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Phase Error Analysis in Polarization Phase-shifting Technique using a Wollaston Prsim and Wave Plates

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The method to obtain four speckle patterns with relative phase shift of $\pi/2$ by passive devices such as two waveplates and a linear polarizer, and to calculate the phase at each point of the speckle pattern in shearography with a Wollaston prism is described. In this paper, we analyze its potential error sources caused by wave plates.

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I. INTRODUCTION

Although shearography has many different applications as a measurement technique, the various methods all rely ultimately on the measurement of phase. In shearography, this phase measurement is accomplished by means of fringes. Different methods can be applied for determining of an interferogram numerically and automatically. These methods can be divided generally into categories: those which take the phase data sequentially and those which take the phase data simultaneously [1]. Methods of the first type are known as time-dependent phase-shifting technique, and those of the second type are known as spatial phase-shifting technique. There are many variations of the time-dependent phase-shifting technique, but for all of them, a temporal phase modulation (or relative phase shift between the object and reference beams in an interferometer) is introduced to perform the measurement. By measuring the interferogram intensity as the phase is shifted, the phase of the wavefront can be determined with the aid of electronics or a digital computer. The spatial phase-shifting techniques can be subdivided into phase stepped methods [2] and spatial carrier methods [3].

A phase shift or modulation in the phase-shifting techniques can be induced by moving a mirror, tilting a glass plate, moving a grating, rotating a half-wave plate or analyzer [4-7].

Shearography with a Wollaston prism has simple structure and is robust to large disturbance from the environment [8,9]. But the shearography has a drawback that the application of the phase-shifting technique is difficult. Recently, to solve the problem, the phase-shifting technique applied to the shearographic system with a Wollaston prism has been reported. The technique uses passive devices such as two wave plates and a linear polarizer to obtain four speckle patterns with relative phase shift of $\pi/2$ by passive devices such as two wave plates and a linear polarizer, and to calculate the phase at each point of the speckle pattern in shearography with a Wollaston prism.[10]

As mentioned above, phase shifting interferometry can be carried out using a variety of arrangements if we measure the intensity in the interference plane for different orientations of the polarization components. However, the errors in the retardation of the phase plates and their azimuth angle errors will influence the accuracy of the phase measurement. The influence of the errors of the retardation and azimuth angle of the polarization components of a polarization phase shifter was reported, on the phase measurement, in the phase shifting interferometry.[6,7] In this paper, the principle of the shearographic system with Wollaston prism[10] is described, and its potential error sources are analyzed.

II. PRINCIPLE OF SHAROGRAPHY WITH A WOLLASTON PRISM

Fig. 1 shows the proposed system, which consists of two wave plates and one linear polarizer. The proposed system obtains the shearing image by a Wollaston prism instead of by the Michelson interferometer. Fig. 1 shows the operation of a Wollaston prism, which has a net effect of splitting a ray polarized at 45 degree into two rays that are out of phase, separated from each other, and orthogonally polarized to each other.

The application of a Wollaston prism yields a pair of laterally sheared images of the investigated object that is observed in the image plane; i.e., the point P_1 on the object's surface is separated into two points P_1' and P_1'' in the image plane, and point P_2 on the objects's surface is divided into two points P_2' and P_2'' in the image plane as well. The rays from two points in an adjusted distance on the object's surface meet in one point P_1' and P_2'' on the image plane. Light waves belonging to points P_1 and P_2 interfere in this position on the image plane. Using the interference of each point in the image plane yields a speckle interferogram. δx is the shearing distance in the object's plane; $\delta x'$ is the shearing distance in the image plane. Since the polarizations of the optical waves that are reflected from points P_1 and P_2, and then pass through the Wollaston prism are orthogonal, two rays orthogonally polarized to each other do not interfere in the image plane. For fringes to appear, a polarizer, whose axis is at 45 degree to the polarization axes of the Wollaston prism, must be placed at the image plane. Two wave plates are inserted in the proposed system to generate a phase shift. In this figure, WP1 and WP2 are wave plates, the slow axis of WP1 coincides with the -x axis and the slow axis of WP2 is rotated by ±45 degree with respect to the -x axis.

The optical waves reflected from the points P_1 and P_2 can be described by the complex exponential functions

$$U(x,y) = \begin{pmatrix} U_1 \\ U_2 \end{pmatrix} = \begin{pmatrix} a_1 e^{-i\theta(x,y)} \\ a_2 e^{-i\theta(x+\delta x,y)} \end{pmatrix}, \tag{1}$$

where $\theta(x,y)$ and $\theta(x+\delta x,y)$ represent the random phase relation of the light from points $P_1(x,y)$ and $P_2(x+\delta x,y)$, respectively, and a_1 and a_2 are the light amplitudes, which are assumed equal for the two neighboring points.

After passing through two wave plates, optical waves reflected from points P_1 and P2 of Fig. 1 are given by

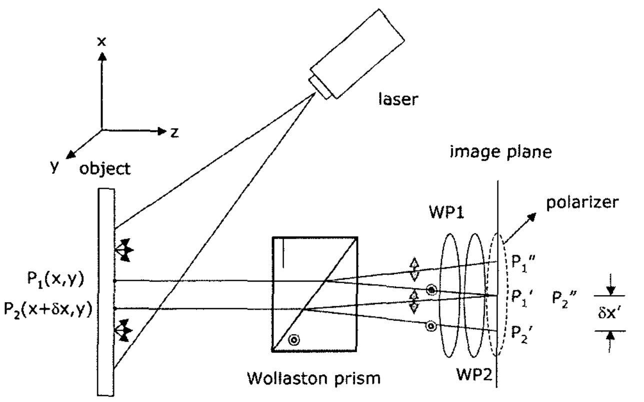

FIG. 1. Shearographic system with a Wollaston prism and two wave plates, and a linear polarizer.

$$\begin{aligned} U &= \begin{pmatrix} \cos\frac{\Gamma_2}{2} & \mp i\sin\frac{\Gamma_2}{2} \\ \mp i\sin\frac{\Gamma_2}{2} & \cos\frac{\Gamma_2}{2} \end{pmatrix} \begin{pmatrix} e^{-i\frac{\Gamma_2}{2}} & 0 \\ 0 & e^{i\frac{\Gamma_2}{2}} \end{pmatrix} \begin{pmatrix} U_2 \\ U_1 \end{pmatrix}, \\ &= e^{-i\frac{\Gamma_2}{2}} \begin{pmatrix} \cos\frac{\Gamma_2}{2} U_2 & \mp i e^{i\Gamma_1} \sin\frac{\Gamma_2}{2} U_1 \\ \mp i\sin\frac{\Gamma}{2} U_2 & + e^{i\Gamma_1}\cos\frac{\Gamma_2}{2} U_1 \end{pmatrix} \end{aligned} \tag{2}$$

where the minus and plus signs correspond to the cases in which the slow axis of WP2 is rotated by 45 degrees and -45 degrees, respectively, with respect to the -x axis, and Γ_1 and Γ_2 are phase retardations of WP1 and WP2, respectively. After passing through a linear polarizer, the complex amplitude of optical waves in the image plane is given by

$$U_{wp2\pm45} = e^{-i\frac{\Gamma_1}{2}} \left\{ \cos\frac{\Gamma_2}{2} U_2 \mp i e^{i\Gamma_1} \sin\frac{\Gamma_2}{2} U_1 \right\}. \tag{3}$$

First, in the case of using two $\lambda/4$ plates, since the phase retardation is $\Gamma_1 = \Gamma_2 = \pi/2$, Eq. (3) results in

$$U_{wp2\pm45} = \frac{\sqrt{2}}{2} e^{-i\frac{\pi}{2}} \{U_2 \pm U_1\}. \tag{4}$$

The intensities corresponding to Eq. (4) are given by

$$I_1 = U_{wp2+45} U_{wp2+45}{}^* = I_0[1+\gamma\cos\phi], \tag{5}$$
$$I_3 = U_{wp2-45} U_{wp2-45}{}^* = I_0[1+\gamma\cos(\phi+\pi)], \tag{6}$$

where $I_0 = (a_1^2 + a_2^2)/2$ is the mean value of the intensity (background brightness I_0), $\gamma = 2a_1a_2/(a_1^2+a_2^2)$ is the modulation of the interference term, and $\phi = \theta_1 - \theta_2$ is the random phase difference.

Second, when the slow axes of WP1 and WP2 are rotated at 0 and 45 degrees (or -45 degrees) with respect to the -x axis, respectively, the complex amplitude of optical waves after passing through a linear polarizer (which is rotated at 45 or -45 degrees) are given by, respectively,

$$U_{p+45} = \frac{1}{2} e^{-i\frac{\Gamma_1}{2}} (\cos\frac{\Gamma_2}{2} - i\sin\frac{\Gamma_2}{2}) \begin{pmatrix} U_2 + e^{i\Gamma_1} U_1 \\ U_2 + e^{i\Gamma_1} U_1 \end{pmatrix}, \tag{7}$$

$$U_{p-45} = \frac{1}{2} e^{-i\frac{\Gamma_1}{2}} (\cos\frac{\Gamma_2}{2} - i\sin\frac{\Gamma_2}{2}) \begin{pmatrix} U_2 - e^{i\Gamma_1} U_1 \\ -U_2 + e^{i\Gamma_1} U_1 \end{pmatrix}. \tag{8}$$

In which case in which $\Gamma_1 = \Gamma_2 = \pi/2$, the intensities corresponding to Eqs. (7) and (8) are given by

$$I_4 = U_{p+45} U_{p+45}{}^* = I_0[1+\gamma\cos(\phi+3\pi/2)], \tag{9}$$
$$I_2 = U_{p-45} U_{p-45}{}^* = I_0[1+\gamma\cos(\phi+\pi/2)]. \tag{10}$$

From Eqs. (5), (6), (9), and (10), we can obtain four intensity patterns with relative phase shift of $\pi/2$ using two wave plates and a linear polarizer. Using combination of $\lambda/2$ plate and $\lambda/4$ plate, we can also obtain four intensity patterns with relative phase shift of $\pi/2$. In the case in which $\Gamma_1 = \pi$, $\Gamma_2 = \pi/2$, the intensity corresponding to Eq. (3) results in

$$U_{\varphi 2 \pm 45} = -\frac{\sqrt{2}}{2} i \{ U_2 \pm i U_1 \}. \qquad (11)$$

The intensities corresponding to Eq. (11) are given by

$$I_4 = U_{\varphi 2+45} U_{\varphi 2+45}{}^{*} = I_0 [1 + \gamma \cos(\phi + 3\pi/2)], \qquad (12)$$
$$I_2 = U_{\varphi 2-45} U_{\varphi 2-45}{}^{*} = I_0 [1 + \gamma \cos(\phi + \pi/2)]. \qquad (13)$$

Second, when the slow axes of WP1 and WP2 are rotated at 0 and 45 degrees (or -45 degrees) with respect to -x axis, respectively, the complex amplitudes of optical waves after passing through a linear polarizer (which is rotated at 45 or -45 degrees) are given by Eqs. (7) and (8), respectively. In which case in which $\Gamma_1 = \pi$, $\Gamma_2 = \pi/2$, the intensities corresponding to Eqs. (7) and (8) are given by

$$I_3 = U_{p+45} U_{p+45}{}^{*} = I_0 [1 + \gamma \cos(\phi + \pi)], \qquad (14)$$
$$I_1 = U_{p-45} U_{p-45}{}^{*} = I_0 [1 + \gamma \cos\phi]. \qquad (15)$$

From mentioned above, we see that we can obtain four speckle patterns with relative phase shift of $\pi/2$ by combination of two $\lambda/4$ plates and a linear polarizer, or $\lambda/2$ plate and $\lambda/4$ plate, and a linear polarizer. From four intensity patterns, we can calculate the phase at each point of the speckle interferogram, as follows:

$$\phi = arctan \frac{(I_4 - I_2)}{(I_1 - I_3)}. \qquad (16)$$

III. PHASE ERROR ANALYSIS

The main potential sources of error in the technique are imperfections of the polarization elements and azimuth angle error of the polarization elements. We shall analyze the effects of these potential error sources one by one.

1. Imperfections of the polarization elements

We shall deal mainly with the errors that are introduced by imperfections in the $\lambda/2$ plate and $\lambda/4$ plate. In this case, we assume that the azimuth angle error of wave plates is zero. In Fig. 1, the Jones matrix of output beam in the output plane [11] is given by

$$E_{out} = A(\varphi_3)\, WP2(\varphi_2)\, WP1(\varphi_1)\, E_{in}, \qquad (17)$$

where E_{in} represents input optical wave, and $A(\phi_3)$, $WP2(\varphi_2)$, $WP1(\varphi_1)$ represent Jones matrices of a polarizer, WP2, and WP1, respectively. The Jones matrices of polarization components are given by, respectively,

$$E_{in} = \begin{pmatrix} U_1 \\ U_2 \end{pmatrix} = \begin{pmatrix} a_1 e^{-j\theta_1} \\ a_2 e^{-j\theta_2} \end{pmatrix}, \qquad (18)$$

$$A(\varphi_3) = \begin{pmatrix} \cos^2\varphi_3 & 1/2 \sin 2\varphi_3 \\ 1/2 \sin 2\varphi_3 & \sin^2\varphi_3 \end{pmatrix}, \qquad (19)$$

$$WPi(\varphi_i) = \begin{pmatrix} 2j\sin^2\varphi_i \sin\frac{\Gamma_i}{2} + e^{-i\frac{\Gamma_i}{2}} & j\sin 2\varphi_i \sin\frac{\Gamma_i}{2} \\ j\sin 2\varphi_i \sin\frac{\Gamma_i}{2} & -2j\sin^2\varphi_i \sin\frac{\Gamma_i}{2} + e^{i\frac{\Gamma_i}{2}} \end{pmatrix}. \qquad (20)$$

where i=1,2 and WPi represents the Jones matrix for WP1 and WP2. φ_3, φ_2, φ_1 represent the azimuth angle of a linear polarizer, WP2, and WP1, respectively.

1.1 The case of two $\lambda/4$ plates

The phase error introduced by imperfections in the $\lambda/4$ plate can be obtained from four intensity patterns as follows.

(1) Intensity for $\varphi_3 = 0$, $\varphi_2 = \pi/4$, $\varphi_1 = 0$

$$I_1 = a_1^2 \cos^2\frac{\Gamma_2}{2} + a_2^2 \sin^2\frac{\Gamma_2}{2} + 2a_1 a_2 \cos\frac{\Gamma_2}{2} \sin\frac{\Gamma_2}{2} \sin(\phi + \Gamma_1). \qquad (21)$$

(2) Intensity for $\varphi_3 = \pi/4$, $\varphi_2 = \pi/4$, $\varphi_1 = 0$

$$I_2 = 1/2[(a_1^2 + a_2^2) + 2a_1 a_2 \cos(\phi + \Gamma_1)]. \qquad (22)$$

(3) Intensity for $\varphi_3 = 0$, $\varphi_2 = -\pi/4$, $\varphi_1 = 0$

$$I_3 = a_1^2 \cos^2\frac{\Gamma_2}{2} + a_2^2 \sin^2\frac{\Gamma_2}{2} - 2a_1 a_2 \cos\frac{\Gamma_2}{2} \sin\frac{\Gamma_2}{2} \sin(\phi + \Gamma_1). \qquad (23)$$

(4) Intensity for $\varphi_3 = \pi/4$, $\varphi_2 = \pi/4$, $\varphi_1 = 0$

$$I_4 = 1/2[(a_1^2 + a_2^2) - 2a_1 a_2 \cos(\phi + \Gamma_1)]. \qquad (24)$$

From Eq. (16) and Eqs. (21)~(24), the phase difference ϕ' of optical waves U_1 and U_2 is given by

$$\tan\phi' = \frac{I_4 - I_2}{I_1 - I_3} = \frac{-\cos(\phi + \Gamma_1)}{\sin\Gamma_2 \sin(\phi + \Gamma_1)}, \qquad (25)$$

where ϕ' includes the error introduced by wave plates. For a nonideal $\lambda/4$ plates we have

$$\Gamma_1 = \pi/2 + \gamma_1, \qquad (26)$$
$$\Gamma_2 = \pi/2 + \gamma_2, \qquad (27)$$

where γ_1 and γ_2 are the errors in the relative retardation introduced by two $\lambda/4$ plates. Substituting Eqs. (26) and (27) into Eq. (25), we obtain

$$\tan\phi' = \frac{\tan\phi + \gamma_1 \sec^2\phi}{1 - 1/2\gamma_2^2}. \quad (28)$$

Since

$$\tan\phi' = \tan(\phi + \Delta\phi) \approx \tan\phi + \Delta \sec^2\phi, \quad (29)$$

we can calculate the error from Eqs. (28) and (29) as follows:

$$\Delta\phi = \gamma_1 + 1/4 \sin(2\phi)\gamma_2^2. \quad (30)$$

The first-order error in Eq. (30) is constant and will vanish in determining the phase differences. Hence, the error in measurement is of second order.

1.2 The case of $\lambda/2$ plate and $\lambda/4$ plate

The phase error introduced by imperfections in the $\lambda/2$ plate and $\lambda/4$ plate can be obtained from four intensity patterns as follows.

(1) Intensity for $\varphi_3 = 0$, $\varphi_2 = \pi/4$, $\varphi_1 = 0$

$$I_2 = a_1^2 \cos^2\frac{\Gamma_2}{2} + a_2^2 \sin^2\frac{\Gamma_2}{2} + 2a_1 a_2 \cos\frac{\Gamma_2}{2} \sin\frac{\Gamma_2}{2} \sin(\phi + \delta_1), \quad (31)$$

where δ_1 and Γ_2 are phase retardations of WP1 and WP2, respectively.

(2) Intensity for $\varphi_3 = \pi/4$, $\varphi_2 = \pi/4$, $\varphi_1 = 0$

$$I_3 = 1/2[(a_1^2 + a_2^2) + 2a_1 a_2 \cos(\phi + \delta_1)]. \quad (32)$$

(3) Intensity for $\varphi_3 = 0$, $\varphi_2 = -\pi/4$, $\varphi_1 = 0$

$$I_2 = a_1^2 \cos^2\frac{\Gamma_2}{2} + a_2^2 \sin^2\frac{\Gamma_2}{2} - 2a_1 a_2 \cos\frac{\Gamma_2}{2} \sin\frac{\Gamma_2}{2} \sin(\phi + \delta_1). \quad (33)$$

(4) Intensity for $\varphi_3 = \pi/4$, $\varphi_2 = \pi/4$, $\varphi_1 = 0$

$$I_3 = 1/2[(a_1^2 + a_2^2) - 2a_1 a_2 \cos(\phi + \delta_1)]. \quad (34)$$

From Eqs. (16), (29) and Eqs. (31)~(34), the phase error is given by

$$\Delta\phi = \gamma - 1/4 \sin(2\phi)\gamma_2^2, \quad (35)$$

where γ and γ_2 are the errors in the relative retardation introduced by $\lambda/2$ plate and $\lambda/4$ plate, respectively. The first-order error in Eq. (35) is constant and will vanish in determining the phase differences. Hence, the error in measurement is of second order.

2. Azimuth angle Error

We shall deal with the errors that are introduced by the azimuth angle error in the $\lambda/2$plate and $\lambda/4$ plate. In this case, we assume that wave plates are ideal.

2.1 The case of two $\lambda/4$ plates

2.1.1 Phase error by the azimuth angle error of WP1

In discussing the azimuth angle error we assume that the azimuth angle errors of all polarization elements are zero except WP1.

(1) Intensity for $\varphi_2 = \pi/4$, $\varphi_3 = 0$

$$I_1 = \frac{1}{2} a_2^2 (1 - \sin 2\varphi_1) + \frac{1}{2} a_1^2 (1 + \sin 2\varphi_1) + a_1 a_2 \cos\phi \cos 2\varphi_1. \quad (36)$$

(2) Intensity for $\varphi_2 = \pi/4$, $\varphi_3 = -\pi/4$

$$I_2 = \frac{1}{2}\{(a_2^2 + a_1^2) - a_1 a_2 \cos\phi + a_1 a_2 \cos\phi (\cos^2 2\varphi_1 - \sin^2 2\varphi_1) - (a_2^2 - a_1^2) \sin 2\varphi_1 \cos 2\varphi_1 - 2a_1 a_2 \sin\phi \cos 2\varphi_1\}. \quad (37)$$

(3) Intensity for $\varphi_2 = -\pi/4$, $\varphi_3 = 0$

$$I_2 = \frac{1}{2} a_2^2 (1 + \sin 2\phi_1) + \frac{1}{2} a_1^2 (1 - \sin 2\phi_1) - a_1 a_2 \cos\phi \cos 2\phi_1. \quad (38)$$

(4) Intensity for $\varphi_2 = \pi/4$, $\varphi_3 = \pi/4$

$$I_4 = \frac{1}{2}\{(a_2^2 + a_1^2) + a_1 a_2 \cos\phi + a_1 a_2 \cos\phi (\sin^2 2\varphi_1 - \cos^2 2\varphi_1) + (a_2^2 - a_1^2) \sin 2\varphi_1 \cos 2\varphi_1 + 2a_1 a_2 \sin\phi \cos 2\varphi_1\}. \quad (39)$$

From Eq. (16) and Eqs. (36)~(39), the phase difference ϕ' of optical waves U_1 and U_2 is given by

$$\tan\phi' = \frac{\frac{1}{2}\cos\phi - \frac{1}{2}\cos\phi\cos 4\varphi_1 - \frac{1}{2}\cot 2\beta \sin 4\varphi_1 + \sin\phi\cos 2\varphi_1}{\cot 2\beta \sin 2\varphi_1 + \cos\phi \cos 2\varphi_1}, \quad (40)$$

where $\cot 2\beta = (a_1^2 - a_2^2)/2a_1 a_2$. In the case of two $\lambda/4$ plates, we assume that azimuth of WP1 is $\varphi_1 = 0 + \epsilon_1$, and ϵ_1 represents the azimuth angle error in WP1. Substituting $\varphi_1 = 0 + \epsilon_1$ into Eq. (40), we obtain the phase error as

$$\Delta\phi = -2\cot 2\beta(\cos\phi + \sin\phi)\epsilon_1. \quad (41)$$

2.1.2 Phase error by the azimuth angle error of WP2

In discussing the azimuth angle error we assume that the azimuth angle errors of all polarization elements are zero except WP2. In an analogous treatment to that used earlier, we obtain the phase error as

$$\Delta\phi = (2\cot 2\beta\cos\phi + \cos^2\phi + 1)\epsilon_2 - \sin^2\phi\epsilon_2', \quad (42)$$

where ϵ_2 and ϵ_2' represent the azimuth angle errors in WP1 for $\varphi_2 = \pi/4 + \epsilon_2$ and $\varphi_2' = -\pi/4 + \epsilon_2'$.

2.1.3 Total phase error by the azimuth angle error of two $\lambda/4$ plates

Total phase error including WP1 and WP2 is written by

$$\Delta\phi = -2\cot 2\beta(\cos\phi + \sin\phi)\epsilon_1 + (2\cot 2\beta\cos\phi + \cos^2\phi + 1)\epsilon_2 - \sin^2\phi\epsilon_2'. \quad (43)$$

If $a_1 = a_2$, then $\cot 2\beta = 0$. In this case

$$\Delta\phi = (\cos^2\phi + 1)\epsilon_2 - \sin^2\phi\epsilon_2'. \quad (44)$$

From Eq. (44), $\Delta\phi$ is indenpent of ϵ_1, we can conclude that, in this case, the azimuth angle error in $\lambda/4$ plate (WP1) has no effect on the measurement of phase (up to the first order).

2.2 The case of $\lambda/2$ plate (WP1) and $\lambda/4$ plate (WP2)

2.2.1 Phase error by the azimuth angle error of WP1

In discussing the azimuth angle error we assume that the azimuth angle errors of all polarization elements are zero except WP1.

(1) Intensity for $\varphi_2 = \pi/4$, $\varphi_3 = 0$

$$I_4 = \frac{1}{2}a_1^2 + \frac{1}{2}a_2^2 + a_1 a_2 \sin\phi. \quad (45)$$

(2) Intensity for $\varphi_2 = \pi/4$, $\varphi_3 = -\pi/4$

$$I_1 = \frac{1}{2}\{(a_1^2 + a_2^2) - (a_2^2 - a_1^2)\sin 4\phi_1 - 2a_1 a_2 \cos\phi\cos 4\phi_1\}. \quad (46)$$

(3) Intensity for $\varphi_2 = -\pi/4$, $\varphi_3 = 0$

$$I_2 = \frac{1}{2}a_1^2 + \frac{1}{2}a_2^2 - a_1 a_2 \sin\phi. \quad (47)$$

(4) Intensity for $\varphi_2 = \pi/4$, $\varphi_3 = \pi/4$

$$I_3 = \frac{1}{2}\{(a_1^2 + a_2^2) + (a_2^2 - a_1^2)sin4\varphi_1 - 2a_1 a_2 cos\phi cos4\varphi_1\}. \quad (48)$$

From Eq. (16) and Eqs. (45)~(48), the phase difference ϕ' of optical waves U_1 and U_2 is given by

$$\tan\phi' = \frac{sin\phi}{cot2\beta sin4\varphi_1 + cos\phi cos4\varphi_1}. \quad (49)$$

In the case of $\lambda/2$ plate and $\lambda/4$ plate, we assume that azimuth of WP1 is $\varphi_1 = 0 + \epsilon_1$, and ϵ_1 represents the azimuth angle error in WP1. Substituting $\varphi_1 = 0 + \epsilon_1$ into Eq. (49), we obtain the phase error as

$$\Delta\phi = -4\cot 2\beta\sin\phi\epsilon_1. \quad (50)$$

2.2.2 Phase error by the azimuth angle error of WP2

Phase error by the azimuth of WP2 is identical with that derived in the section 2.1.2.

2.2.3 Total phase error by the azimuth angle error of $\lambda/2$ plate and $\lambda/4$ plate

Total phase error including WP1 and WP2 is written by

$$\Delta\phi = -4\cot 2\beta\sin\phi\epsilon_1 + (2\cot 2\beta\cos\phi + \cos^2\phi + 1)\epsilon_2 - \sin^2\phi\epsilon_2'. \quad (51)$$

If $a_1 = a_2$, then $\cot 2\phi = 0$. In this case

$$\Delta\phi = (\cos^2\phi + 1)\epsilon_2 - \sin^2\phi\epsilon_2'. \quad (52)$$

From Eq. (52), $\Delta\phi$ is indenpent of ϵ_1, we can conclude that, in this case, the azimuth angle error in $\lambda/2$ plate (WP1) has no effect on the measurement of phase (up to the first order).

IV. Conclusion

In this paper, we described the principle and the theory of the method to obtain four speckle interferograms with relative phase shift of $\lambda/2$ by passive devices such as two wave plates and a linear polarizer.

We also analyzed the potential errors of the system. The retardation errors of the wave plates do not influence the measured value of phase differences up to first order. Under the condition of equality of the amplitudes of the interfering beams, which are easily achieved, the azimuth angle error of WP1 does not influence the measured value of phase differences up to first order.

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REFERENCES

[1] D. W. Robinson and G. T. Reid, *Interferogram analysis-digital fringe pattern measurement techniques*, Institute of Physics Publishing 1993, Chaps. 4 and 5.

[2] O. Y. Kwon and D. M. Shough, "Multichannel grating phase-shift interferometry," *Proc SPIE*, vol. 599, pp. 273-279, 1985.

[3] T. Kreis, "Digital holographic interference-phase measurement using the Fourier-transform method," *J. Opt. Soc. Am A*, vol. 3, pp. 847-855, 1986.

[4] W. H. Stevenson, "Optical frequency shifting by means of a rotating diffraction grating," *Appl. Opt*, vol. 9, pp. 649-652, 1970.

[5] R. N. Shagam and J. C. Wyant, "Optical frequency shifter for heterodyne interferometers using multiple rotating polarization retards," *Appl. Opt*, 17, 3034-3035, 1978.

[6] M. P. Kothiyal and C. Delisle, "Polarization component phase shifters in phase shifting interferometry: error analysis," *Optica Acta* vol. 33, pp. 787-793, 1986.

[7] H. Z. Hu, "Polarization heterodyne interferometry using a simple roattaing analyzer. 1: theory and error analysis," *Appl. Opt.* vol. 22, pp. 2052-2056, 1983.

[8] Y. Y. Hung, "Shearography fo non-destructive evaluation of composite structures," *Optics and Lasers in Engineering* vol. 24, pp. 161-182, 1996.

[9] S. Dilhaire, S. Jorez, A. Cornet, L.D. Patino Lopez, W. Claeys, "Measurement of the thermomechnical strain of electronic devices by shearography," *Microelectronics Reliability* vol. 40, pp. 1509-1514, 2000.

[10] S.-G. Kim "Polarization phase-shifting technique in shearographic system with a Wollaston prism," *J. Opt. Soc. Korea*, vol. 8, pp. 122-126, 2004.

[11] A. Yariv and P. Yeh, *Optical Waves in Crystals* (Wiley, New York, 1984), Chap. 5.