International Journal of Management Science
Vol 11, No 2, November 2005

A Heuristic Approach for Arrangement of Footwear Boxes to Maximize Space Utilization and Related Business Issues

Prasun Das*

SQC & OR Division, Indian Statistical Institute
Kolkata, India

(Received Mar. 2005 ; Revised Jun. 2005 ; Accepted Aug. 2005)

ABSTRACT

This paper considers a special case of the two-dimensional bin-packing problem for identical items. The objective of this work is to maximize the space utilization. The main contribution of the paper is to suggest a new heuristic algorithm keeping in view the existing complexity of racking system for the footwear boxes in the compartments of different sizes for a warehouse. The results show that a significant improvement can be obtained. An economic choice of compartments is also estimated using the criteria for maximizing space utilization. A non-linear mathematical model was presented based on the constraints of racking dynamics.

Keywords: Box, Compartment, Heuristic method, Space utilization

* Corresponding author, Email: dasprasun@rediffmail.com

1. INTRODUCTION

A reputed footwear company spreads its business through exclusive retail outlets, dealers and wholesalers amongst the various states of India. With strong foothold in the market for 40 years, the company has reached a landmark of 100 crores turnover through its committed channel networks. In recent past, the company has also constructed a central warehouse with a much higher capacity and decided to close down all the three separate warehouses located at three different locations in order to simplify the distribution logistics across the country.

This study is carried out with a view to adopt a proper scientific method of racking system to handle the cumulative load of product complexity in the warehouse. It is decided to explore the feasibility of solutions in one of the existing well-structured warehouses, which caters around 3,500 running items of footwear on a regular basis to its customers.

This paper is organized as follows. In *Section 2*, the objective of adopting this new heuristic approach is explained based on literature review. In *Section 3*, the footwear box stacking orientations are described. The heuristic models, respective algorithms and software developed for obtaining solutions are described in *Section 4*. In *Section 5*, the results obtained are given followed by justification of the heuristic approach in *Section 6*. An optimal choice of compartment is investigated in *Section 7* vis-à-vis maximizing space utilization and box stacking orientations. The respective cost benefit analysis is experienced in *Section 8*. *Section 9* presents the concluding remarks and discusses future scope of work in formulating a non-linear mathematical model for the optimal allocation of footwear boxes based on certain restrictions.

2. LITERATURE REVIEW

There exist several literatures about on-line and off-line algorithms to solve the two-dimensional bin-packing problem. Historically, *Gilmore and Gomory* [1-3] developed the models for packing problem with a variant to cutting stock problem. Till then, lot of work has been done considering some additional constraints with the objective of developing efficient algorithms for optimal / sub-optimal solutions. Some references are cited on this [4-6]. *Lodi et al.*, 1999, 2002 highlighted some recent advances on two-dimensional bin packing problems with special emphasis on off-line algorithms and effective heuristics and metaheuristic approaches [7-9]. This type of problem has enough opportunity for practical applications, especially in cutting and packing industries. The real-life problem considered in this paper is to allocate identical boxes, without overlapping, in a compartment to maximize the space utilization when one dimension of the boxes remains fixed during orientation in the compartment. The solution to this problem can be obtained by optimally positioning the boxes through generation of optimal shelving patterns. This may be a special case of two-dimensional cargo loading and bin packing problems. The proposed heuristic actually transferring to two one-dimensional knapsack problems with two variables, is simple, quick and easier to compute.

3. ORIENTATIONS OF BOX ; DEFINITION

The orientation of box, while stacking inside the compartment, is described here, including the existing method of shelving system in the warehouse. Two types of orientations are defined.

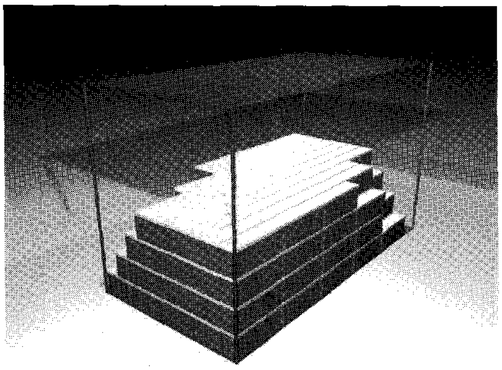


Figure 1(a). Horizontal orientation of boxes

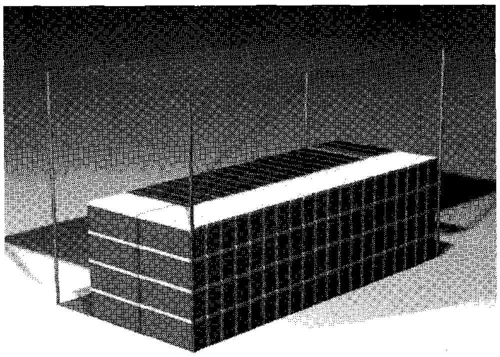


Figure 1(b). Vertical orientation of boxes

In *Figure 1(a)*, all boxes in a compartment are placed horizontally along the length of the compartment. This means that the length of the box is along the length of the compartment. In *Figure 1(b)*, all boxes in a compartment are placed vertically along the length of the compartment. This means that the length of the box is along the width of the compartment. Presently, the warehouse adopts this system of shelving due to ease of loading and unloading activities including smooth identification of items.

4. THE HEURISTIC METHOD

Suppose, the dimension i.e, (length, width, height) of a compartment is (X, Y, Z) and that of a footwear box is (X_1, Y_1, Z_1).

Then,

(a) Maximum number of boxes that can be kept in the compartment =

$$\left\lfloor \frac{X \times Y \times Z}{X_1 \times Y_1 \times Z_1} \right\rfloor = K, \text{ say}$$

(b) Let, all the boxes be arranged along length of the compartment, i.e. X_1 along X Y_1 along Y and Z_1 along Z. Then,
Maximum number of boxes that can be kept in the compartment is

$$\left\lfloor \frac{X}{X_1} \right\rfloor \times \left\lfloor \frac{Y}{Y_1} \right\rfloor \times \left\lfloor \frac{Z}{Z_1} \right\rfloor = K_1, \text{ say}$$

(c) Next, let, all the boxes are arranged along width of the compartment, i.e. Y_1 along X, X_1 along Y and Z_1 along Z. Then,
Maximum number of boxes that can be kept in the compartment is

$$\left\lfloor \frac{X}{Y_1} \right\rfloor \times \left\lfloor \frac{Y}{X_1} \right\rfloor \times \left\lfloor \frac{Z}{Z_1} \right\rfloor = K_2, \text{ say}$$

Now,

i) If $K = K_1$, then the optimal solution is to arrange all the boxes along the length of the compartment. Again, if $K = K_2$, then the optimal solution is to arrange all the boxes along the width of the compartment;

ii) If $K \neq K_1$ and $K \neq K_2$, then the ***maximum*** additional quantity of boxes is $\{K - \max(K_1, K_2)\}$ which can be arranged either along the length or along the width. For this, the following integer-programming problem is to be solved.

$$\begin{aligned} &\text{Maximize } L = (n_1 X_1 + n_2 Y_1) \\ &\text{subject to } n_1 X_1 + n_2 Y_1 \le X \qquad \text{(I)} \\ &n_1, n_2 \text{ are non-negative integers} \end{aligned}$$

and,

$$\begin{aligned} &\text{Maximize } W = (n_3 X_1 + n_4 Y_1) \\ &\text{subject to } n_3 X_1 + n_4 Y_1 \le Y \qquad \text{(II)} \\ &n_3, n_4 \text{ are non-negative integers} \end{aligned}$$

where,

n_1 = no. of boxes placed along its length (X_1) through length (X) of the compartment.

n_2 = no. of boxes placed along its width (Y_1) through length (X) of the compartment.

n_3 = no. of boxes placed along its length (X_1) through width (Y) of the compartment.

n_4 = no. of boxes placed along its width (Y_1) through width (Y) of the compartment.

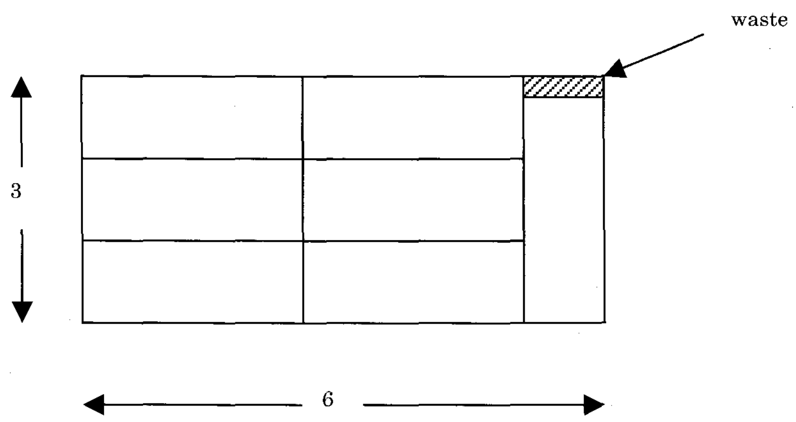
Two strong assumptions are made here. First, no footwear-box stands upside as it is practically not feasible, therefore Z_1 will always be considered along Z. Secondly, in one compartment, only one particular size of box is kept, as practiced in the warehouse for ease of identification of footwear.

4.1 An Example

Suppose, the dimension of a compartment is $(X, Y, Z) = (6, 3, 2)$ and the dimension of a box is $(X_1, Y_1, Z_1) = (2.5, 1, 0.5)$. Then, using the formulae for K, K_1 and K_2 described earlier, we get

$$K = 28, K_1 = 24, K_2 = 24$$

which means, a maximum number of extra 4 (= 28 − max[24,24]) boxes can be placed. Solving the problems **(I)** and **(II)** gives the following arrangement along the length and width of the compartment.



Since, 4 (= 2/0.5) layers of boxes can be placed along the height of the compartment, the total number of boxes is 28 (= 7 × 4), and using the equations **(I)** and **(II)**, we find

$$n_1 = 2, n_2 = 1, n_3 = 0, n_4 = 3.$$

Before stating the algorithm for solving the problems **(I)** and **(II),** let us now define the following top view of a compartment comprising of four sectors.

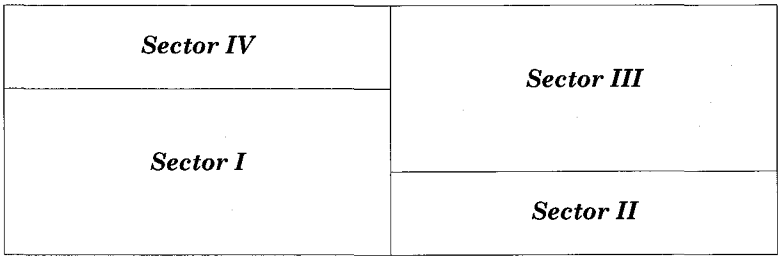


Figure 2. Top view of a compartment

4.2 A Statement of the Algorithm

An algorithm developed for the above heuristic method is presented below (as Z_1 is always considered along Z so the problem can be considered as a two-dimensional shelving optimization problem where the height of the compartment is not considered).

Step 0 : Initialise i = 1. (i : index of the box)

Step 1 : Consider the dimensions (X, Y) ≡ (length, width) of compartment i.

Step 2 : Solve the problems **(I)** and **(II)** and obtain the optimal solutions (n_1^0, n_2^0) and (n_3^0, n_4^0). If $n_1^0 = 0$ and $n_3^0 = 0$ then go to Step 4, otherwise go to Step 3.

Step 3 : Lay up ($n_2^0 * n_3^0$) boxes in vertical position in Sector I and ($n_1^0 * n_4^0$) boxes in horizontal position in Sector II. Get the dimension of the Sector II as $(X - Y_1.n_2^0)*(Y - Y_1.n_4^0)$. Design the boxes either vertically or horizontally in Sector III. The choice of orientation depends upon the number of boxes that can be obtained. Calculate the dimension of the vertical left over portion in Sector III.

If $Y_1.n_4^0 \le X_1.n_3^0$, then include that vertical left over portion in Sector IV, go to Step 5. Otherwise, the dimension of Sector IV is $(X - X_1.n_1^0)*(Y - X_1.n_4^0)$, go to Step 5.

Step 4 : Calculate the number of boxes possible to accommodate in Sector I either vertically or horizontally which will give rise to optimal (i.e. maximum in number) solution, go to Step 6.

Step 5 : Take horizontal and vertical dimensions of Sector IV as A and B respectively. If max(X, Y) < X_1 or min(X, Y) < Y_1, go to Step 6, otherwise go to Step 2.

Step 6 : Optimal design of boxes on compartment 'i' is completed. Set $i = i+1$; if $i \leq 5$, go to Step 1, otherwise go to Step 7.

Step 7 : Terminate the algorithm.

5. DATABASE, ANALYSIS AND RESULTS

5.1 Prevalent Information at the Warehouse

Presently, the operating infrastructure of the warehouse is as follows. There are six (6) types of rectangular steel frame compartments (shelves), each of different dimensions, arranged in altogether thirteen (13) rows in the entire warehouse. The total number of available compartments is 1336 ***(ref.* Annexure 1)**. The compartments, numbered CS-1 to CS-6, are measured in feet but converted into inches for necessary calculations. Around 3,500 types of footwear are packed in 28 types of footwear-boxes of different dimensions, and placed in all the compartments of the warehouse. An estimate of number of footwear-boxes of different dimensions is given ***(ref.* Annexure 2)**. Box dimensions are given in unit of inches. The existing average stock per day is around 78,000 pairs of footwear.

5.2 Results

A software is developed to find heuristically the optimal number of boxes *(sector wise)*, the corresponding empty space (*or, waste*) and the space utilization for any (box, compartment) combination, given the dimensions of (box, compartment) as input. The following table (***ref.* Table** 1) illustrates the output for maximum number of boxes to be kept, using the methods described in ***Sec 4.0***, for vertical, horizontal and mix *(i.e., both vertical and horizontal)* arrangement of boxes. This is computed for each of the 28 footwear-boxes of different dimensions as compared with 6 standard compartments.

Table 1. Maximum number of boxes in a compartment

	Orientation of Boxes																	
	Vertical (Existing)						Horizontal						Mix (Horz. & Vert.) - Heuristic					
	Compartment No.																	
	1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6
Box Code																		
B01	384	288	240	180	126	90	480	384	300	240	196	140	496	400	310	250	210	150
B02	270	216	180	144	144	96	270	225	180	150	135	90	297	234	198	156	144	96
B03	234	192	156	120	90	60	270	192	180	108	108	72	297	216	198	144	126	84
B04	270	216	180	144	144	96	270	192	180	120	108	72	288	225	192	150	144	96
B05	192	140	120	90	63	45	240	168	150	120	84	60	240	192	150	120	91	65
B06	208	160	130	100	70	50	192	144	120	90	84	60	208	160	130	100	91	65
B07	216	150	135	105	98	70	192	120	120	80	84	60	216	168	135	105	98	70
B08	224	162	128	96	72	48	224	162	128	96	90	60	238	182	136	104	96	64
B09	224	162	128	96	72	48	224	162	128	96	90	60	224	168	128	96	90	60
B10	208	160	130	100	70	50	192	144	120	90	84	60	208	160	130	100	91	65
B11	144	100	90	70	49	35	160	120	100	80	84	60	192	152	120	95	84	60
B12	140	108	80	64	48	32	140	108	80	60	54	36	154	126	88	72	66	44
B13	126	90	72	56	42	28	112	90	64	48	54	36	126	98	72	56	60	40
B14	112	72	64	48	36	24	112	72	64	48	36	24	126	98	72	56	42	28
B15	112	84	64	48	36	24	112	84	64	48	36	24	112	84	64	48	36	24
B16	112	72	64	48	36	24	112	72	64	48	36	24	126	84	72	48	36	24
B17	108	80	72	56	42	28	96	75	64	48	54	36	108	84	72	56	54	36
B18	112	84	64	48	36	24	112	84	64	48	36	24	112	84	64	48	36	24
B19	108	80	72	56	42	28	96	75	64	48	54	36	108	84	72	56	54	36
B20	112	72	64	48	36	24	112	72	64	48	36	24	112	84	64	48	36	24
B21	96	72	64	48	36	24	96	72	64	48	36	24	96	72	64	48	36	24
B22	84	54	48	32	24	16	84	54	48	36	36	24	91	63	52	36	36	24
B23	84	63	48	40	30	20	84	70	48	36	36	24	84	70	48	40	36	24
B24	84	54	48	32	24	16	84	54	48	36	18	12	84	63	48	36	24	16
B25	36	36	24	20	30	20	54	30	36	24	24	16	54	36	36	24	30	20
B26	60	42	36	24	20	12	60	36	36	27	30	18	60	45	36	27	30	18
B27	28	18	16	12	18	12	42	24	24	16	12	8	49	35	28	20	18	12
B28	30	21	18	12	20	12	45	24	27	18	10	6	45	35	27	21	20	12

6. JUSTIFICATION

Claim-1. Only laying up the boxes vertically inside the compartment is no better than the heuristic approach.

Proof. The claim is proved graphically. To prove this, two intermediate configurations are considered as shown in ***Fig. (iii)*** and ***Fig. (iv)***. In ***Fig. (ii),*** all boxes are arranged vertically and ***Fig. (v)*** represents the configuration based on our heuristic method. We argue that the wastage decreases as we go on from ***Fig. (ii)*** to ***Fig. (v)***. ***Fig. (iii)*** differs from ***Fig. (ii)*** in the sense that the left portion of length $Y_1.n_2^0$ remains the same but in the right portion up to the height of $Y_1.n_4^0$, all boxes are designed horizontally and above that in the vertical position a maximum number of boxes are arranged. Since, (n_1^0, n_2^0) and (n_3^0, n_4^0) are the optimal solutions of **(I)** and **(II),** it can be seen that the wastage on the right portion is no more than that in ***Fig. (ii)***. ***Fig. (iv)*** differs from ***Fig. (iii)*** only in the top part of the height $(Y - X_1.n_1^0)$ of left portion. Using the optimal solutions it can be seen that ***Fig. (iii)*** is no better than ***Fig. (iv)***. ***Fig. (v)*** differs from ***Fig. (iv)*** only in the top part of the height $(Y - Y_1.n_4^0)$ of right portion. Both figures are the same if horizontal positioning is better in Sector III, otherwise ***Fig. (v)*** gives more boxes than ***Fig. (iv).*** Hence the claim.

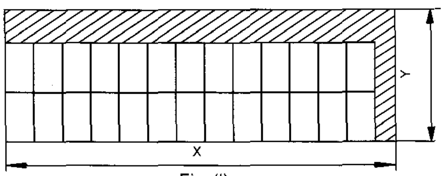


Fig. (i)

- WASTAGE

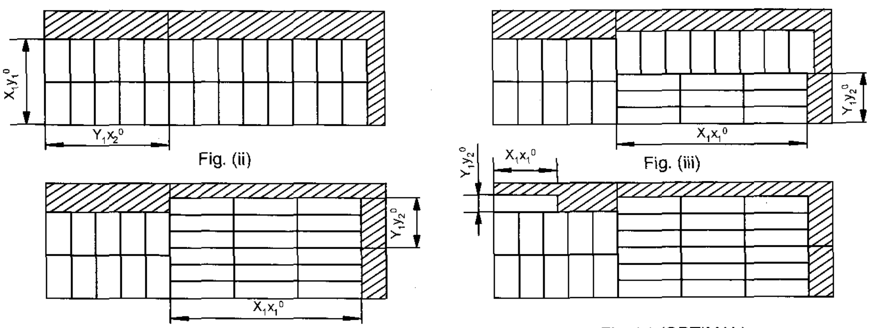


Fig. (ii)

Fig. (iii)

Fig. (iv)

Fig. (v) (OPTIMAL)

- WASTAGE

Claim-2. Only laying up the boxes horizontally inside the compartment is no better than the heuristic approach.

Proof. This claim can also be verified in a similar manner. However, the following figures ensure the proof graphically.

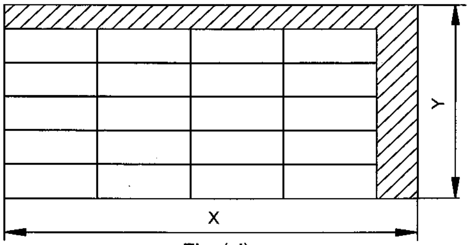


Fig. (vi)

- WASTAGE

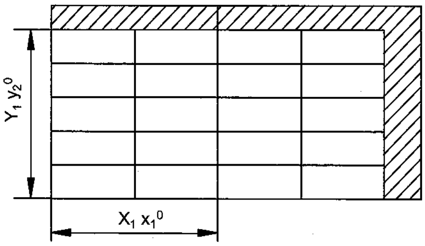


Fig. (vii)

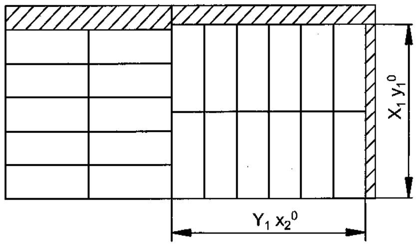


Fig. (viii) (OPTIMAL)

- WASTAGE

The next table ***(ref.* Table 2)** confirms the above claims under the practical situation as experienced by the company. The table contains the following.

i) A comparative picture of the number of footwear boxes if they are stacked inside the compartments either in horizontal, or in vertical and also by applying the heuristic approach (mix orientation) developed. A (*) sign mark in the table indicates the respective gain in the number of boxes with respect to a compartment if they are placed horizontally in place of the existing vertical shelving system. A (**) sign mark indicates the respective gain in the number of boxes with respect to a compartment through heuristic method.

ii) The reduction in empty space (%) as well as the increase in space utilization (%), if the allocation changes from the existing shelving system to horizontal system and to heuristic mix orientation from either of the two (vertical / horizontal) layouts.

Table 2. A Sample Summary Analysis (for 2 box sizes)

Box Size (H × W × L)	PARAMETERS	CS-1	CS-2	CS-3	CS-4	CS-5	CS-6
3.5 × 2 × 8.5	Vertical (V)	384	288	240	180	126	90
	Horizontal (H)	480*	384*	300*	240*	196*	140*
	Mix (M)	496**	400**	310**	250**	210**	150**
	%empty space ↓ (V → H)	55.30	73.73	45.55	60.73	52.04	54.81
	%empty space ↓ (V/H → M)	20.62	46.78	13.94	25.77	21.70	24.26
	%space utilization↑ (V→H)	17.22	22.96	16.14	21.52	26.87	27.59
	%space utilization↑ (V/H→M)	2.87	3.83	2.69	3.59	5.37	5.52
3 × 4.5 × 7	Vertical (V)	270	216	180	144	144	96
	Horizontal (H)	270	225*	180	150*	135	90
	Mix (M)	297**	234**	198**	156**	144	96
	%empty space ↓ (V → H)	0.00	19.02	0.00	19.02	-44.95	-33.14
	%empty space ↓ (V/H → M)	33.30	23.49	33.30	23.49	0.00	0.00
	%space utilization↑ (V→H)	0.00	3.42	0.00	3.42	-5.49	-5.26
	%space utilization↑ (V/H→M)	7.69	3.42	7.69	3.42	0.00	0.00

The necessary calculations for Empty space% and Space utilization are given in **Annexure-3**. The results of Empty space% and Space utilization for some sample box sizes are displayed in **Annexure-4(a)**. The average gain in number of boxes to be kept by changing their orientations for all the available compartments is shown in **Annexure-4(b)**.

7. ESTIMATION OF OPTIMAL CHOICE OF COMPARTMENTS

In order to have an economic procurement policy, the management feels interest to know about the different combinations of compartments to be procured for shelving of footwear boxes in different orientations, provided the types of the boxes remain same. An estimate of this is obtained using the criteria for maximizing space utilization. The results are given below.

Table 3. Optimum Utilization of Compartments

	Vertical	Horizontal	Ver. & Hor. (Selective)	Mix (Heuristic)
CS-1	78.15%	80.63%	82.43%	85.69%
CS-2	75.87%	76.14%	77.93%	87.01%**
CS-3	71.20%	73.48%	75.19%	78.14%
CS-4	71.88%*	72.44%	75.10%	79.37%*
CS-5	63.12%	68.67%	67.70%	75.62%
CS-6	60.94%	66.49%	65.54%	73.13%
Overall	72.20%	74.59%	75.90%	81.38%

Necessary calculations for the above findings are illustrated in **Annexure-5** for selective (vertical & horizontal) box stacking orientation. The following points are observed from the above table.

i) It is advisable to procure Compartment No.-1 to maximize overall space utilization. The next choice of procurement is, of course, Compartment No.-2, irrespective of the orientation of the boxes.
ii) The Compartment No.-4 *(marked with * above)* is preferable than Compartment No.-3, if the boxes are arranged either in 'Vertical' direction or in 'Mix' orientation. Similarly, the Compartment No.-2 *(marked with ** above)* is preferable than Compartment No.-1, if the boxes are arranged either in 'Mix' orientation.
iii) The overall maximum space utilization increases from 'Vertical' orientation to 'Mix' orientation.
iv) For any particular combination of box types *(in this case, 28 types of boxes)*, the estimated average space utilization (*calculated in terms of number of compartments required for a given stock of boxes*) will always be less than or equal to the maximum space utilization. This is evident from **Annexure-5** and **Annexure-6** for the selective *(vertical and horizontal both, whenever there is a gain)* case. However, if stock pattern changes within the box types, then average space utilization will also change but it will never exceed the maximum space utilization. Further, if the combination of box types changes, then both the calculations *(estimated average and maximum space utilization)* will be redone.

8. COST BENEFIT ANALYSIS

Based on the practical situation as experienced by the company in the warehouse, the reduction in the number of compartments is estimated by considering all types of orientations. The results are displayed in **Annexure-7**, which summarizes the following three situations.

Situation-1. If the entire arrangement is changed from 'Vertical' to 'Horizontal', then for a demand of 75,629 boxes of 28 types, on an average, 45 less compartments will be required.

Situation-2. If the arrangements are changed selectively (i.e., only those box types for which there is an average gain, if changed from 'Vertical' to 'Horizontal'), then for a demand of 75,629 boxes of 28 types, arrangements will be changed for 15 types of boxes and for that, on an average, 65 less compartments will be required.

Situation-3. If the entire arrangements are changed from 'Vertical / Horizontal' to 'Mix' orientation, then for a demand of 75,629 boxes of 28 types, on an average, 87 less compartments will be required.

Considering the **Situation-2,**

a) Scope of storage of excess inventory – for the existing Warehouse :

Space Utilization = (Total volume of 75,629 boxes / Total volume of 1336 compartments)
= (16852017.88 / 33421800.96) x 100
= 50.42% *(existing)*

Now, the question is, how much extra inventory in terms of pairs of boxes can be stored in 65 compartments ?

- Considering the estimated stock level (75,629), the overall average *(with respect to compartment type)* space utilization becomes 72.06% (ref. **Annexure-6**). This implies, another 21.64% (= 72.06 – 50.42) utilization is possible in the existing set-up of the warehouse.
- Potential increase in volume of boxes that can be stored = 21.64% of total compartment volume = (33421800.96 × 0.2164) = 7232477.73 cubic inch
- Average volume of a box = (16852017.88 / 75629) = 222.82 cubic inch

- Potential increase in average no. of boxes = (7232477.73 / 222.82) = 32458.84 = 32459
- Total possible pairs of footwear to be stored in 1336 compartments = (75629 + 32459) = 108088.

Now, on an average, in 65 compartments, which is a potential savings in less number of compartments required, 5,259 (= [108088 / 1336] × 65) pairs can be stored which costs around Rupees (Rs.) 10,51,800/- (= 5259 × Rs.200/-).

b) Less amount of procurement of compartments – for the newly built-up central warehouse :

- Cost of shelves per cubic feet = Rs.50.38/-
- Average volume of a compartment = 33421800.96 / 1336 = 25016.32 c.inch ≅ 14.48 c.ft.
- Average savings in less procurement of 65 compartments = (65 × 14.48 × 50.38) = Rs. 47,417.66/-

Similar cost benefit analysis can be carried out for the other two situations. However, **Situation-2** is the most practical situation for which the management can think for first stage improvement of the shelving process.

9. CONCLUSION AND FUTURE SCOPE OF WORK

The solution of the problem is to provide the maximum number of boxes of a particular dimension that can be stored inside a compartment of any specific size. This can be obtained by optimally positioning the boxes through generation of optimal shelving patterns, using heuristic algorithm, to increase the overall space utilization of the warehouse. Subsequently, it will help to identify the minimum requirements of economic shelving in the warehouse as well as to accommodate the pressure of the volume of the business in the centralized warehouse. The distribution logistics from warehouse to retail outlets will be smooth and continuous to protect from periodic lack of sufficient storage space, stock out position and variation in order cycle time. The cost improvement analysis justifies the financial impact of shelving leading to an overall improvement in retail logistics business environment.

Since the receipt of footwear from suppliers and issue of stock of footwear to the customers is very much dynamic, the variation in stock level of different types of boxes containing around 3,500 types of footwear is too high. Further, since the footwear business is highly connected with the fashion world, the level of uncertainty is huge, in general, to survive, compete and grow over time. Under such circumstances, the following areas of work are thought of.

a) Assessment of stochastic behaviour of stocking pattern in the warehouse and then finding the way of increasing space utilization. Not too much work has been done so far.
b) Estimation of an optimal allocation rule for shelving of a new box of footwear inside the compartment.

An attempt is made to formulate a mathematical model for the optimal allocation of footwear boxes. It is based on the available capacity of shelving, the receipt quantity of footwear boxes (*static, at any time point*) along with certain restrictions imposed on shelving (*which is already in practice*). The problem formulated as a non-linear programming (NLP) where maximization of space utilization is of utmost concern.

9.1 Formulation of NLP Problem

Variables Involved

Assume

M = Total types of Boxes
N = Total types of Compartments
n_j = Total no. of j^{th} type compartments available; j = 1 to N
m_j = No. of j^{th} type of compartment used.
x_{ijk} = No. of i^{th} type boxes in k^{th} compartment of j^{th} type
i = 1 to M; j = 1 to N; k = 1 to n_j
t_{ijk} = 1, if i^{th} type box is stacked in k^{th} compartment of j^{th} type.
= 0, otherwise
V_j = Volume of j^{th} type of compartment
C_{ij} = Maximum no. of i^{th} box that can be kept horizontally/vertically in j^{th} compartment.
BV_i = Volume of i^{th} type of boxes.
d_i = Receipt (purchased) quantity of i^{th} type of boxes.

Objective Function for NLP

$$\text{Maximize } Z = \sum_{i=1}^{M}\sum_{j=1}^{N}\left[\frac{\sum_{k=1}^{m_j} x_{ijk} \times BV_i}{m_j \times V_j}\right]$$

Constraints Imposed:

$$\sum_{j=1}^{N}\sum_{k=1}^{n_j} x_{ijk} \times t_{ijk} = d_i, \forall i$$

$$\sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{n_j} t_{ijk} \le \sum_{j=1}^{N} m_j$$

$$\sum_{i=1}^{M} t_{ijk} \le 1, \ \forall j,k$$

$$x_{ijk} \times t_{ijk} \le C_{ij}, \ \forall i,j,k$$

Bounds:

$$x_{ijk} \ge 0, \ \text{int}, \ \forall i,j,k$$

$$t_{ijk} = 0, \ 1 \forall i, \ j,k$$

It must be mentioned here that the dynamic (stochastic) situation of the problem, specially, with respect to distribution of receipt quantity of footwear boxes and the filled-up condition (empty / filled / semi-filled) of compartments needs to be taken into consideration while solving the above non-linear programming problem for an economic solution for the company.

REFERENCES

[1] Gilmore, P. C. and R. E. Gomory, "A linear programming approach to the cutting stock problem," *Operations Research* 9 (1961), 849-859.

[2] Gilmore, P. C. and R. E. Gomory, "A linear programming approach to the cutting stock problem-part II," *Operations Research* 9 (1963), 863-888.

[3] Gilmore, P. C. and R. E. Gomory, "Multistage cutting problems of two and more dimensions," *Operations Research* 13 (1965), 94-119.

[4] Bengsston, B. E., "Packing Rectangular Pieces – A Heuristic Approach,"

The Computer Journal 25, 3 (1982), 353-357.

[5] George, J. A. and D. F. Robinson, “A Heuristic for Packing Boxes into a Container,” *Comp. Operations Research* 7 (1980), 147-156.

[6] Hausman, W. H., L. B. Schwarz, and S. C. Graves, “Optimal Storage Assignment in Automatic Warehousing Systems,” *Management Science* 22 (1976), 629-638.

[7] Lodi, A., S. Martello, and D. Vigo, “Approximation algorithms for the oriented two-dimensional bin packing problem,” *European Journal of Operations Research* 112 (1999), 158-166.

[8] Lodi, A., S. Martello, and D. Vigo, “Heuristics and metaheuristic approaches for a class of two-dimensional bin packing problems,” *INFORMS J. Comput.* 11(1999), 345-357.

[9] Lodi, A., S. Martello, and D. Vigo, “Recent advances on two-dimensional bin packing problems,” *Discrete Applied Mathematics* 123 (2002), 379-396.

ANNEXURE-1

Distribution of Compartments in the Warehouse

WAREHOUSE ARRANGEMENT	COMPARTMENT SIZE (CS) – [in feet]						
(in rows)	Length (X) × Width (Y) × Height (Z)						TOTAL
ROW No.	4×2×2.4	3×2×2.4	4×2×1.6	3×2×1.6	3×1.3×2.3	3×1.3×1.6	
	CS-1	CS-2	CS-3	CS-4	CS-5	CS-6	
1					52	52	**104**
2		87		29			**116**
3	9	75	3	25			**112**
4	54	15	18	5			**92**
5	12	66	4	22			**104**
6	63	9	21	3			**96**
7	60	12	20	4			**96**
8	60	12	20	4			**96**
9	12	66	4	22			**104**
10	54	15	18	5			**92**
11	9	78	3	26			**116**
12		87		29			**116**
13	57	12	19	4			**92**
TOTAL	**390**	**534**	**130**	**178**	**52**	**52**	**1336**

ANNEXURE-2

Distribution of Existing Box Sizes in the Warehouse

Sl. No.	Category	Box Code	Box Size (height × width × length)	Volume (c.inch)	Stock Quantity
1	Hawai	B01	3.5 × 2 × 8.5	59.5	1228
2	Kids	B02	3 × 4.5 × 7	94.5	3704
3	School Canvas	B03	3 × 3.5 × 9.5	99.75	682
4	School Days	B04	3 × 4.5 × 7.5	101.25	1832
5	Rangilla	B05	3.5 × 4 × 9	126	200
6	School Canvas	B06	3.5 × 3.5 × 10.5	128.625	354
7	Kids	B07	3.5 × 5 × 7.5	131.25	5783
8	Hawai	B08	4 × 3 × 11	132	1382
9	Hawai	B09	4 × 3 × 11.5	138	1326
10	School Canvas	B10	3.5 × 3.5 × 11.5	140.875	398
11	Kids	B11	3.5 × 5 × 9	157.5	421
12	School Days	B12	4 × 4.5 × 10.5	189	1813
13	Ladies	B13	4 × 5 × 10.5	210	11842
14	Flotas	B14	4 × 6 × 10	240	1765
15	Sharon	B15	4 × 5.5 × 11	242	7366
16	Ladies	B16	4 × 6 × 10.5	252	4365
17	Gents	B17	4.5 × 5 × 11.5	258.75	16637
18	Gents	B18	4 × 5.5 × 12	264	3665
19	Ladies	B19	4.5 × 5 × 12	270	670
20	Gents/Ladies	B20	4 × 6 × 11.5	276	5724
21	Easy Walk	B21	4.5 × 5.5 × 11.5	284.625	749
22	Sharon	B22	4 × 7.5 × 11	330	354
23	Gents/Shoe	B23	4 × 7 × 12	336	1078
24	Shoe	B24	4 × 8 × 11	352	192
25	Gents	B25	4.5 × 7 × 12.5	393.75	402
26	Gents	B26	5 × 7.5 × 11.5	431.25	1205
27	Shoe	B27	4 × 10 × 12.5	500	306
28	Shoe	B28	5 × 8 × 13	520	192

ANNEXURE-3

Calculations for Empty space% and Space utilization

- Empty space%↓ (V → H) :

$$\left[\frac{Vol(H)-Vol(V)}{Vol(C)-Vol(V)}\right]\times 100\%$$

- Empty space%↓(V/H →M) :

$$\left[\frac{Vol(M)-MaxVol(V/H)}{Vol(C)-MaxVol(V/H)}\right]\times 100\%$$

- Space utilization↑(V → H) :

$$\left[\frac{Vol(H)-Vol(V)}{Vol(C)}\right]\times 100\%$$

- Space utilization↑(V/H → M) :

$$\left[\frac{Vol(M)-MaxVol(V/H)}{Vol(C)}\right]\times 100\%$$

ANNEXURE-4(a)

Summary Table : A Sample

Sl. No.	Box Size (in inch) (H × W × L)	Parameters	CS-1	CS-2	CS-3	CS-4	CS-5	CS-6
1	3.5 × 2 × 8.5	Vertical (V)	384	288	240	180	126	90
		Horizontal (H)	480*	384*	300*	240*	196*	140*
		Mix (M)	496**	400**	310**	250**	210**	150**
		%empty space ↓ (V → H)	55.30	73.73	45.55	60.73	52.04	54.81
		%empty space ↓ (V/H → M)	20.62	46.78	13.94	25.77	21.70	24.26
		%space utilization↑ (V→H)	17.22	22.96	16.14	21.52	26.87	27.59
		%space utilization↑ (V/H→M)	2.87	3.83	2.69	3.59	5.37	5.52
2	3 × 4.5 × 7	Vertical (V)	270	216	180	144	144	96
		Horizontal (H)	270	225*	180	150*	135	90
		Mix (M)	297**	234**	198**	156**	144	96
		%empty space ↓ (V → H)	0.00	19.02	0.00	19.02	-44.95	-33.14
		%empty space ↓ (V/H → M)	33.30	23.49	33.30	23.49	0.00	0.00
		%space utilization↑ (V→H)	0.00	3.42	0.00	3.42	-5.49	-5.26
		%space utilization↑ (V/H→M)	7.69	3.42	7.69	3.42	0.00	0.00
3	3 × 3.5 × 9.5	Vertical (V)	234	192	156	120	90	60
		Horizontal (H)	270*	192	180*	108	108*	72*
		Mix (M)	297**	216**	198**	144**	126**	84**
		%empty space ↓ (V → H)	36.51	0.00	36.51	-25.92	27.53	24.95
		%empty space ↓ (V/H → M)	43.13	41.77	43.13	51.83	37.98	33.24
		%space utilization↑ (V→H)	10.82	0.00	10.82	-7.22	11.58	11.10
		%space utilization↑ (V/H→M)	8.12	9.62	8.12	14.43	11.58	11.10
4	3 × 4.5 × 7.5	Vertical (V)	270	216	180	144	144	96
		Horizontal (H)	270	192	180	120	108	72
		Mix (M)	288**	225**	192**	150**	144	96
		%empty space ↓ (V → H)	0.00	-80.65	0.00	-120.97	-396.13	-228.66
		%empty space ↓ (V/H → M)	31.21	30.24	31.21	30.24	0.00	0.00
		%space utilization↑ (V→H)	0.00	-9.77	0.00	-14.65	-23.52	-22.54
		%space utilization↑ (V/H→M)	5.49	3.66	5.49	3.66	0.00	0.00

ANNEXURE-4(b)

Gain in Number of boxes – Compartment wise

Box Type	Box Size (in inch) (H × W × L)	Change of Orientation	CS-1	CS-2	CS-3	CS-4	CS-5	CS-6
B01	3.5 × 2 × 8.5*	Vertical → Horizontal	✓	✓	✓	✓	✓	✓
		Vertical/Horizontal → Mix	✓	✓	✓	✓	✓	✓
B02	3 × 4.5 × 7	Vertical → Horizontal		✓		✓		
		Vertical/Horizontal → Mix	✓	✓	✓	✓		
B03	3 × 3.5 × 9.5*	Vertical → Horizontal	✓		✓		✓	✓
		Vertical/Horizontal → Mix	✓	✓	✓	✓	✓	✓
B04	3 × 4.5 × 7.5	Vertical/Horizontal → Mix	✓	✓	✓	✓		
B05	3.5 × 4 × 9*	Vertical → Horizontal	✓	✓	✓	✓	✓	✓
		Vertical/Horizontal → Mix		✓			✓	✓
B06	3.5 × 3.5 × 10.5*	Vertical → Horizontal					✓	✓
		Vertical/Horizontal → Mix					✓	✓
B07	3.5 × 5 × 7.5	Vertical/Horizontal → Mix		✓				
B08	4 × 3 × 11*	Vertical → Horizontal					✓	✓
		Vertical/Horizontal → Mix	✓	✓	✓	✓	✓	✓
B09	4 × 3 × 11.5*	Vertical → Horizontal					✓	✓
		Vertical/Horizontal → Mix		✓				
B10	3.5 × 3.5 × 11.5	Vertical → Horizontal					✓	✓
		Vertical/Horizontal → Mix					✓	✓
B11	3.5 × 5 × 9*	Vertical → Horizontal	✓	✓	✓	✓	✓	✓
		Vertical/Horizontal → Mix	✓	✓	✓	✓		
B12	4 × 4.5 × 10.5*	Vertical → Horizontal					✓	✓
		Vertical/Horizontal → Mix	✓	✓	✓	✓	✓	✓
B13	4 × 5 × 10.5*	Vertical → Horizontal					✓	✓
		Vertical/Horizontal → Mix		✓			✓	✓
B14	4 × 6 × 10	Vertical/Horizontal → Mix	✓	✓	✓	✓	✓	✓
B16	4 × 6 × 10.5	Vertical/Horizontal → Mix	✓	✓	✓			
B17	4.5 × 5 × 11.5*	Vertical → Horizontal					✓	✓
		Vertical/Horizontal → Mix		✓				
B19	4.5 × 5 × 12*	Vertical → Horizontal					✓	✓
		Vertical/Horizontal → Mix		✓				
B20	4 × 6 × 11.5	Vertical/Horizontal → Mix		✓				
B22	4 × 7.5 × 11*	Vertical → Horizontal				✓	✓	✓
		Vertical/Horizontal → Mix	✓	✓	✓			
B23	4 × 7 × 12*	Vertical → Horizontal		✓			✓	✓
B24	4 × 8 × 11	Vertical → Horizontal				✓		
		Vertical/Horizontal → Mix		✓				
B25	4.5 × 7 × 12.5*	Vertical → Horizontal	✓		✓	✓		
B26	5 × 7.5 × 11.5*	Vertical → Horizontal				✓	✓	✓
		Vertical/Horizontal → Mix		✓				
B27	4 × 10 × 12.5	Vertical → Horizontal	✓	✓	✓	✓		
		Vertical/Horizontal → Mix	✓	✓	✓	✓		
B28	5 × 8 × 13	Vertical → Horizontal	✓	✓	✓	✓		
		Vertical/Horizontal → Mix		✓	✓	✓		

* indicates overall average gain with respect to all compartments, if arranged horizontally.

Annexure-5

Orientation : Selective (Vert + Horz.)

Sl. No.	Vol (Box)	CS-1	CS-2	CS-3	CS-4	CS-5	CS-6	Vol-1	Vol-2	Vol-3	Vol-4	Vol-5	Vol-6
1*	59.5	480	384	300	240	196	140	28560	22848	17850	14280	11662	8330
2	94.5	270	216	180	144	144	96	25515	20412	17010	13608	13608	9072
3*	99.75	270	192	180	108	108	72	26933	19152	17955	10773	10773	7182
4	101.25	270	216	180	144	144	96	27338	21870	18225	14580	14580	9720
5*	126	240	168	150	120	84	60	30240	21168	18900	15120	10584	7560
6	128.625	208	160	130	100	70	50	26754	20580	16721	12863	9003.8	6431.25
7	131.25	216	150	135	105	98	70	28350	19688	17719	13781	12863	9187.5
8*	132	224	162	128	96	90	60	29568	21384	16896	12672	11880	7920
9*	138	224	162	128	96	90	60	30912	22356	17664	13248	12420	8280
10	140.875	208	160	130	100	70	50	29302	22540	18314	14088	9861.3	7043.75
11*	157.5	160	120	100	80	84	60	25200	18900	15750	12600	13230	9450
12*	189	140	108	80	60	54	36	26460	20412	15120	11340	10206	6804
13	210	126	90	72	56	42	28	26460	18900	15120	11760	8820	5880
14	240	112	72	64	48	36	24	26880	17280	15360	11520	8640	5760
15	242	112	84	64	48	36	24	27104	20328	15488	11616	8712	5808
16	252	112	72	64	48	36	24	28224	18144	16128	12096	9072	6048
17	258.75	108	80	72	56	42	28	27945	20700	18630	14490	10868	7245
18	264	112	84	64	48	36	24	29568	22176	16896	12672	9504	6336
19	270	108	80	72	56	42	28	29160	21600	19440	15120	11340	7560
20	276	112	72	64	48	36	24	30912	19872	17664	13248	9936	6624
21	284.625	96	72	64	48	36	24	27324	20493	18216	13662	10247	6831
22*	330	84	54	48	36	36	24	27720	17820	15840	11880	11880	7920
23*	336	84	70	48	36	36	24	28224	23520	16128	12096	12096	8064
24	352	84	54	48	32	24	16	29568	19008	16896	11264	8448	5632
25*	393.75	54	30	36	24	24	16	21263	11813	14175	9450	9450	6300
26*	431.25	60	36	36	27	30	18	25875	15525	15525	11644	12938	7762.5
27*	500	42	24	24	16	12	8	21000	12000	12000	8000	6000	4000
28*	520	45	24	27	18	10	6	23400	12480	14040	9360	5200	3120
		Volume of the Boxes (CS-wise)						765758	542968	465670	348830	293820	197871
		Total Volume of the Boxes											2614916
								Vol-CS-1	**Vol-CS-2**	**Vol-CS-3**	**Vol-CS-4**	**Vol-CS-5**	**Vol-CS-6**
								33177.6	24883.2	22118	16588.8	15500.2	10782.72
		Total Volume of the Compartments											123050.9
		Max Space Utilization (CS-wise)						82.43%	77.93%	75.19%	75.10%	67.70%	65.54%
		Overall Max Space Utilization											**75.90%**

* Arranged in 'Horizontal' directions, 'Vol' stands for volume.

Annexure-6

Estimation of Average Space Utilization for Selective arrangements of boxes

Box Size	Stock Level	*Volume*	Minimum No. of required compartments for (Ver. / Horz.) arrangements						Total Vol (Box)	Total Vol (Comp)	Avg Vol (Comp)	Avg. Util%
(H × W × L)	*random*	Box	#CS-1	#CS-2	#CS-3	#CS-4	#CS-5	#CS-6			*per comp. type*	*per box type*
3.5 × 2 × 8.5*	1228	59.5	3	4	5	6	7	9	73066	614736	102456	71.31
3 × 4.5 × 7	3704	94.5	14	18	21	26	26	39	350028	2631709.44	438618.24	79.80
3 × 3.5 × 9.5*	682	99.75	3	4	4	7	7	10	68029.5	619989.12	103331.52	65.84
3 × 4.5 × 7.5	1832	101.25	7	9	11	13	13	20	185490	1332305.28	222050.88	83.53
3.5 × 4 × 9*	200	126	1	2	2	2	3	4	25200	249989.76	41664.96	60.48
3.5 × 3.5 × 10.5*	354	128.625	2	3	3	4	5	6	45533.25	415912.32	69318.72	65.69
3.5 × 5 × 7.5	5783	131.25	27	39	43	56	60	83	759018.75	5571279.36	928546.56	81.74
4 × 3 × 11*	1382	132	7	9	11	15	16	24	182424	1455114.24	242519.04	75.22
4 × 3 × 11.5*	1326	138	6	9	11	14	15	23	182988	1379064.96	229844.16	79.61
3.5 × 3.5 × 11.5	398	140.875	2	3	4	4	6	8	56068.25	475096.32	79182.72	70.81
3.5 × 5 × 9*	421	157.5	3	4	5	6	6	8	66307.5	588453.12	98075.52	67.61
4 × 4.5 × 10.5*	1813	189	13	17	23	31	34	51	342657	2954223.36	492370.56	69.59
4 × 5 × 10.5*	11842	210	106	132	186	247	220	329	2486820	21970414.08	3661735.68	67.91
4 × 6 × 10	1765	240	16	25	28	37	50	74	423600	3958951.68	659825.28	64.20
4 × 5.5 × 11	7360	242	66	88	115	154	205	307	1781120	15965562.24	2660927.04	66.94
4 × 6 × 10.5	4365	252	39	61	69	91	122	182	1099980	9701026.56	1616837.76	68.03
4.5 × 5 × 11.5*	16637	258.75	174	222	260	347	309	463	4304823.75	32586019.2	5431003.2	79.26
4 × 5.5 × 12	3665	264	33	44	58	77	102	153	967560	7980698.88	1330116.48	72.74
4.5 × 5 × 12*	670	270	7	9	11	14	13	19	180900	1338111.36	223018.56	81.11
4 × 6 × 11.5	5724	276	52	80	90	120	159	239	1579824	12738798.72	2123133.12	74.41
4.5 × 5.5 × 11.5	749	284.625	8	11	12	16	21	32	213184.125	1740528	290088	73.49
4 × 7.5 × 11*	354	330	5	7	8	10	10	15	116820	999648	166608	70.12
4 × 7 × 12*	1078	336	13	16	23	30	30	45	362208	2786054.4	464342.4	78.00
4 × 8 × 11	192	352	3	4	4	6	8	12	67584	640465.92	106744.32	63.31
4.5 × 7 × 12.5*	402	393.75	8	14	12	17	17	26	158287.5	1705069.44	284178.24	55.70
5 × 7.5 × 11.5*	1205	431.25	21	34	34	45	41	67	519656.25	4399228.8	733204.8	70.87
4 × 10 × 12.5	306	500	11	17	20	26	17	26	153000	2205498.24	367583.04	41.62
5 × 8 × 13	192	520	7	10	11	16	10	16	99840	1317323.52	219553.92	45.47
TOTAL	75629								16852017.88		23386878.72	72.06

* indicates box type (with respect to size) where less compartments are needed, if arranged horizontally.

Annexure-7

Estimation of Gain in reduction of number of compartments

Box Size	Stock Level	Minimum no. of required Compartments for (Vertical, Horizontal, Mix) arrangements							Avg. Gain / CS type	Selective [Gain > 0]	Avg. Gain / CS type
(H × W × L)	random	#CS-1	#CS-2	#CS-3	#CS-4	#CS-5	#CS-6	Total #CS	all (V→H)	(V→H)	all (V/H→M)
3.5 × 2 × 8.5*	1228	(4,3,3)	(5,4,4)	(6,5,4)	(7,6,5)	(10,7,6)	(14,9,9)	(46,34,31)	2	2	1
3 × 4.5 × 7	3704	(14,14,13)	(18,17,16)	(21,21,19)	(26,25,24)	(26,28,26)	(39,42,39)	(144,147,137)	-1		2
3 × 3.5 × 9.5*	682	(3,3,3)	(4,4,4)	(5,4,4)	(6,7,5)	(8,7,6)	(12,10,9)	(38,35,31)	1	1	1
3 × 4.5 × 7.5	1832	(7,7,7)	(9,10,9)	(11,11,10)	(13,16,13)	(13,17,13)	(20,26,20)	(73,87,72)	-3		1
3.5 × 4 × 9*	200	(2,1,1)	(2,2,2)	(2,2,2)	(3,2,2)	(4,3,3)	(5,4,4)	(18,14,14)	1	1	0
3.5 × 3.5 × 10.5*	354	(2,2,2)	(3,3,3)	(3,3,3)	(4,4,4)	(6,5,4)	(8,6,6)	(26,23,22)	1	1	1
3.5 × 5 × 7.5	5783	(27,31,27)	(39,49,35)	(43,49,43)	(56,73,56)	(60,69,60)	(83,97,83)	(308,368,304)	-10		1
4 × 3 × 11*	1382	(7,7,6)	(9,9,8)	(11,11,11)	(15,15,14)	(20,16,15)	(29,24,22)	(91,82,76)	2	2	1
4 × 3 × 11.5*	1326	(6,6,6)	(9,9,8)	(11,11,11)	(14,14,14)	(19,15,15)	(28,23,23)	(87,78,77)	2	2	1
3.5 × 3.5 × 11.5	398	(2,3,2)	(3,3,3)	(4,4,4)	(4,5,4)	(6,5,5)	(8,7,7)	(27,27,25)	0		1
3.5 × 5 × 9*	421	(3,3,3)	(5,4,3)	(5,5,4)	(7,6,5)	(9,6,6)	(13,8,8)	(42,32,29)	2	2	1
4 × 4.5 × 10.5*	1813	(13,13,12)	(17,17,15)	(23,23,21)	(29,31,26)	(38,34,28)	(57,51,42)	(177,169,144)	2	2	5
4 × 5 × 10.5*	11842	(94,106,94)	(132,132,121)	(165,186,165)	(212,247,212)	(282,220,198)	(423,329,297)	(1308,1220,1087)	15	15	23
4 × 6 × 10	1765	(16,16,15)	(25,25,19)	(28,28,25)	(37,37,32)	(50,50,43)	(74,74,64)	(230,230,198)	0		6
4 × 5.5 × 11	7360	(66,66,66)	(88,88,88)	(115,115,115)	(154,154,154)	(205,205,205)	(307,307,307)	(935,935,935)	0		0
4 × 6 × 10.5	4365	(39,39,35)	(61,61,52)	(69,69,61)	(91,91,91)	(122,122,122)	(182,182,182)	(564,564,543)	0		4
4.5 × 5 × 11.5*	16637	(155,174,155)	(208,222,199)	(232,260,232)	(298,347,298)	(397,309,309)	(595,463,463)	(1885,1775,1656)	19	19	20
4 × 5.5 × 12	3665	(33,33,33)	(44,44,44)	(58,58,58)	(77,77,77)	(102,102,102)	(153,153,153)	(467,467,467)	0		0
4.5 × 5 × 12*	670	(7,7,7)	(9,9,8)	(10,11,10)	(12,14,12)	(16,13,13)	(24,19,19)	(78,73,69)	1	1	1
4 × 6 × 11.5	5724	(52,52,52)	(80,80,69)	(90,90,90)	(120,120,120)	(159,159,159)	(239,239,239)	(740,740,729)	0		2
4.5 × 5.5 × 11.5	749	(8,8,8)	(11,11,11)	(12,12,12)	(16,16,16)	(21,21,21)	(32,32,32)	(100,100,100)	0		0
4 × 7.5 × 11*	354	(5,5,4)	(7,7,6)	(8,8,7)	(12,10,10)	(15,10,10)	(23,15,15)	(70,55,52)	3	3	1
4 × 7 × 12*	1078	(13,13,13)	(18,16,16)	(23,23,23)	(27,30,27)	(36,30,30)	(54,45,45)	(171,157,154)	3	3	1
4 × 8 × 11	192	(3,3,3)	(4,4,4)	(4,4,4)	(6,6,6)	(8,11,8)	(12,16,12)	(37,44,37)	-2		0
4.5 × 7 × 12.5*	402	(12,8,8)	(12,14,12)	(17,12,12)	(21,17,17)	(14,17,14)	(21,26,21)	(97,94,84)	1	1	2
5 × 7.5 × 11.5*	1205	(21,21,21)	(29,34,27)	(34,34,34)	(51,45,45)	(61,41,41)	(101,67,67)	(297,242,235)	10	10	2
4 × 10 × 12.5	306	(11,8,7)	(17,13,9)	(20,13,11)	(26,20,16)	(17,26,17)	(26,39,26)	(117,119,86)	-1		6
5 × 8 × 13	192	(7,5,5)	(10,8,6)	(11,8,8)	(16,11,10)	(10,20,10)	(16,32,16)	(70,84,55)	-3		3
TOTAL	**75629**								**45**	**65**	**87**

* indicates box type (with respect to size) where less compartments are needed, if arranged horizontally.