Transformer Core Model and Parameter Estimation for ATP

Sung-Don Cho†

Abstract - Power transformers would appear to be simple. However, due to their nonlinear and frequency-dependent behaviors, they can be one of the most complex system components to model. It is imperative that the applied models be appropriate for the range of frequencies and excitation levels that the system experiences. Transformer modeling is not a mature field and newer improved models must be made available in ATP packages. Further, there is a lack of published guidance on recommended modeling approaches. And there is typically not enough detailed design or test information available to determine the parameters for a given model. The purpose of this paper is to develop improved transformer core models for ATP and parameter estimation methods that can efficiently utilize the limited available information such as factory test reports.

Keywords: ATP, core model, EMTP, optimization, TACS, transformer

1. Introduction

The core model includes the saturable magnetizing inductance, hysteresis loss, and eddy current loss of core.

There are three nonlinear inductances available in ATP. The Type-93 is a true nonliear inductance. Operation is always on the proper λ-i segment of the charateristic and hence may allow much better results [1]. The Type-96 hysteretic inductance and the Type-98 pseudo-nonlinear inductances are not as robust, due to different implementation methods.

Core losses can be modeled in a simplistic manner as a separate linear resistance in parallel with the nonlinear magnetizing inductance. However, this is only valid for the steady-state sinusoidal applied voltage of a given RMS magnitude.

Actually the core loss is nonlinear and frequency-dependent and the use of a linear resistance can result in errors for some types of simulations. Therefore, the core loss needs to be modeled using a more sophisticated description. Unfortunately, there is a lack of a suitable nonlinear resistance element in ATP to model the constricted (non-sigmoid, non-monotonic) flux-current loop.

The core loss description must be a function of frequency and voltage, and must ultimately be implemented in the time domain in ATP.

2. Transformer Test Data

Typical transformer factory test reports available from manufacturers consist of data like that presented in Table 1. The available data are no-load kW losses and true RMS exciting current at 100% and 110% of rated voltage. However, it is valid only for the frequency at which the nameplate data was obtained.

Core losses from the test data are employed to calculate the model parameters. Using the 100% and 110% excitation data from the factory test report, the RMS magnetizing current is obtained by removing the core loss component from the exciting current as in Equation (1).

I-peak and λ-peak can be calculated from Irms-Vrms using the Saturation subroutine of ATP.

$$I_m = \sqrt{I_{ex}^2 - I_c^2} \quad (1)$$

Table 1 Open Circuit Test Data

	13.8kV@100%V	15.18kV@110%V
Exciting Current (I_{ex})	54.338 Arms	122.261Arms
Core Loss	99.20kW	134.08kW
Core Loss Current (I_c)	7.188Arms	8.833Arms
Magnetizing Current (I_m)	53.861Arms, 76.17Apeak	121.941Arms, 278.90Apeak
λ_{peak}	51.76 Wb-t	56.94 Wb-t

3. Magnetic Core Saturation

Factory test report data typically provides excitation data at the 100% and 110% levels. However, the 110% magnetic flux level was being exceeded in many of the cases investigated and higher levels of excitation data are

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not available in most factory test reports.

One of the traditionally used representations for the core saturation curve is the Frolich equation, Equation (2). This equation gives a smooth single-valued anhysteretic curve relating the flux density B to the magnetizing force H. Only two data points on the curve are needed to fit this equation [2].

$$B = \frac{H}{a + b \cdot |H|} \tag{2}$$

Saturation data for Armco M4 Steel is given as H(A/m)= [0 2 46 8.4 11.1 14.4 23 55 130 416] at B(T)= [0 .2 .4 .6 .8 1. 1.2 1.4 1.6 1.7 1.8]. For example, if two points, H=[14.4, 55] and B=[1.2, 1.6], are chosen, then a fit of "a" = 4.0640 and "b"= 0.5511 for Equations (2) can be obtained. Fig. 1 shows the B-H curve obtained from the above method and it matches well with the given saturation data.

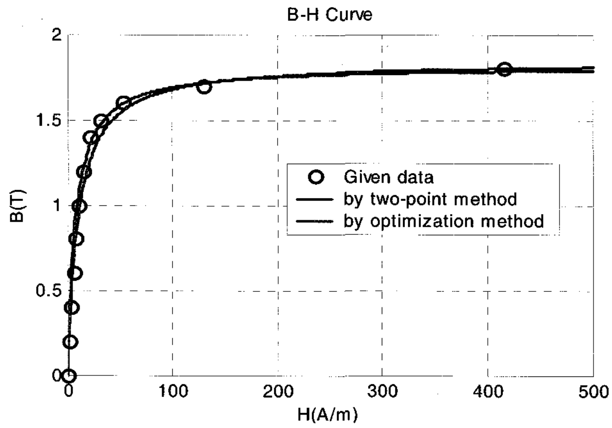


Fig. 1 Saturation Curve Fitting using Frolich Equation

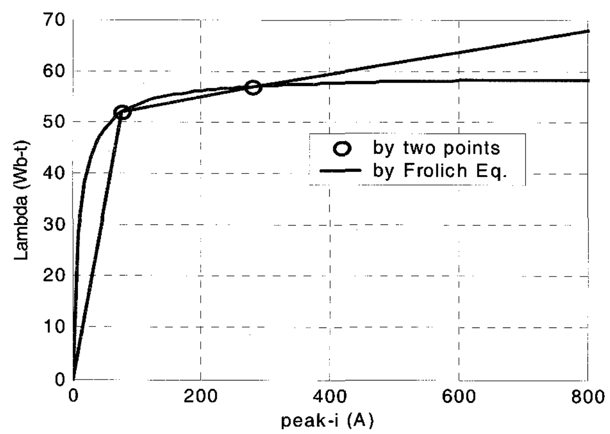


Fig. 2 Derived Saturation Curve

If both the B-H curve and the dimension data are unknown (general case), x=0.1842, y=0.0169 for λ-i curve are directly obtained from two data points (Table 1) using Equation (3). Fig. 2 indicates the λ-i curves obtained from the above methods and it matches well with Fig. 1.

$$\lambda = \frac{i}{a \cdot \frac{L}{AN} + b \cdot \frac{|i|}{AN}} = \frac{i}{x + y \cdot |i|} \tag{3}$$

where A= Cross-Sectional Area, L= Length, N= Turns

4. Nonlinear Core Loss

4.1 Separation of eddy current and hysteresis losses

The modeling of eddy currents and hysteresis has been approximate and difficult, because of the lack of information. In the approach developed here, parameters for the transformer model are estimated using basic transformer test data and optimization techniques.

For the detailed model, separation of the core losses is necessary as below.

P_C(core loss) = P_H(hysteresis loss) +P_E(eddy current loss)

The ratio between P_E and P_H is about 1 to 3, but is usually not given in a factory test report, since these two parts cannot be separated in the factory's excitation tests.

Generally, P_E is proportional to λ^2 and f^2 in the low-frequency range. In the high-frequency range, it changes to about $f^{1.5}$ because of the skin effect in the laminations. P_H is proportional to λ^{X2} and f. "X_2" is generally larger than 2 and close to 3 for grain-oriented steel. [1]

$$\text{Thus, } P_C = P_H + P_E = X_1 \cdot \lambda^{X2} \cdot f + X_3 \cdot \lambda^2 \cdot f^{X4}. \tag{4}$$

Core losses (P_C) at 100%V and 110%V are usually given from factory test reports as shown in Table 1. Core loss at 120 Hz and 200%V (frequency and voltage are both changed in order to keep the flux magnitude constant) is assumed as 292.7 kW (99.2 kW x 1.48/0.51) from Table 2.

There are four unknowns (X_1, X_2, X_3, X_4). From Table 3, three known conditions are:

$$1.0 = X_1 + X_3 \text{ at } \lambda = 1\text{pu and } f = 1\text{pu (60Hz)} \tag{5}$$

$$1.35 = X_1 \cdot (1.1)^{X2} + X_3 \cdot (1.21) \text{ at } \lambda = 1.1\text{pu and } f = 1\text{pu} \tag{6}$$

$$2.95 = (2) \cdot X_1 + X_3 \cdot (2)^{X4} \text{ at } \lambda = 1 \text{ pu and } f = 2\text{pu} \tag{7}$$

Applying optimization techniques to find the solutions of a constrained nonlinear multivariable function,

$$F(X) = (2 \cdot X_1 + X_3 \cdot 2^{X4} - 2.95)^2 \text{ as objective function}$$

$1 = X_1 + X_3$ *as linear constraint*

$1.35 = X_1 \cdot (1.1)^{X2} + 1.21 \cdot X_3$ *as nonlinear constraint.*

The result is X = [0.5245 4.1132 0.4755 1.9998]. From Equations (5)~(7), PC = [1.0000 1.3516 2.9506] and %error= [0.0000 0.0000 0.0000]

Using the above result, separated loss functions for the example transformer can be obtained:

$$P_H = 0.5245 \cdot \lambda^{4.1132} \cdot f \text{ and } P_E = 0.4755 \cdot \lambda^2 \cdot f^{1.9998} \text{ (pu)} \quad (8)$$

$$I_H = P_H/V = (0.5245 \cdot \lambda^{4.1132} \cdot f)/(\lambda \cdot f) = 0.5245 \cdot \lambda^{3.1132} \text{ (pu)} \quad (9)$$

$$I_E = V/R_E = V/2.103 \text{ (pu)} \quad (10)$$

$$R_E = V^2/P_E = \lambda^2 \cdot f^2/(0.4755 \cdot \lambda^2 \cdot f^{1.9998}) \approx 2.103 \text{(pu)} \quad (11)$$

$$R_H = V^2/P_H = \lambda^2 \cdot f^2/(0.5245 \cdot \lambda^{4.1132} \cdot f) = 1.9066 \cdot \lambda^{-2.1132} \cdot f \text{ (pu)} \quad (12)$$

Table 2 Core Loss for M4 (B= 1.5T) [3]

Frequency (Hz)	60Hz	120Hz	180Hz	300Hz	1000Hz
Core loss (W/lb)	0.51	1.48	2.85	6.7	56
Ratio @ 60-Hz	1.0	2.95	5.7	13.4	112

Table 3 Given Core Loss Data from Tables 1 and 2

	100%V, 60Hz	110%V, 60Hz	200%V, 120Hz
λ (Wb-t)	51.77	56.95	51.77
Pc(W) (pu)	99,200 (1.0)	134,080(1.35)	292,700 (2.95)
Ic (rmsA)	7.1884	8.8327	10.6051

Table 4 Separated Core Loss using Functions

	100%V, 60Hz	110%V, 60Hz	200%V, 120Hz
P_H (W, pu)	52,030(.5245)	77,000 (.7763)	104,060 (1.0490)
P_E (W, pu)	47,170 (.4755)	57,080 (.5754)	188,650 (1.9017)
I_H (A,rms)	3.7703	5.0727	3.7703
I_E (A,rms)	3.4181	3.7599	6.8352

In the case of $X_4 = 2$, P_E can be modeled by a resistance R_E. In the case of $X_4 \neq 2$, frequency dependency needs to be considered. P_H can be represented by a resistance (R_H) in Equation (11). In this case, R_H for hysteresis loss is nonlinear and frequency dependent.

However, I_H is also nonlinear and frequency independent. If the hysteresis loss can be modeled by I_H current injection, the frequency-dependency can be implemented as a time-varying current injection. The enclosed area of a λ-i_H plot shown in Fig. 3 is the hysteresis loss per cycle. Hysteresis loss at rated frequency might be represented by a two-slope *v-i* curve, defined by Fig. 3. If the RMS currents of the transformer are converted to the peak currents by the SATURATION routine, I_H is 5.332 peak-A at 100%V and 9.179 peak-A at 110%V. Fig. 3 indicates the waveforms for the v-i_H and λ-i_H at 110%V and 60 Hz. However, actual hysteresis loss is dependent on maximum flux, not voltage [34].

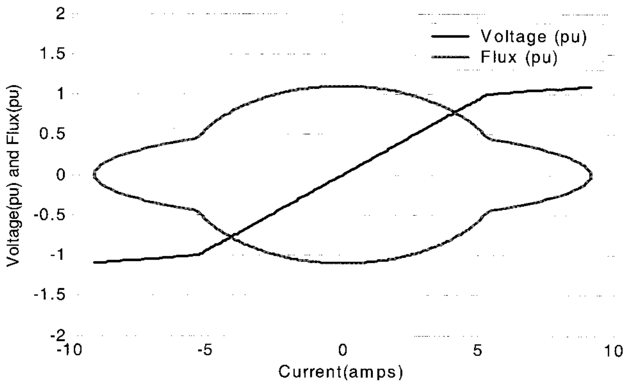


Fig. 3 v-i_H and λ-i_H Plot at 110%V and 60Hz

4.2 Hysteresis Loop Model

The λ-i hysteresis loop gives the instantaneous relationship between current and flux linked for near-DC periodic excitation. The spine of the λ-i hysteresis loop provides the normal magnetic saturation curve shown in Fig. 4. Hysteresis loss can be thought of as a nonlinear frequency-dependent resistance. Hysteresis loss is not directly a function of voltage, but of λ. Therefore, the matching of average losses for 60 Hz excitation does not mean that correct flux-current trajectory is being followed in the time domain. Residual flux of a transformer is another important aspect, in fact a critical one for inrush simulations. Therefore, a correct hysteresis loop trajectory is a necessary part of a correct time domain core model. Note that (*Hctop*, *Bctop*) is defined here as the coercive force and flux density corresponding to the maximum known excitation level.

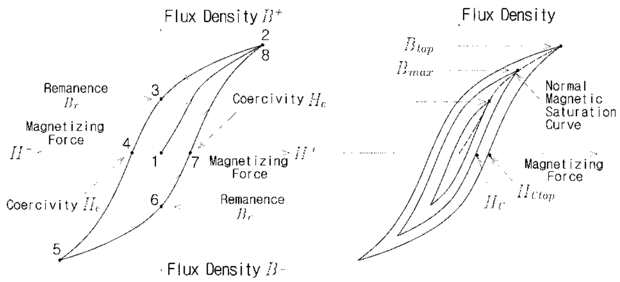


Fig. 4 Example of Hysteresis Loop [4]

In this paper, the λ-i hysteretic loop is based on the saturation curve given from the open circuit test in

Section 3. The loop is modeled by the left and right displacement. The enclosed area for each cycle is the energy loss from hysteresis. Multiplying this area by the frequency, results in the power loss. As a check, the area of the λ-i loop for a given λmax should equal the average power loss at the given λmax.

The right displacement (i.e. the right curve of hysteresis loop minus the core saturation curve at $B > 0$) is linear and is assumed as Equation (14). The left displacement (i.e. the left curve of hysteresis loop minus the core saturation curve at $B > 0$) is nonlinear and increases slowly for low flux, more rapidly for bigger flux, and decays to zero for maximum flux Bmax [5]. Thus, the left displacement is assumed as Equation (15). At zero flux, both displacements must be equivalent. This is a coercive force (H_C) and is assumed as Equation (13) because of its nonlinearity (see Fig. 5). The coercive force for each loop should be determined to meet the power loss at the Bmax given for each loop in Equation (16). In the case of Fig. 5 from ARMCO M4 [3], approximation using an exponential fit, "*K*" for Equation (13) is about 0.5.

$$\text{Coercive force } H_C = (Bmax/Btop)^K \times Hctop \quad (13)$$

$$\text{Right displacement } R_{HD} = (1-f)*H_C \quad (14)$$

$$\text{Left displacement } L_{HD} = -H_C \cdot (a+1/a)/[(1-f)/a+a/(1-f)] \quad (15)$$

$$\text{Power Loss at each loop} = \int_0^{B_{max}} 2 \cdot (R_{HD} - L_{HD}) dB \quad (16)$$

where

Bmax = Maximum Flux density at each minor loop
Btop = Maximum Flux density for major loop
a = (Btop-Bmax) / Btop and f = B / Bmax
Hctop = Maximum Coercive force for major loop

Using Equations (14) and (15), the displacements for each Bmax are presented in Fig. 6 and the obtained hysteresis loop for $B > 0$ is shown in Fig. 7. The entire DC hysteresis loop is indicated in Fig. 8.

5. ATP Implementation of the Model

Finally, the complete core model implemented in ATP is shown in Fig. 9. "I_sat" represents the anhysteretic saturation curve and is modeled using a Type-60 current source controlled by TACS or a Type-93 element. "I_eddy" and "I_hyster" are modeled using a Type-60 current source controlled by TACS.

"I_hyster" represents the resistive hysteresis current for DC hysteresis loss. The left (or right) displacements of resistive hysteresis current are changed with the right (or left) displacements at the reversing point of flux linkages. The sign of the displacement current is determined by the sign of the flux.

"I_eddy" represents the resistive current for the eddy current loss of core. This current is approximated by dividing a given voltage by a linear resistance using TACS. This implementation is more flexible for future enhancements and avoids unwanted interactions between components, which may occur when a linear resistor is used.

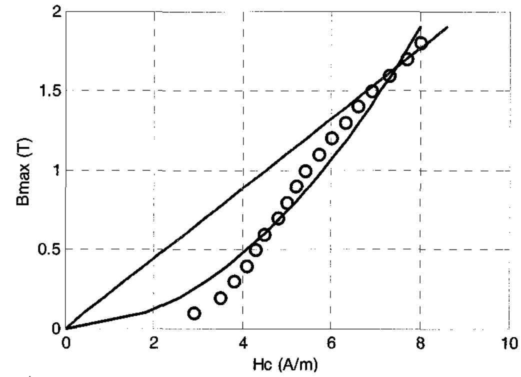


Fig. 5 H_C and Bmax (bold line=square root)

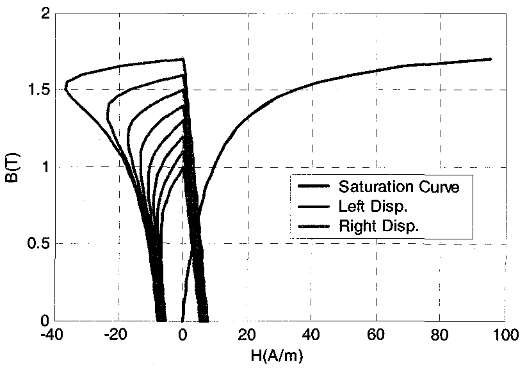


Fig. 6 Left and Right Displacements of Hysteresis Current

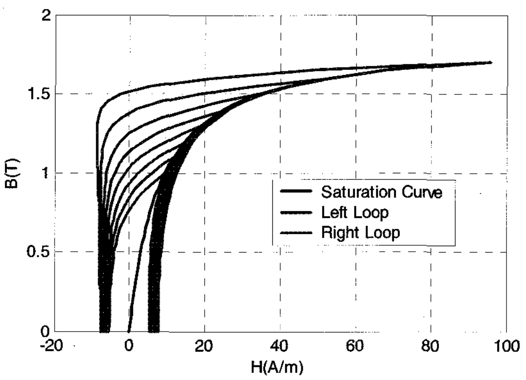


Fig. 7 DC Hysteresis Loop Generated by the Model

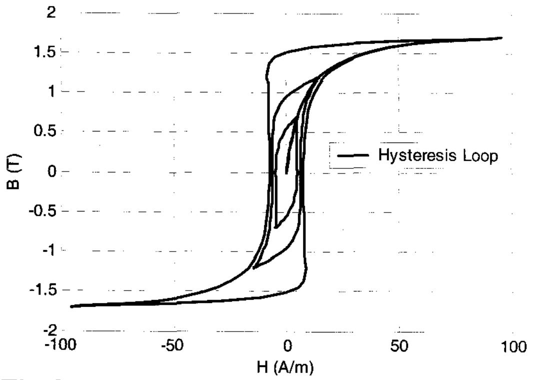


Fig. 8 DC Hysteresis Loop Generated by the Model

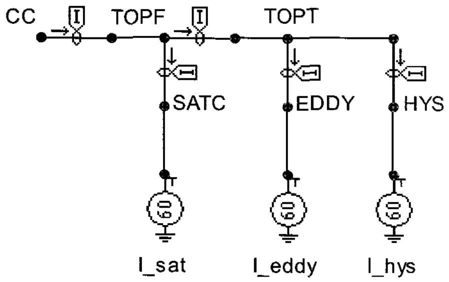
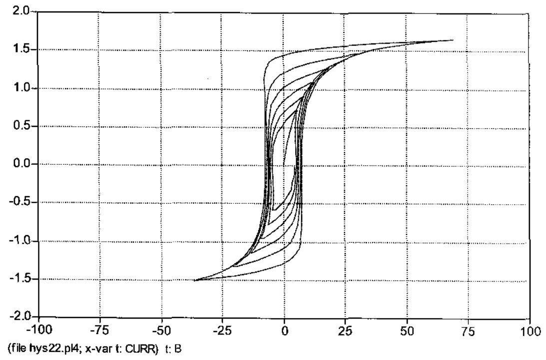


Fig. 9 Core Model for ATP Implementation

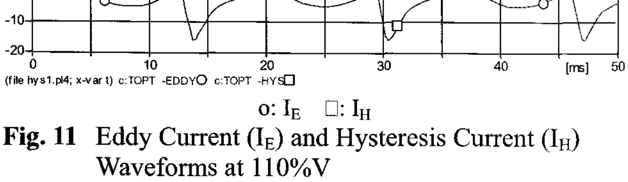
The hysteresis losses and eddy current losses of core are modeled using a Type-60 current source controlled by TACS.

Fig. 10 illustrates hysteresis loops generated by decaying B with time made by the core model implemented in ATP. Fig. 11 shows the current of the eddy current loss and the resistive hysteresis current. Fig. 12 reveals the mag-netizing current waveform.



X-axis: H in A/m Y-axis: B in T

Fig. 10 DC Hysteresis Loop by Decaying B with Time



o: I_E □: I_H

Fig. 11 Eddy Current (I_E) and Hysteresis Current (I_H) Waveforms at 110%V

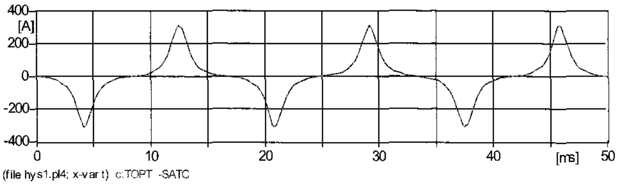


Fig. 12 Magnetizing Current Waveform at 110%V

6. Conclusion

To verify the transformer model developed using TACS, results from simulated open circuit tests were compared to the transformer test report. After the model was implemented and run with ATP, the results of the open-circuit test were close to the test report.

The DC hysteresis loop and eddy current loss of the core could be modeled using a Type-60 current source controlled by TACS in ATP. TACS was effective to incorporate the hysteresis loop model. The assumptions that the right displacement for each hysteresis loop is linear and the left displacement is nonlinear and increases slowly for low flux and more quickly for bigger flux, then decays to zero for maximum flux, were very effective and the obtained curves matched well with actual hysteresis loops. Parameter estimation methods were developed to deter-mine the parameters of a given model in cases where incomeplete information is available. The accuracy of ATP simulation for power systems is thus enhanced with the detailed transformer model.

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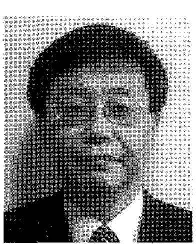

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He received his Ph.D. degree from Michigan Tech University in 2002. His research interests are transient analysis, EMTP and ATP application.