Optimization of Channel Capacity in MIMO Systems

Van-Su Pham, Minh Tuan Le, and Giwan Yoon, *Member, KIMICS*

Abstract—In this paper, a new method to get the optimum channel capacity of a Multiple-Input Multiple-Output (MIMO) system is presented. The proposed method exploits the diagonal structure of channel matrix to maximize the channel capacity. The diagonal format of the channel matrix is formed by multiplying the transmitted signal with the pre-compensated channel (PCC) matrix. Numerical simulations show that the proposed method exploiting the diagonal structure of channel matrix could significantly increase the system capacity compared with the system without applying the diagonal structure of channel matrix.

Index Terms—MIMO system, channel capacity, wireless communications

I. INTRODUCTION

Recently, the next-generation wireless systems are required to have high quality of service and to support the multimedia communication as well. In other words, the systems are supposed to have better quality, larger coverage, and be more powerful and very bandwidth-efficient. A lot of technologies have been proposed to meet the demand of more advanced systems in both quality of service and capacity efficiency. And recent information theory results have demonstrated an enormous capacity potential of the wireless communications in which multiple antennas at both transmitter and receiver are used [1] (so called Multiple-Input Multiple-Output system - MIMO). In particular, the MIMO systems, using the multiple antennas at both sides, can efficiently exploit not only the time and frequency domain but also spatial domain [2]. Thus, the data rate is literally multiplied, at no cost in bandwidth or power, by exploiting a simultaneous transmission. Consequently, several antennas can send independent signals over the same frequency and time slots and the systems promise a linear increase of the system capacity according to the increase of number of transmitter/receiver (TX/RX) antennas. Several MIMO systems have been examined in [2]-[6]. In [5], the MIMO system exploits the singular value decomposition to increase the system performance. The MIMO system capacity has been considered with the aid of Space-Time Code in [6] where a relatively complex signal processing, known as BLAST algorithm, is implemented to increase the system capacity and performance as well.

In this paper, we exploit the diagonal structure of the channel matrix to get the optimal high channel capacity. The main point of the proposed approach is to transform the channel matrix into diagonal one by multiplying the transmitted signal with a weight matrix. The remainder of the paper is organized as follows. The System model and signal formula are described in section II. In section III, the proposed pre-compensated channel matrix is presented. The numerical simulations for verifying the proposed approach are illustrated in section IV. Finally, section V concludes this paper.

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II. SYSTEM MODEL

Let us consider a MIMO wireless system as depicted in Fig.1, in which the number of transmitter (TX) antennas is N and that of receiver (RX) antennas is M. In general, the signals of the TX antennas are transmitted over M different single-input multiple-output channels, *i.e.* the signals from one of N TX antennas are transmitted over M different paths to reach to the RX antennas. Let us denote the vector $\underline{h}_n(t,\tau)$ as the communication channel vector at the time t from the n-th TX antenna to every RX antenna, so we can rewrite it as follows:

$$\underline{h}_n(t,\tau) = [h_{n,0}(t,\tau) \quad h_{n,1}(t,\tau) \quad \cdots \quad h_{n,M-1}(t,\tau)]^T \qquad (1)$$

In the equation (1), $h_{n,i}(t,\tau), i = 0,1,...,M-1$ is channel gain between the n-th TX antenna and the i-th RX antenna, and τ is delay time of transmission over that channel.

The received signal vector is achieved by a superposition of signals transmitted over N TX antennas as the following equation:

$$x(t) = \sum_{n=0}^{N} \sum_{k=-\infty}^{\infty} s_n(k)\underline{h}_n(t, t-kT_s) + i(t) + n(t) \qquad (2)$$

where $i(t)$ represents the interference and $n(t)$ is the noise vector, T_s is the symbol duration, $s_n(k)$ denotes the transmitted symbol from the n-th TX antenna.

After sampling the received signal, the discrete time representation of (2) is given:

$$x(nT_s) = \sum_{n=0}^{N} \sum_{l=-\infty}^{\infty} \hat{s}_n(lT_s)\underline{h}_n(nT_s,(n-l)T_s) + i(nT_s) + n(nT_s) \quad (3)$$

The equation (3) can be re-expressed in matrix formulation as follows:

$$\underline{x}(n) = \underline{\underline{H}}\underline{s}(n) + \underline{i}(n) + \underline{n}(n) \quad (4)$$

Herein, $\underline{\underline{H}}(n)$ represents the channel matrix given as:

$$\underline{\underline{H}}(n) = [\underline{h}_1(n) \quad \underline{h}_2(n) \quad \cdots \quad \underline{h}_N(n)]^T \quad (5)$$

where the elements of matrix in the equation (5) defines the channel vector through which the signal is transmitted from the n-th TX antenna to all RX antennas as given in (1).

The general expression for MIMO channel capacity on the system of N TX antennas and M RX antennas with AWGN is given [1], [3]:

$$C = \log_2[\det(I_M + \frac{1}{\sigma_n^2}\underline{\underline{H}}\underline{\underline{Q}}\underline{\underline{H}}^*)] \text{ (bits/s/Hz)} \quad (6)$$

where $\underline{\underline{Q}}$ is the autocorrelation matrix of received signal, I_M is the unity matrix, $\underline{\underline{H}}$ is channel matrix given in (5), and σ_n^2 is the noise power. If the transmitted power has uniform distribution, the capacity can be rewritten as [3]:

$$C' = \log_2[\det(I_M + \frac{\rho}{N}\underline{\underline{H}}\underline{\underline{H}}^*)] \text{ (bits/s/Hz)} \quad (7)$$

In the above equation, (*) denotes the complex conjugate transpose and ρ is the signal to noise ratio (SNR).

III. PROPOSED METHOD

Although the MIMO channel capacity is much higher than that of Single-Input and Single-Output (SISO) channel, it is dramatically decreased by the fading effects.

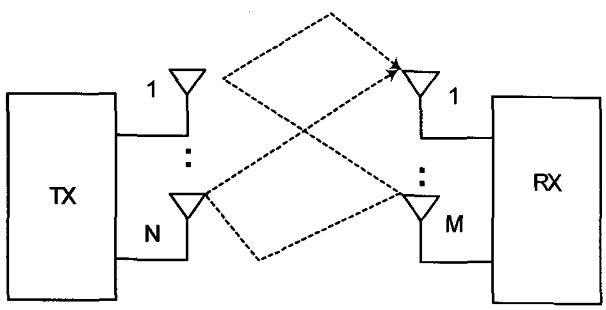

Fig.1 Wireless MIMO systematic scheme

From the model of MIMO system as illustrated in Fig.1, if the channel matrix is a diagonal one, *i.e.* each transmit antenna has its own link to the corresponding receive one, the channel capacity will be likely increased in proportion with the number of antennas. As a result, the system capacity can be several-fold of the SISO. Therefore, if we can find out a matrix (so called pre-compensated channel (PCC) matrix) whose combination with the channel matrix is a diagonal one, we can improve the channel capacity.

We define the PCC matrix is $\underline{\underline{M}}$, and then the condition is needed to satisfy our approach given as:

$$\underline{\underline{H}} * \underline{\underline{M}} = diag(\lambda_1 \quad \lambda_2 \quad \cdots \quad \lambda_T) \quad (8)$$

where λ_i (i=1,2...T) is a certain real number, and T is the rank of channel matrix H. In our proposed method, λ_i's are chosen to be unit.

Then MIMO system can be formulated as follows:

$$\underline{x}(n) = \underline{\underline{I}}_T(n)\underline{s}(n) + \underline{n}(n) \quad (9)$$

where $\underline{\underline{I}}_T(n)$ is unity matrix with dimension of minimum of the number of TX/RX antennas

In our proposed approach, the solution of (8) is found out by normal arithmetic algorithm. It is assumed that the channel matrix $\underline{\underline{H}}$ is perfectly known at the transmitter. In practice, we can divide the communication process into the following two sessions: (i) the training session and (ii) transmitting session. The training session is to be implemented in a short time before or periodically during the communication process. In this duration, the channel matrix is estimated to solve the equation (8) to get PCC matrix. After a short time of training, the real transmitting data is to be implemented. In this duration, the transmitting data is first multiplied to the PCC, and then transmitted over channel.

In brief, the proposed method can be summarized as the following steps:

1. Training duration:
   - The training sequence is transmitted over channel to estimate the channel matrix
   - The arithmetic algorithm is applied to find out PCC matrix
2. Transmitting duration:
   - The transmit signal is multiplied with PCC first
   - Then, the pre-compensated signals are transmitted over the channel

IV. NUMERICAL SIMULATIONS

In the simulation, we apply the proposed method for the MIMO wireless system shown in the Section II. The modulation scheme is assumed to be binary, *i.e.* BPSK. The system capacity with and without applying PCC is presented in **Fig. 2** In this figure, the different formats of channel matrix are formed by using PCC. It can be seen that the system without using PCC (random channel, *i.e.* all elements of channel matrix are randomly generated) has the smallest capacity, whereas the system with perfectly compensated PCC (perfect diagonal channel matrix) has the largest one. If the PCC is used to combine with channel matrix to form a matrix in which all the diagonal elements are units where the other elements are Gaussian-distributed, the system capacity is also increased slightly as compared to the case without using PCC.

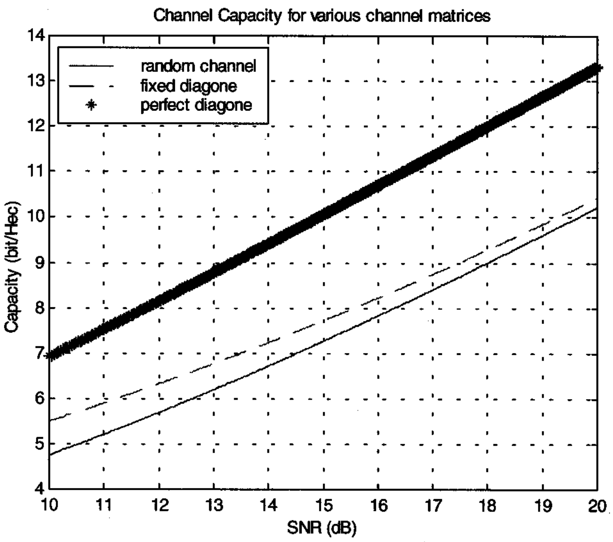

Fig. 2 MIMO Channel capacity comparison between the cases with/without using PCC matrix. The number of TX and RX antennas are 2 and 2 respectively

In **Fig. 3**, we consider the system capacity as a function of SNR when the number of TX/RX antennas varies. As expected, for the MIMO system with more TX and RX antenna elements, the system capacity increases dramatically. For instance, when the number of the TX antennas and the number of RX antennas are 3 and 3, the system capacity is roughly twice as large as the case when the corresponding parameters are 2 and 2. However, we also can see that the reasonable drawback of the proposed approach is that the system capacity increases just slightly if the number of TX (or RX) increases while that of RX (or TX) stays the same.

The performance of the proposed approach is also investigated with the sufferings of the multi-path fading environment considered. The multi-path fading model is shown in **Fig. 4** In this simulation, the different values of Ricean factors (K) are taken into consideration. As expected, from the **Fig. 5**, in Ricean multi-path fading channel, the more dominant the direct signal (Light of Sight) is, the lower BER values the system gains. For example, when the signal over interference is –3dB (K1) the system performance is nearly the same with the case 3 dB (K2), while when that is 10 dB (K3) the system performance increases moderately. If the case K is negative infinitive, *i.e.* the Rayleigh multi-path fading channel is considered, the system performance is degraded severely.

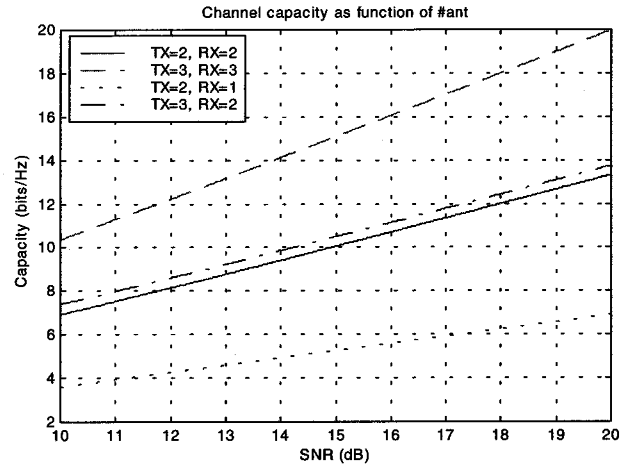

Fig. 3 MIMO channel capacity with using PCC matrix for a variety of numbers of antennas

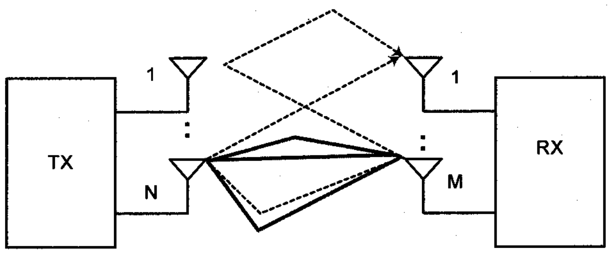

Fig.4 Multi-path scheme

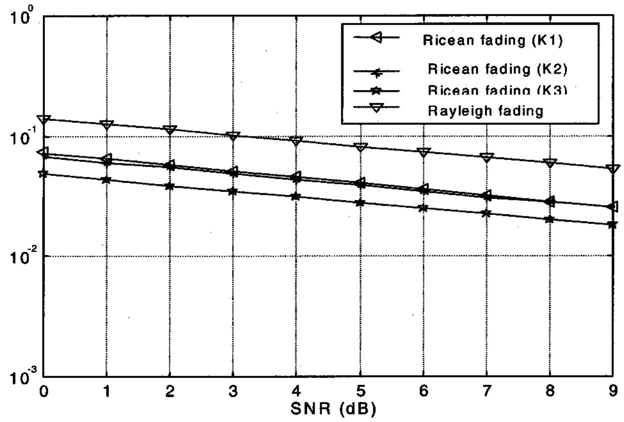

Fig. 5 System performance in terms of Bit Error Rate (BER) vs. SNR with regards to multi-path fading channel

V. CONCLUSION

The diagonal structure of channel matrix is exploited to increase the channel capacity. By using the pre-compensated channel matrix, we can form independent parallel channels over MIMO system, thus several antennas can transmit simultaneously independent signals and the system can gain the optimal high capacity. A drawback of the proposed method is the complexity associated with the solving process of the equation (8). However, it is expected that more powerful processor developed recently may be able to eliminate this issue easily. Therefore, the proposed method seems highly feasible and promising for the future real-time wireless applications.

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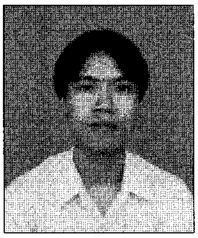

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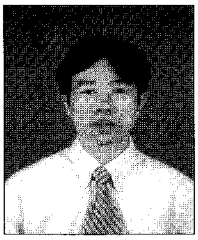

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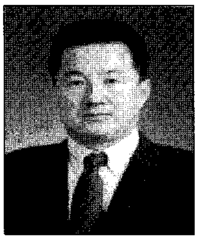

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