta_{\ell^e}]$, so is zero. Thus by (i) the statement follows.

(iii) follows immediately from (ii). □

Corollary 3.6. *Let K, E and Δ be as in Proposition 3.5. Suppose $[K : k]$ is a power of a prime ℓ and $|\Delta| = \ell$. Then Theorem 3.1 holds for $E \subset K$.*

Proof. It suffices to show that C_K/C_E is cyclic over $\mathbb{Z}[G]$. By Corollary 3.3, it is enough to show that $Z_{F,K}$ is cyclic over $\mathbb{Z}[G]$. Note that $Y_{F,K}$ is $\mathbb{Z}[G]$-cyclic (generated by g_F^K). If F is not a power of prime, $Z_{F,K} = Y_{F,K}$ is $\mathbb{Z}[G]$-cyclic. If F is a power of prime, $Z_{F,K} = I_G \cdot Y_{F,K}$. Since G is a cyclic group, I_G is a $\mathbb{Z}[G]$-cyclic. Thus $Z_{F,K} = I_G \cdot Y_{F,K}$ is a $\mathbb{Z}[G]$-cyclic. □

One may assume in the proof of Theorem 3.1 that $[K : E]$ is a prime ℓ. Furthermore a standard argument shows that C_K^{Δ}/C_E is annihilated by $\ell = |\Delta|$. It is enough to show that

$$(\mathbb{Z}_\ell \otimes C_K)^\Delta = \mathbb{Z}_\ell \otimes C_E.$$

As above write $G = G' \times H$, where G' is a ℓ-group and $\gcd(|H|, \ell) = 1$. In addition we set $K' = K^H, L = K^{G'}$, so that K is the compositum $K = K'L$. For any ℓ-adic character ψ of H and $\mathbb{Z}_\ell[H]$-module M, one defines $M_\psi = \mathbb{Z}_\ell(\psi) \otimes_{\mathbb{Z}_\ell[H]} M$. It is well known that for any $\mathbb{Z}_\ell[H]$-module M one has

$$M \simeq \bigoplus_\psi M_\psi \quad (\text{as } \mathbb{Z}_\ell[H]\text{-modules}),$$

where ψ runs over all ℓ-adic characters of H modulo $\mathbb{Q}_\ell$-conjugation. Now we consider $M = C_K/C_E$. It suffices to show that M is cyclic over $\mathbb{Z}[G]$.

Proposition 3.7. *For any ℓ-adic character ψ of H, M_ψ is cyclic over $\mathbb{Z}_\ell(\psi)[G']$.*

Proof. Let $K^\psi = K^{\ker \psi}$. Then $L \subseteq K^\psi$. Let $F(\psi)$ be the conductor of K^ψ. The core of the argument is now in the following lemma.

Lemma 3.8. $(C_K)_\psi = (C_E)_\psi (Z_{F(\psi),K})_\psi$.

Proof. We begin the proof with the following representation $C_K = \mathbb{F}_q^* \cdot \prod_{D|F} Z_{D,K}$. Since $F(\psi)|F$, this representation shows that ($\supseteq$) holds. Note that $E = (E \cap K')K$ and $E \cap K'$ is the largest proper subfield of K'. We must show that for any $D \in \mathbb{A}_+$ with $D|F$, $(Z_{D,K})_\psi$ is contained in the right. If $K' \not\subseteq k(\Lambda_D)$, then $K \cap k(\Lambda_D) = (K'L) \cap k(\Lambda_D) = (K' \cap k(\Lambda_D))(L \cap k(\Lambda_D)) = (E \cap K' \cap k(\Lambda_D))(L \cap k(\Lambda_D)) = ((E \cap K')L) \cap k(\Lambda_D) = E \cap k(\Lambda_D)$. Thus $Z_{D,K} = Z_{D,E}$, and so we are done. Now assume that $K' \subseteq k(\Lambda_D)$. Let $\Gamma = \ker \psi = Gal(K/K^{\ker \psi})$. Then $\psi(s(\Gamma)) = |\ker \psi|$ is a unit in $\mathbb{Z}_\ell$, therefore $(Z_{D,K})_\psi = (s(\Gamma) Z_{D,K})_\psi$. In addition one has $s(\Gamma) Z_{D,K} \subseteq U_{k(\Lambda_D)} \cap U_{K^\psi} = U_{k(\Lambda_D) \cap K^\psi}$. If $\Gamma' = Gal(k(\Lambda_D) \cap K / k(\Lambda_D) \cap K')$, then $(Z_{D,K})_\psi = (s(\Gamma') Z_{D,K})_\psi$, and corresponding statements holds with Y in place of Z, i.e., $(Y_{D,K})_\psi = \langle (N_{k(\Lambda_D)/k(\Lambda_D) \cap K^\psi}(\lambda_D))_\psi \rangle_{\mathbb{Z}[G]}$.

(Case 1) D is not a multiple of $F(\psi)$. Then $K^\psi \not\subseteq k(\Lambda_D)$, i.e., $K^\psi \cap k(\Lambda_D) \subsetneq K^\psi$. Thus there exists $\sigma \in H - \ker \psi$ which is trivial on $K^\psi \cap k(\Lambda_D)$. Since $\psi(\sigma) - 1 \in \mathbb{Z}_\ell(\psi)$ is not zero divisor, it follows that $(U_{k(\Lambda_D) \cap K^\psi})_\psi = 0$, thus $(Z_{D,K})_\psi = 0$.

(Case 2) D is a multiple of $F(\psi)$. Then $K^\psi \subseteq k(\Lambda_{F(\psi)}) \subseteq k(\Lambda_D)$, and so one has

$$(Z_{D,K})_\psi = (U_K)_\psi \cap \langle (N_{k(\Lambda_D)/K^\psi}(\lambda_D))_\psi \rangle_{\mathbb{Z}[G]},$$
$$(Z_{F(\psi),K})_\psi = (U_K)_\psi \cap \langle (N_{k(\Lambda_{F(\psi)})/K^\psi}(\lambda_{F(\psi)}))_\psi \rangle_{\mathbb{Z}[G]}.$$

Since $N_{k(\Lambda_D)/k(\Lambda_{F(\psi)})}(\lambda_D)$ is contained in $\langle \lambda_{F(\psi)} \rangle_{\mathbb{Z}[G]}$ it follows that $(Z_{D,K})_\psi \subset (Z_{F(\psi),K})_\psi$ and lemma is proved. □

By Lemma 3.8 and the proof of Corollary 3.6, $(C_K)_\psi/(C_E)_\psi$ is cyclic as surjective image of $(Z_{F(\psi),K})_\psi$ over $\mathbb{Z}_\ell(\psi)[G']$. Therefore Proposition 3.7 and Theorem 3.1 are proved. □

References

[AJ] J. Ahn and H. Jung, Cyclotomic units and divisibility of the class number of function fields. J. Korean Math. Soc. 39 (2002), no. 5, 765–773.

[BJ] S. Bae and H. Jung, Bases for cyclotomic units over function fields. J. Aust. Math. Soc. 70 (2001), no. 1, 56–70.

[G] C. Greither, Uber relativ-invariante Kreiseinheiten und Stickelberger-Elemente. Manuscripta Math. 80 (1993), no. 1, 27–43.

[GK] R. Gold and J. Kim, Bases for cyclotomic units. Compositio Math. 71 (1989), no. 1, 13–27.

[H] F. Harrop, Circular units of function fields. Trans. Amer. Math. Soc. 341 (1994), no. 1, 405–421.

[R] M. Rosen, Number theory in function fields. Graduate Texts in Mathematics, 210. Springer-Verlag, New York, 2002.

[S] W. Sinnott, On the Stickelberger ideal and the circular units of an abelian field. Invent. Math. 62 (1980/81), no. 2, 181–234.

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