Bull. Korean Math. Soc. **42** (2005), No. 4, pp. 777–787

SECTIONAL CURVATURE OF CONTACT CR-SUBMANIFOLDS OF AN ODD-DIMENSIONAL UNIT SPHERE

HYANG SOOK KIM AND JIN SUK PAK

ABSTRACT. In this paper we study $(n+1)$-dimensional compact contact CR-submanifolds of $(n-1)$ contact CR-dimension immersed in an odd-dimensional unit sphere S^{2m+1}. Especially we provide necessary conditions in order for such a submanifold to be the generalized Clifford surface

$$S^{2n_1+1}(((2n_1+1)/(n+1))^{\frac{1}{2}}) \times S^{2n_2+1}(((2n_2+1)/(n+1))^{\frac{1}{2}})$$

for some portion (n_1, n_2) of $(n-1)/2$ in terms with sectional curvature.

1. Introduction

Let S^{2m+1} be a $(2m+1)$-unit sphere. For any point $z \in S^{2m+1}$ we put $\xi = Jz$, where J denotes the complex structure of the complex $(m+1)$-space $\mathbb{C}^{m+1}$. We consider the orthogonal projection $\pi : T_z\mathbb{C}^{m+1} \rightarrow T_zS^{2m+1}$. Putting $\phi = \pi \circ J$, we can see that the aggregate (ϕ, ξ, η, g) is a Sasakian structure on S^{2m+1}, where η is a 1-form dual to ξ and g the standard metric tensor induced on S^{2m+1}. So S^{2m+1} can be considered as a Sasakian manifold of constant ϕ-holomorphic sectional curvature 1, that is, of constant curvature 1 (cf. [1, 2, 7]).

Let M be an $(n+1)$-dimensional submanifold tangent to the structure vector field ξ of S^{2m+1} and denote by $\mathcal{D}_x$ the ϕ-invariant subspace

Received July 12, 2004.

2000 Mathematics Subject Classification: 53C40, 53C15.

Key words and phrases: Sasakian manifold, odd-dimensional unit sphere, contact CR-submanifold, contact CR-dimension, minimal real hypersurface, sectional curvature.

The first author is supported by the 2003 Inje University Research Grant and the second author is supported by the 2004 Kyungpook National University Research Fund.

$T_xM \cap \phi T_xM$ of the tangent space T_xM of M at x in M. Then ξ cannot be contained in $\mathcal{D}_x$ at any point x in M (cf. [4, 5]). Thus the assumption $\dim \mathcal{D}_x^\perp$ being constant and equal to 2 at any point x in M yields that M can be given with a contact CR-submanifold in the sense of Yano-Kon [7], where $\mathcal{D}_x^\perp$ denotes the complementary orthogonal subspace to $\mathcal{D}_x$ in T_xM. In fact, if there exists a non-zero vector U which is orthogonal to ξ and contained in $\mathcal{D}_x^\perp$, then ϕU must be normal to M.

In this paper we shall study $(n+1)$-dimensional contact CR-submanifolds M of $(n-1)$ contact CR-dimension immersed in S^{2m+1} and, especially, provide necessary conditions in order that M is locally isometric to a Riemannian product of $M_1 \times M_2$ in terms with sectional curvature, where M_1 and M_2 belong to some $(2n_1+1)$ and $(2n_2+1)$-dimensional spheres.

Manifolds, submanifolds, geometric objects and mappings we discuss in this paper will be assumed to be differentiable and of class C^∞.

2. Preliminaries

Let $\overline{M}$ be a $(2m+1)$-dimensional almost contact metric manifold with structure (ϕ, ξ, η, g). Then by definition it follows that

$$\begin{aligned} &\phi^2 X = -X + \eta(X)\xi, \quad \phi\xi = 0, \quad \eta(\phi X) = 0, \quad \eta(\xi) = 1, \\ &g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y), \quad \eta(X) = g(X, \xi) \end{aligned} \tag{2.1}$$

for any vector fields X, Y tangent to $\overline{M}$. We consider a Riemannian manifold isometrically immersed in $\overline{M}$ with induced metric tensor field g.

First of all we note that an n-dimensional submanifold normal to the structure vector field ξ of $\overline{M}$ is anti-invariant with respect to ϕ, that is, $\phi T_xM \subset T_xM^\perp$ for each point x of M, and $m \geq n$ (for details, see [7]).

In the sequel we assume that M is an $(n+1)$-dimensional submanifold tangent to the structure vector field ξ of a $(2m+1)$-dimensional almost contact metric manifold $\overline{M}$. We now denote by $\mathcal{D}_x$ the ϕ-invariant subspace defined by $T_xM \cap \phi T_xM$ and by $\mathcal{D}_x^\perp$ the complementary orthogonal to $\mathcal{D}_x$ in T_xM. Then the structure vector field ξ is contained in $\mathcal{D}_x^\perp$. In fact, if $\xi \in \mathcal{D}_x$, then there is a vector field X tangent to M such that $\xi = \phi X$, from which applying the operator ϕ and using (2.1), we have $X = \eta(X)\xi$. Thus it follows that $\xi = 0$, which is a contradiction. Hence $\xi \in \mathcal{D}_x^\perp$ at each point x of M. Moreover by definition we can easily see that $\phi \mathcal{D}_x^\perp \subset T_xM^\perp$ for each point x of M.

If the ϕ-invariant subspace $\mathcal{D}_x$ has constant dimension for x in M, then M is called a *contact CR-submanifold* ([1, 7]) and the constant is called *contact CR-dimension* of M([4, 5]). Especially, a contact CR-submanifold M of which contact CR-dimension is equal to $\dim M$ is called an *invariant submanifold* (cf. [1, 7]). Real hypersurfaces tangent to the structure vector field ξ are typical examples of contact CR-submanifolds.

3. Fundamental properties of contact CR-submanifolds

Let M be an $(n+1)$-dimensional contact CR-submanifold of $(n-1)$ contact CR-dimension in a $(2m+1)$-dimensional almost contact metric manifold $\overline{M}$. Then by definition $\dim \mathcal{D}_x^{\perp} = 2$ for any x in M, and so there is a unit vector field U contained in $\mathcal{D}^{\perp}$ which is orthogonal to ξ. Since $\phi\mathcal{D}^{\perp} \subset TM^{\perp}$, ϕU is a unit normal vector field to M, which will be denoted by N_1, that is,

$$N_1 = \phi U. \tag{3.1}$$

Moreover, it is clear that $\phi TM \subset TM \oplus \mathrm{Span}\{N_1\}$. Hence we have, for any tangent vector field X and for a local orthonormal basis $\{N_\alpha,\ \alpha = 1, \ldots, p\}$ $(p = 2m-n)$ of normal vectors to M, the following decomposition in tangential and normal components:

$$\phi X = FX + u^1(X)N_1, \tag{3.2}$$

$$\phi N_\alpha = -U_\alpha + PN_\alpha, \quad \alpha = 1, \ldots, p. \tag{3.3}$$

It is easily shown that F and P are skew-symmetric linear endomorphisms acting on T_xM and $T_xM^{\perp}$, respectively. Since the structure vector field ξ is tangent to M, (2.1) implies

$$g(FU_\alpha, X) = -u^1(X)g(N_1, PN_\alpha), \tag{3.4}$$

$$g(U_\alpha, U_\beta) = \delta_{\alpha\beta} - g(PN_\alpha, PN_\beta). \tag{3.5}$$

We also have

$$g(U_\alpha, X) = u^1(X)\delta_{1\alpha} \tag{3.6}$$

and consequently

$$g(U_1, X) = u^1(X), \quad U_\alpha = 0, \quad \alpha = 2, \ldots, p. \tag{3.7}$$

Furthermore from (3.2), it is clear that

$$F\xi = 0, \quad u^1(\xi) = 0, \quad FU = 0, \quad u^1(U) = 1. \tag{3.8}$$

Next, applying ϕ to (3.1) and using (2.1) and (3.3), we have

$$U_1 = U, \quad PN_1 = 0. \tag{3.9}$$

Applying ϕ to (3.2) and using (2.1), (3.2), (3.3), and (3.9), we also have

$$F^2X = -X + \eta(X)\xi + u^1(X)U, \quad u^1(FX) = 0. \tag{3.10}$$

On the other hand, it follows from (3.3), (3.7), and (3.9) that

$$\phi N_1 = -U, \quad \phi N_\alpha = PN_\alpha, \quad \alpha = 2, \ldots, p \tag{3.11}$$

and moreover we may put

$$PN_\alpha = \sum_{\beta=2}^{p} P_{\alpha\beta} N_\beta, \quad \alpha = 2, \ldots, p, \tag{3.12}$$

where $(P_{\alpha\beta})$ is a skew-symmetric matrix which satisfies

$$\sum_{\beta=2}^{p} P_{\alpha\beta} P_{\beta\gamma} = -\delta_{\alpha\gamma}. \tag{3.13}$$

We denote by $\overline{\nabla}$ and ∇ the Levi-Civita connection on $\overline{M}$ and M, respectively and denote by $\nabla^\perp$ the normal connection induced from $\overline{\nabla}$ in the normal bundle $TM^\perp$ of M. The Gauss and Weingarten equations are

$$\overline{\nabla}_X Y = \nabla_X Y + h(X, Y), \tag{3.14}$$

$$\overline{\nabla}_X N_\alpha = -A_\alpha X + \nabla^\perp_X N_\alpha, \quad \alpha = 1, \ldots, p \tag{3.15}$$

for any tangent vector fields X, Y to M. Here h denotes the second fundamental form and A_α is the shape operator corresponding to N_α. They are related by

$$h(X,Y) = \sum_{\alpha=1}^{p} g(A_\alpha X, Y) N_\alpha.$$

Furthermore, we put

$$\nabla^{\perp}_{X} N_\alpha = \sum_{\beta=1}^{p} s_{\alpha\beta}(X) N_\beta, \tag{3.16}$$

where $(s_{\alpha\beta})$ is the skew-symmetric matrix of connection forms of $\nabla^{\perp}$.

Finally the equation of Gauss, Codazzi and Ricci are

$$\begin{aligned} g(\overline{R}(X,Y)Z,W) = g(R(X,Y)Z,W) &\\ + \sum_{\alpha} \{ g(A_\alpha X, Z) g(A_\alpha Y, W) &\\ - g(A_\alpha Y, Z) g(A_\alpha X, W) \}, & \end{aligned} \tag{3.17}$$

$$\begin{aligned} g(\overline{R}(X,Y)Z,N_\alpha) = g((\nabla_X A_\alpha)Y - (\nabla_Y A_\alpha)X, Z) &\\ + \sum_{\beta} \{ g(A_\beta Y, Z) s_{\beta\alpha}(X) &\\ - g(A_\beta X, Z) s_{\beta\alpha}(Y) \}, & \end{aligned} \tag{3.18}$$

$$g(\overline{R}(X,Y)N_\alpha, N_\beta) = g(R^{\perp}(X,Y)N_\alpha, N_\beta) + g([A_\beta, A_\alpha]X, Y) \tag{3.19}$$

for any tangent vector fields X, Y, Z to M, where $\overline{R}$ and R denote the Riemannian curvature tensors of $\overline{M}$ and M, respectively, and $R^{\perp}$ is the curvature tensor of the normal connection $\nabla^{\perp}$ (cf. [2]).

4. The special case of an ambient Sasakian manifold $\overline{M}$

In this section we specialize to the case of an ambient Sasakian manifold $\overline{M}$, that is,

$$\overline{\nabla}_X \xi = \phi X, \tag{4.1}$$

$$(\overline{\nabla}_X \phi) Y = -g(X,Y)\xi + \eta(Y) X. \tag{4.2}$$

Then, by differentiating (3.2) and (3.11) covariantly and by comparing the tangential and normal parts, we have

$$(4.3)\quad (\nabla_Y F)X = -g(Y,X)\xi + \eta(X)Y - g(A_1Y,X)U + u^1(X)A_1Y,$$

$$(4.4)\quad (\nabla_Y u^1)X = g(FA_1Y,X),$$

$$(4.5)\quad \nabla_X U = FA_1X,$$

$$(4.6)\quad g(A_\alpha U,X) = -\sum_{\beta=2}^{p} s_{1\beta}(X)P_{\beta\alpha}, \quad \alpha = 2,\dots,p$$

On the other hand, since ξ is tangent to M, (4.1) gives

$$(4.7)\quad \nabla_X\xi = FX,$$

$$(4.8)\quad g(A_1\xi,X) = u^1(X), \text{ that is, } A_1\xi = U,$$

$$(4.9)\quad A_\alpha\xi = 0, \ \alpha = 2,\dots,p.$$

In what follows we assume that $\overline{M}$ *is a Sasakian manifold of constant curvature* 1 *and that* N_1 *is parallel with respect to the normal connection* $\nabla^\perp$. Hence it follows from (3.16) that

$$(4.10)\quad s_{1\beta} = 0, \quad \beta = 2,\dots,p,$$

which and (4.6) give

$$(4.11)\quad A_\alpha U = 0, \quad \alpha = 2,\dots,p.$$

Next, since the curvature tensor $\overline{R}$ has the form

$$\overline{R}(\overline{X},\overline{Y})\overline{Z} = g(\overline{Y},\overline{Z})\overline{X} - g(\overline{X},\overline{Z})\overline{Y}$$

for $\overline{X}$, $\overline{Y}$, $\overline{Z}$ tangent to $\overline{M}$, the equations (3.17), (3.18), and (3.19) imply

$$(4.12)\quad \begin{aligned} g(R(X,Y)Z,W) = {} & g(Y,Z)g(X,W) - g(X,Z)g(Y,W) \\ & + \sum_\alpha \{g(A_\alpha Y,Z)g(A_\alpha X,W)\} \\ & - g(A_\alpha X,Z)g(A_\alpha Y,W), \end{aligned}$$

$$(4.13)_{(a)}\quad (\nabla_X A_1)Y - (\nabla_Y A_1)X = 0,$$

$$(4.13)_{(b)}\quad \begin{aligned} (\nabla_X A_\alpha)Y - (\nabla_Y A_\alpha)X = {} & \sum_{\beta=2}^{p}\{s_{\beta\alpha}(Y)A_\beta X - s_{\beta\alpha}(X)A_\beta Y\}, \\ & \alpha = 2,\dots,p, \end{aligned}$$

$$(4.14)\quad [A_1, A_\alpha] = 0, \quad \alpha = 2,\dots,p$$

with the help of (4.10) and (4.11).

Finally we introduce some lemmas for later use :

LEMMA 4.1. *Let M be an $(n+1)$-dimensional contact CR-submanifold M of $(n-1)$ contact CR-dimension immersed in a Sasakian manifold of constant curvature 1 and let the distinguished normal vector field N_1 be parallel with respect to the normal connection. Then the commutativity condition*

$$A_1F = FA_1$$

holds on M if and only if

$$\nabla A_1 = 0.$$

Moreover, in this case

$$A_1{}^2 = u^1(A_1U)A_1 + I, \quad A_1U = \xi + u^1(A_1U)U \tag{4.15}$$

and the function $u^1(A_1U)$ is locally constant.

Proof. We first assume that $\nabla A_1 = 0$. Differentiating (4.8) covariantly along M and using (4.5), (4.7), and $\nabla A_1 = 0$, we can easily see that $A_1F = FA_1$ holds on M.

The proofs of the converse and (4.15) have been given in [4, Lemma 5.1, p.433].

LEMMA 4.2. *Let M be as in Lemma 4.1. Then*

$$FA_\alpha + A_\alpha F = 0, \quad trA_\alpha = 0, \quad \alpha = 2, \dots, p. \tag{4.16}$$

Proof. Differentiating (4.9) covariantly and using (4.7), we have

$$(\nabla_X A_\alpha)\xi + A_\alpha FX = 0,$$

or equivalently

$$g((\nabla_X A_\alpha)Y, \xi) + g(A_\alpha FX, Y) = 0 \tag{4.17}$$

for any vector fields X, Y tangent to M. By means of (4.9) and $(4.13)_{(b)}$, it can be easily verified from (4.17) that

$$(A_\alpha F + FA_\alpha)X = 0, \quad \alpha = 2, \dots, p$$

hold on M. Inserting FX back into the above equation and using (3.10), (4.9) and (4.11), we have $A_\alpha X = FA_\alpha FX$, which implies $\mathrm{tr}A_\alpha = 0$.

5. Main theorems

In this section we assume that the ambient manifold is a $(2m+1)$-dimensional unit sphere S^{2m+1}. Suppose that the distinguished normal vector field N_1 is parallel with respect to the normal connection $\nabla^{\perp}$ and that the trace of the shape operator A_1 vanishes, that is,

$$\mathrm{tr} A_1 = 0. \tag{5.1}$$

Then, from $(4.13)_{(a)}$ and (5.1), we have

$$\sum (\nabla_i A_1) e_i = 0, \tag{5.2}$$

where $\{e_i\}_{i=1,\dots,n+1}$ is an orthonormal basis of tangent vectors to M and $\nabla_i := \nabla_{e_i}$. Hence it follows from $(4.13)_{(a)}$ and (5.2) that

$$\sum (\nabla_i \nabla_i A_1) X = \sum (R(e_i, X) A_1) e_i$$

for any vector X tangent to M, and consequently we have

$$g(\nabla^2 A_1, A_1) = \sum_{i,j} g((R(e_i, e_j) A_1) e_i, A_1 e_j). \tag{5.3}$$

Thus we have

THEOREM 5.1. *Let M be an $(n+1)$-dimensional compact contact CR-submanifold of $(n-1)$ contact CR-dimension immersed in S^{2m+1} and let the distinguished normal vector field N_1 be parallel with respect to the normal connection. Suppose that the trace of the shape operator A_1 in direction of N_1 vanishes and that the minimum of sectional curvatures of M is zero. Then M is minimal and $\nabla A_1 = 0$ on M.*

Proof. The minimality of M is easily followed by our assumptions and Lemma 4.2.

Taking account of the Laplacian of $\mathrm{tr} A_1^2$, we have

$$\int_M \|\nabla A_1\|^2 * 1 = -\int_M g(\nabla^2 A_1, A_1) * 1,$$

which together with (5.3) yields

$$0 \le \int_M \|\nabla A_1\|^2 * 1 = -\int_M \sum_{i,j} g((R(e_i, e_j) A_1) e_i, A_1 e_j) * 1. \tag{5.4}$$

Now we choose an orthonormal frame $\{e_j\}$ of M such that

$$A_1 e_j = \lambda_j e_j \quad (j = 1, \ldots, n+1).$$

Then it is clear that

$$\begin{aligned} &\sum_{i,j} g((R(e_i, e_j)A_1)e_i, A_1 e_j) \\ &= \sum_{i,j} \{g((R(e_i, e_j)A_1 e_i, A_1 e_j) - g(A_1 R(e_i, e_j)e_i, A_1 e_j)\} \\ &= \frac{1}{2}\sum_{i,j} (\lambda_i - \lambda_j)^2 K_{ij}, \end{aligned}$$

where K_{ij} denotes the sectional curvature of the plane section spanned by $\{e_i, e_j\}$. Hence, if the minimum of the sectional curvature of M is zero, the above equation and (5.4) imply $\nabla A_1 = 0$.

By means of Theorem 5.1 we can obtain the following theorem under additional condition:

THEOREM 5.2. *Let M be an $(n+1)$-dimensional compact contact CR-submanifold of $(n-1)$ contact CR-dimension in S^{2m+1} and assume that there exists an orthonormal basis $\{N_1, N_\alpha\}_{\alpha=2,\ldots,p}$ of normal vectors to M each of which is parallel with respect to the normal connection. If the trace of the shape operator A_1 in direction of N_1 vanishes and if the minimum of sectional curvatures of M is zero, then there is an $(n+2)$-dimensional totally geodesic unit sphere S^{n+2} of S^{2m+1} such that $M \subset S^{n+2}$.*

Proof. Under our assumptions it follows from Theorem 5.1 that

$$\operatorname{tr} A_\alpha = 0, \quad \alpha = 2, \ldots .p.$$

Moreover, it is clear from $(4.13)_{(b)}$ that, for any vector fields X, Y tangent to M,

$$(\nabla_X A_\alpha)Y - (\nabla_Y A_\alpha)X = 0$$

since $s_{\alpha\beta} = 0,\ 1 \leq \alpha, \beta \leq p$, and consequently

$$\sum (\nabla_i A_\alpha) e_i = 0,$$

where $\{e_i\}_{i=1,\dots,n+1}$ is an orthonormal basis of tangent vectors to M. Taking account of the Laplacian of $\mathrm{tr}A_\alpha^2$ and using the quite similar method as shown in the proof of Theorem 5.1, we can easily see that

$$\nabla_X A_\alpha = 0, \quad \alpha = 2, \dots, p \tag{5.5}$$

for any vector field X tangent to M.

Differentiating (4.9) covariantly and using (4.7) and (5.5), we have

$$A_\alpha FX = 0$$

for any vector fields X, Y tangent to M. Inserting FX instead of X in this equation and using (3.10), (4.9), and (4.11), we have

$$A_\alpha = 0, \quad \alpha = 2, \dots, p.$$

Hence the first normal space of M is contained in Span$\{N_1\}$, which is invariant under parallel translation with respect to the normal connection from our assumption. Thus we may apply Erbacher's reduction theorem [3], which gives the proof of our theorem.

Combining Theorem 5.2 and a theorem provided in [4, Theorem 6.2, p.436], we have

THEOREM 5.3. *Let M be an $(n+1)$-dimensional compact contact CR-submanifold of $(n-1)$ contact CR-dimension in S^{2m+1} and assume that there exists an orthonormal basis $\{N_1, N_\alpha\}_{\alpha=2,\dots,p}$ of normal vectors to M each of which is parallel with respect to the normal connection. If the trace of the shape operator A_1 in direction of N_1 vanishes and if the minimum of sectional curvatures of M is zero, then M is isometric to a generalized Clifford surface:*

$$S^{2n_1+1}(((2n_1+1)/(n+1))^{\frac{1}{2}}) \times S^{2n_2+1}(((2n_2+1)/(n+1))^{\frac{1}{2}})$$

for some portion (n_1, n_2) of $(n-1)/2$.

Proof. By means of Theorem 5.2, M can be regarded as a real minimal hypersurface of S^{n+2} which is a totally geodesic invariant submanifold of S^{2m+1}. Moreover, it is clear from (4.15) that M has exactly two constant eigenvalues ρ_1, ρ_2 with

$$\rho_1 = (\lambda + \sqrt{\lambda^2+4})/2, \quad \rho_2 = (\lambda - \sqrt{\lambda^2+4})/2,$$

where $\lambda = g(A_1U, U)$. In fact, since ${\rho_k}^2 - \lambda\rho_k - 1 = 0$ $(k = 1, 2)$, (4.8) and (4.15) imply

$$A_1(\rho_1 U + \xi) = \rho_1(\rho_1 U + \xi), \quad A_1(\rho_2 U + \xi) = \rho_2(\rho_2 U + \xi).$$

Therefore, since $\nabla A_1 = 0$ and $A_1F = FA_1$, we can easily see that M is isometric to a Riemannian product of odd-dimensional spheres, that is,

$$S^{2n_1+1}(r_1) \times S^{2n_2+1}(r_2)$$

for some portion (n_1, n_2) of $(n-1)/2$ and some r_1, r_2 with $r_1^2 + r_2^2 = 1$ (for details, see [4]). In fact, since M is minimal, $r_1 = ((2n_1+1)/(n+1))^{\frac{1}{2}}, r_2 = ((2n_2+1)/(n+1))^{\frac{1}{2}}$. Taking account of (4.15), we can verify that the minimum of sectional curvatures of those hypersurfaces is zero.

COROLLARY 5.4. *Let M be a compact, minimal real hypersurface tangent to the structure vector fields ξ of an odd-dimensional unit sphere S^{n+2} . If the minimum of sectional curvatures of M is zero, then M is isometric to*

$$S^{2n_1+1}(((2n_1+1)/(n+1))^{\frac{1}{2}}) \times S^{2n_2+1}(((2n_2+1)/(n+1))^{\frac{1}{2}})$$

for some portion (n_1, n_2) of $(n-1)/2$.

References

[1] A. Bejancu, *Geometry of CR-submanifolds*, D. Reidel Publishing Company, Dordrecht, Boston, Lancaster, Tokyo, 1986.

[2] B. Y. Chen, *Geometry of submanifolds*, Marcel Dekker Inc., New York, 1973.

[3] J. Erbacher, *Reduction of the codimension of an isometric immersion*, J. Differential Geom. **5** (1971), 333-340.

[4] J.-H. Kwon and J. S. Pak, *On some contact CR-submanifolds of an odd-dimensional unit sphere*, Soochow J. Math. **26** (2000), 427-439.

[5] J. S. Pak, J.-H. Kwon, H. S. Kim, and Y.-M. Kim, *Contact CR-submanifolds of an odd-dimensional unit sphere*, accepted in Geom. Dedicata.

[6] P. J. Ryan, *Homogeneity and some curvature conditions for hypersurfaces*, Tôhoku Math. J. **21** (1969), 363-388.

[7] K. Yano and M. Kon, *CR submanifolds of Kaehlerian and Sasakian manifolds*, Birkhäuser, Boston, Basel, Stuttgart, 1983.

HYANG SOOK KIM, DEPARTMENT OF COMPUTATIONAL MATHEMATICS, SCHOOL OF COMPUTER AIDED SCIENCE, INJE UNIVERSITY, KIMHAE 627-749, KOREA
E-mail: mathkim@inje.ac.kr

JIN SUK PAK, DEPARTMENT OF MATHEMATICS, KYUNGPOOK NATIONAL UNIVERSITY, DAEGU 702-701, KOREA
E-mail: jspak@mail.knu.ac.kr