

A study on Generalized Synchronization in the State-Controlled Cellular Neural Network(SC-CNN)

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Abstract

In this paper, we introduce a generalized synchronization method and secure communication in the State-Controlled Cellular Neural Network (SC-CNN). We make a SC-CNN using the n-double scroll. A SC-CNN is created by applying identical n-double scroll or non-identical n-double scroll and Chua's oscillator with weak coupled method to each cell. SC-CNN synchronization was achieved using GS(Generalized Synchronization) method between the transmitter and receiver about each state variable in the SC-CNN. In order to secure communication, we have synthesizing the desired information with a SC-CNN circuit by adding the information signal to the hyper-chaos signal using the SC-CNN in the transmitter. And then, transmitting the synthesized signal to the ideal channel, we confirm secure communication by separating the information signal and the SC-CNN signal in the receiver.

1. Introduction

Recently, there has been interest in studying the behavior of chaotic dynamics. Chaotic systems are characterized by sensitive dependence on initial conditions, making long term prediction impossible, self-similarity, and a continuous broad-band power spectrum, etc. Chaotic systems have a variety of applications, including chaos synchronization and chaos secure communication [1-6].

Chaos synchronization and secure communication has been a topic of intense research in the past decade. However, secure communication or cryptographic using chaos has several problems [7]. First, almost all chaos-based secure communication or cryptographic algorithms use dynamical systems defined on the set of real number, and therefore are difficult for practical realization and circuit implementation. Second, security and performance of almost all proposed chaos-based methods are not analyzed in terms of the techniques developed in cryptography. Moreover, most of the proposed methods generate cryptographically weak and slow algorithms.

To address these problems, we need a SC-CNN circuit to increase the complexity in secure communication or cryptographic communication. In this paper, we introduce a SC-CNN synchronization method called GS (Generalized synchronization). We make a SC-CNN using the n-double scroll [8] and Chua's oscillator.

In order to make a SC-CNN, we used identical n-double scroll or non-identical n-double scroll and Chua's oscillator-with weak coupled method to each cell. Then we accomplished a SC-CNN synchronization using GS method between the transmitter and receiver about each state in the SC-CNN. Then we accomplished a Sc-CNN synchronization using GS (Generalized synchronization) method between the transmitter and receiver. We accomplish secure communication by synthe-

sizing the desired information with a SC-CNN circuit by embedding the information signal to the hyper-chaos signal, using only one state variable of the SC-CNN in the transmitter. After transmitting the synthesized signal to the ideal channel, we confirmed the actuality of secure communication by separating the information signal and the SC-CNN signal in the receiver [10, 11].

2. SC-CNN model[12,13]

In [12, 13], the follow generalized cell was introduced:

$$\dot{x}_j = x_j + a_j y_j + G_o + G_s + i_j \tag{1}$$

where j is the cell index, x_j the state variable, y_j the cell output given as

$$y_j = 0.5(|x_j + 1| - |x_j - 1|) \tag{2}$$

where, a_j a constant parameter and i_j a threshold value. In equation (1), G_o is linear combination of the outputs and G_s is state variable of the connected cells.

Generalizing the output nonlinearity (2), the following new output PWL equation is considered

$$y_j = \frac{1}{2} \sum_{k=1}^{2n-1} n_k (|x + b_k| - |x - b_k|) \tag{3}$$

where b_k are the break point and the coefficients n_k are related to the slopes of segments.

SC-CNN cells required to generate the n-double scroll in accordance with the state equation (1) and output equation (3) are given by

$$\dot{x}_1 = -x_1 + a_1 y_1 + a_{12} y_2 + a_{13} y_3 + \sum_{k=1}^3 s_{1k} x_k + i_1$$

$$\dot{x}_2 = -x_2 + a_{21}y_1 + a_{22}y_2 + a_{23}y_3 + \sum_{k=1}^3 s_{2k}x_k + i_2 \quad (4)$$

$$\dot{x}_3 = -x_3 + a_{31}y_1 + a_{32}y_2 + a_{33}y_3 + \sum_{k=1}^3 s_{3k}x_k + i_3$$

3. The Synchronization of SC-CNN using generalized Synchronization

In order to apply to generalized synchronization theory in the hyper-chaos, we compromised to state equation of dimensionless type of SC-CNN is written as follows:

The state equation of transmitter

$$\begin{aligned} \dot{x} &= Ax + g(x), \\ g(x) &= [g(x_1), 0, 0, g(x_4), 0, 0]^T \\ \dot{x}' &= A'x' + g'(x') + F(x, x') \end{aligned} \quad (5)$$

The state equation of receiver

$$\begin{aligned} \dot{y} &= Ay + g(y), \\ g(y) &= [g(y_1), 0, 0, g(y_7), 0, 0]^T \\ \dot{y}' &= A'y' + g'(y') + F(y, y') \end{aligned} \quad (6)$$

where, $x = [x_1, \dots, x_6]^T$, $y = [y_1, \dots, y_6]^T$ are state variable of 2-double scroll circuit, and $x = [x'_1, \dots, x'_6]^T$, $y = [y'_1, \dots, y'_6]^T$ are Chua's oscillator, $g(x)$ represented as equation (4) is nonlinear element. The Matrix A and A' have the following structures:

$$A = \begin{bmatrix} -\alpha & \alpha & 0 & 0 & 0 & 0 \\ 1 & -1-K & 1 & 0 & K & 0 \\ 0 & -\beta & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\alpha & \alpha & 0 \\ 0 & K & 0 & 1 & -1-K & 1 \\ 0 & 0 & 0 & 0 & -\beta & 0 \end{bmatrix} \quad (7)$$

$$A' = \begin{bmatrix} -\alpha' & \alpha' & 0 & 0 & 0 & 0 \\ 1 & -1-K' & 1 & 0 & K' & 0 \\ 0 & -\beta' & -\gamma' & 0 & 0 & 0 \\ 0 & 0 & 0 & -\alpha' & \alpha' & 0 \\ 0 & K' & 0 & 1 & -1-K' & 1 \\ 0 & 0 & 0 & 0 & -\beta' & -\gamma' \end{bmatrix} \quad (8)$$

The function vector $F(x, x')$ is used to assure the GS [14] between the 2-double scroll and Chua's oscillator by means of a proper linear feedback action. This action is characterized by the difference between selected state variables of 2-double scroll and Chua's oscillator and by a feedback gain M .

$$\begin{aligned} F(x, x') &= [M(x_1 + x_4 - x'_1), 0, 0, \\ &M(x_1 + x_4 - x'_1 - x'_4), 0, 0]^T \end{aligned} \quad (9)$$

$$\begin{aligned} F(y, y') &= [M(y_1 + y_4 - y'_1), 0, 0, \\ &M(y_1 + y_4 - y'_1 - y'_4), 0, 0]^T \end{aligned} \quad (10)$$

where in order to solve M , we consider equation (5) and (6), have different initial conditions. Following the auxiliary system approach, the time evolution of the differences

$$d_i = x'_i - x''_i, i = 1, 2, \dots, 6 \quad (11)$$

are described by equations.

$$\begin{aligned} \dot{d}_1 &= \alpha'(d_2 - d_1) - \alpha'[g(x'_1) - g(x''_1)] - Md_1 \\ \dot{d}_2 &= d_1 - d_2 + d_3 + K'(d_5 - d_2) \\ \dot{d}_3 &= -\beta d_2 - \gamma d_3 \\ \dot{d}_4 &= \alpha'(d_5 - d_4) - \alpha'[g(x'_4) - g(x''_4)] - Md_1 - Md_4 \\ \dot{d}_5 &= d_4 - d_5 + d_6 + K'(d_2 - d_5) \\ \dot{d}_6 &= -\beta' d_5 - \gamma' d_6 \end{aligned} \quad (12)$$

The asymptotical stability of the response system (7) occurs if the dynamical system (12) possess a stable fixed point at the origin $d=0$, where $d = [d_1, d_2, d_3, d_4, d_5, d_6]^T$

After choosing the following positive definite Lyapunov function as [15]

$$V(d) = d_1^2 + \alpha' d_2^2 + \frac{\alpha'}{\beta'} d_3^2 + d_4^2 + \alpha' d_5^2 + \frac{\alpha'}{\beta'} d_6^2 \quad (13)$$

Thus the derivative of $V(d)$ along the system trajectory can be expressed as:

$$\begin{aligned} \dot{V}(d) &\leq (-\alpha' - M + \alpha' \max\{|a|, |b|\}) d_1^2 \\ &+ 2\alpha' d_1 d_2 - \alpha' d_2^2 - \gamma' \frac{\alpha'}{\beta'} d_3^2 \\ &+ (-\alpha' - M + \alpha' \max\{|a|, |b|\}) d_4^2 \\ &+ 2\alpha' d_4 d_5 - \alpha' d_5^2 - \gamma' \frac{\alpha'}{\beta'} d_6^2 - Md_1 d_4 \\ &- \alpha' K' d_2^2 - 2\alpha' K' d_2 d_5 + \alpha' K' d_5^2 \end{aligned} \quad (14)$$

By considering the worst case, equation (14) can be rewritten as a quadratic form [16]

$$\dot{V}(d) = d^T \Psi d \quad (15)$$

where $\Psi \in \mathbb{R}^{6 \times 6}$ is a symmetric matrix given by:

$$\Psi = \begin{bmatrix} M-p & -\alpha' & 0 & 0 & 0 & 0 \\ -\alpha' & \alpha'(k+1) & 0 & 0 & -\alpha'K' & 0 \\ 0 & 0 & -\gamma' \frac{\alpha'}{\beta'} & 0 & 0 & 0 \\ M & 0 & 0 & M-p & -\alpha' & 0 \\ 0 & -\alpha'K' & 0 & -\alpha' & \alpha'(K'+1) & 0 \\ 0 & 0 & 0 & 0 & 0 & -\gamma' \frac{\alpha'}{\beta'} \end{bmatrix} \quad (16)$$

with $p = -\alpha' + \alpha' \max\{|a|, |b|\}$.

Equation (16) can be rewritten as:

$$\dot{V}(d) = -d_1^T \Psi d_1 - \gamma' \frac{\alpha'}{\beta'} d_3^2 - \gamma' \frac{\alpha'}{\beta'} d_6^2 \quad (17)$$

Where $d_r = [d_1, d_2, d_4, d_5]^T$ is a reduced difference vector and $\Psi^1 \in \mathbb{R}^{4 \times 4}$ is the symmetric submatrix obtained from Ψ by deleting the third and the sixth rows and the third and sixth columns, respectively.

By imposing that the matrix Ψ^1 be positive definite, i.e. that all principal minors be strictly positive [17], the parameter M can be derived. Finally, it can be concluded that $\dot{V}(d)$ is strictly negative for every $d_i (i = 1, 2, \dots, 6)$ when

$$M > \frac{\alpha'}{K'+1} + \alpha' \max\{|a|, |b|\} \quad (18)$$

The block diagram of the proposed SC-CNN synchronization system is shown in Fig. 1 and the result of SC-CNN synchronization is shown in Fig. 2 respectively.

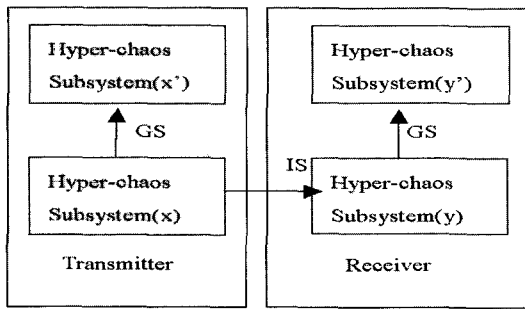
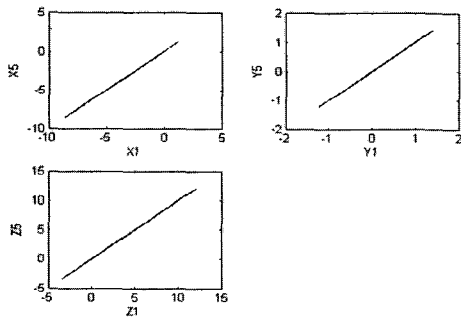
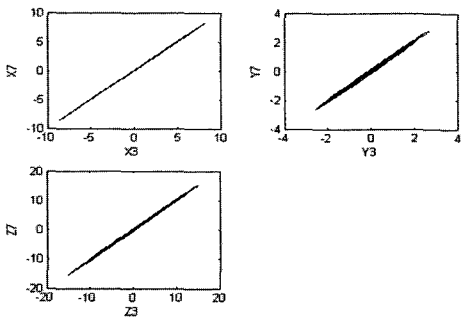


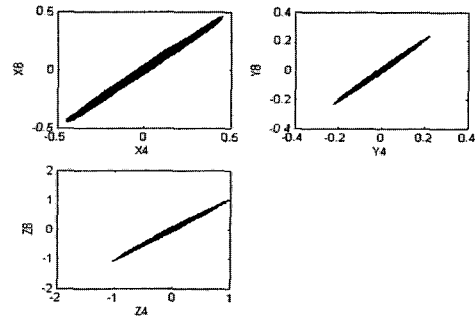
Fig. 1 The Block diagram of Sc-CNN synchronization.



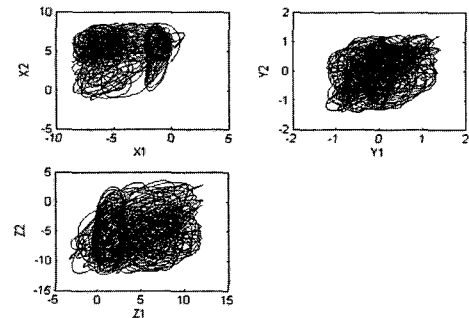
(a) Master subsystem $(x(x_1, x_2, x_3))$ of transmitter vs. master subsystem $(y(y_1, y_2, y_3))$ of receiver



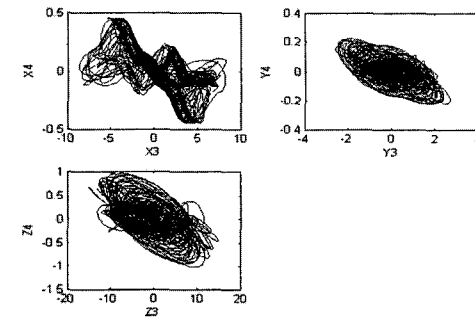
(b) Slave subsystem $(x'(x'_1, x'_2, x'_3))$ of transmitter vs. slave subsystem $(y'(y'_1, y'_2, y'_3))$ of receiver



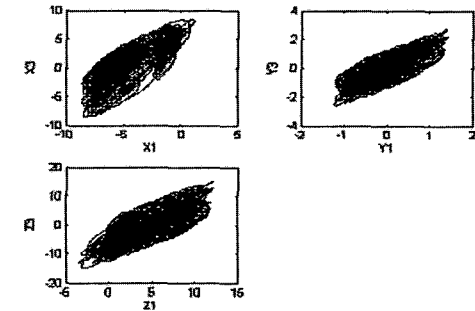
(c) Slave subsystem $(x'(x'_4, x'_5, x'_6))$ of transmitter vs. slave subsystem of $(y'(y'_4, y'_5, y'_6))$ receiver



(d) Phase plane of master subsystems (x_1, x_2, x_3) vs. (y_1, y_2, y_3)



(e) Phase plane of slave subsystem of transmitter (x'_1, x'_2, x'_3) vs. (y'_1, y'_2, y'_3)



(f) Phase plane of master subsystem (x_1, x_2, x_3) vs. slave subsystem (y'_1, y'_2, y'_3)

Fig. 2 The synchronization result

In Fig. 2, we confirmed that effective synchronization result between the transmitter and receiver in the SC-CNN circuit.

3. The Secure Communication of SC-CNN using embedding Method

The method we used to accomplish the secure communication was to synthesize the desired information with the SC-CNN circuit by adding sinusoidal signal as an information signal to the SC-CNN. After transmitting the synthesized signal to the ideal channel, we confirmed secure communication by separating the information signal and the hyper-chaos signal in the receiver [10,11].

In order to achieve the secure communication, we propose that method using only one state variable embedding instead of use to all state variable driven-synchronization method in the transmitter [11]. To information signal embedding, we chosen x_1 and x_3 term as a state variable in the transmitter state equation with SC-CNN and written as follows:

The state equation of transmitter

$$\begin{aligned} \dot{x} &= Ax + g(w) \\ g(w) &= [g(x_1 + 0.1 \sin(2\pi f)) \ 0 \ 0 \ g(x_4) \ 0 \ 0] \quad (19) \\ \dot{x}' &= Ax' + g(x') + F(x, x') \end{aligned}$$

The state equation of receiver

$$\begin{aligned} \dot{y} &= Ay + g(y) \\ g(y) &= [g(y_1) \ 0 \ 0 \ g(y_7) \ 0 \ 0] \quad (20) \\ \dot{y}' &= Ay' + g(y') + F(y, y') \end{aligned}$$

Proposed secure communication diagram of hyper-chaos is shown in Fig. 3.

In Fig. 4, we use sinusoidal signal as an information signal and shown Fig.5, and add it to state variables x_1 and x_3 in the SC-CNN.

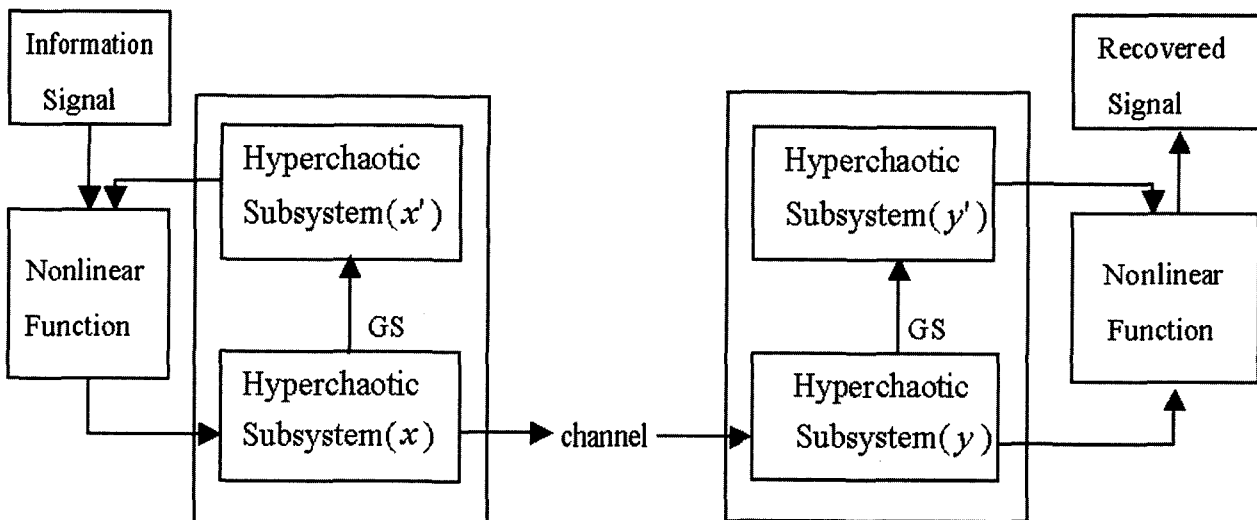


Fig. 3 Block diagram of SC-CNN secure communication

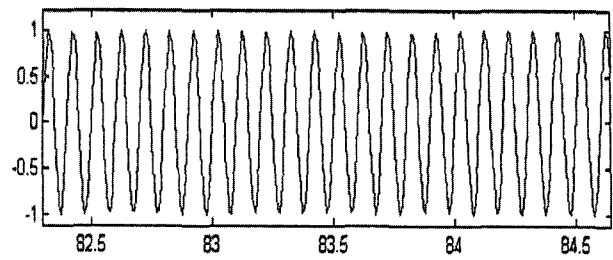


Fig. 4 Information signal

Fig. 5 and 6 are shown that the result of adding the information signal to state variable x_1 and x_3 .

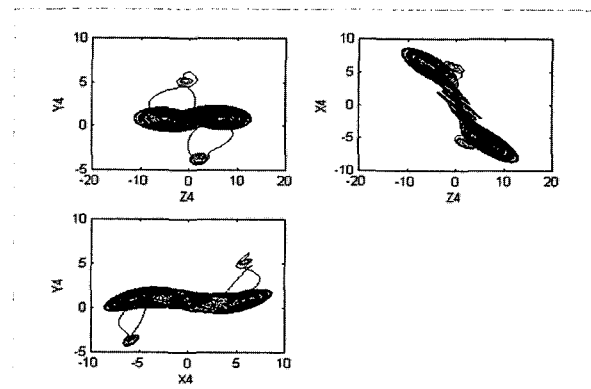


Fig. 5 The result of adding the information signal to state variable x_1

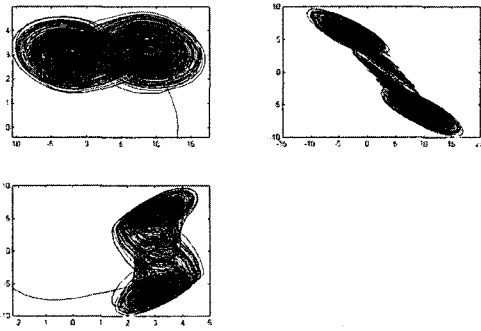


Fig. 6 The result of adding the information signal to state variable

After synchronizing the transmitter and receiver in a hyper-chaos circuit through the ideal channel, we separate the information signal and the hyper-chaos signal in the demodulation part. Recover signals in the demodulation part are shown in Fig. 7 and 8, respectively.

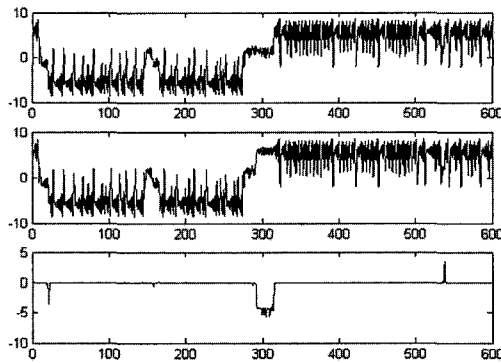


Fig. 7 The result of recovery information signal of state variable x_1

In Fig. 7, the first part shows state x_1 with information signal embedding, the second part shows the result in the receiver, and the third part shows the recover signal.

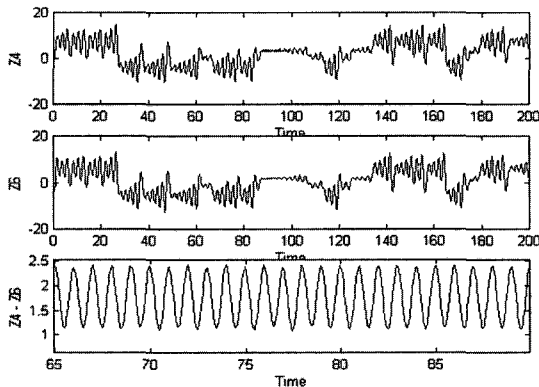


Fig. 8 The result of recovery information signal of state variable x_3

In Fig. 8, the first part shows state x_3 with information signal embedding, the second part shows the result in the receiver, and the third part shows the recover signal

We show that the superiority of the recovery signal for state x_3 to state x_1 . This is significant because we can not use the current component i_L in Chua's circuit or Chua's oscillator, which is replaced by x_3 in the hyper-chaos circuit using the SC-CNN. It is clear that state variable x_3 is superior to state x_1 or x_2 as a carrier signal in the SC-CNN. In order to increase secure communication complexity, we can choose better transmitter signal which is x_3 when it is compare with x_1 and x_2 .

4. Conclusion

In this paper, we introduced a SC-CNN communication method which is called GS (Generalized synchronization) and embedding secure communication. The method in which after we accomplished synchronization between the transmitter and receiver in the SC-CNN using GS method, we used to accomplish the secure communication was to synthesizing the desired information with a SC_CNN circuit by embedding the information signal to the SC-CNN signal by only one state variable x_3 embedding from the SC-CNN to the transmitter. As a computer simulation result, we confirm embedding secure communication method by separating the information signal and the SC-CNN signal in the receiver.

REFERENCE

- [1] L. O. Chua "Chua's circuit 10 Years Later", Int. J. Circuit Theory and Application, vol. 22, pp 79-305, 1994
- [2] M. Itoh, H. Murakami and L. O. Chua, "Communication System Via Chaotic Modulations" IEICE. Trans. Fundamentals. vol. E77-A, no. 6, pp. 1000-1005, 1994.
- [3] L. O. Chua, M. Itoh, L. Kocarev, and K. Eckert, "Chaos Synchronization in Chua's Circuit" J. Circuit. Systems and computers, vol. 3, no. 1, pp. 93-108, 1993.
- [4] M. Itoh, K. Komeyama, A. Ikeda and L. O. Chua, "Chaos Synchronization in Coupled Chua Circuits", IEICE. NLP. 92-51. pp. 33-40. 1992.
- [5] K. M. Short, "Unmasking a modulated chaotic communications scheme", Int. J. Bifurcation and Chaos, vol. 6, no. 2, pp. 367-375, 1996.
- [6] K. M. Cuomo, "Synthesizing Self - Synchronizing Chaotic Arrays", Int. J. Bifurcation and Chaos, vol. 4, no. 3, pp. 727-736, 1993.
- [7] L. Kocarev, "Chaos-based cryptography: A brief overview", IEEE, Vol. pp. 7-21. 2001.
- [8] J.A.K. Suykens, "n-Double Scroll Hypercubes in 1-D CNNs" Int. J. Bifurcation and Chaos, vol. 7, no. 8, pp. 1873-1885, 1997.
- [9] L. M. Pecora and T. L. Carroll "Synchronization in

Chaotic System" *Phys. Rev. Lett.*, vol. 64, no. 8, pp. 821-824, 1990.

- [10] L. Kocarev, K. S. Halle, K. Eckert and L. O. Chua, "Experimental Demonstration of Secure Communication via Chaotic Synchronization" *Int. J. Bifurcation and Chaos*, vol. 2, no. 3, pp. 709-713, 1992.
 - [11] K. S. Halle, C. W. Wu, M. Itoh and L. O. Chua, "Spread Spectrum communication through modulation of chaos" *Int. J. Bifurcation and Chaos*, vol. 3, no. 2, pp. 469-477, 1993.
 - [12] P. Arena, P. Baglio, F. Fortuna & G. Manganaro, "Generation of n-double scrolls via cellular neural networks", *Int. J. Circuit Theory Appl*, 24, 241-252, 1996.
 - [13] P. Arena, S. Baglio, L. Fortuna and G. Manganaro, "Chua's circuit can be generated by CNN cell", *IEEE Trans. Circuit and Systems I, CAS-42*, pp. 123-125, 1995.
 - [14] L. Kocarev, L. & U. Parlitz, "Generalized synchronization, predictability and equivalence of unidirectionally coupled dynamical systems", *Phys. Rev. Lett.*, vol. 76, no. 11, pp. 1816-1819, 1996.
 - [15] M. Brucoli, D. Cafagna, L. Camimeo & G. Grassi, "An efficient technique for signal masking using synchronized hyperchaos circuits", *Proc. 5th Int. workshop on Nonlinear Dynamics of Electronic Systems(NDES '97)*, Moscow, Russia, June 26-27, pp. 229-232, 1997.
 - [16] J.A.k. Suyken, P.F. Curran & L.O. Chua, "Master-slave synchronization using dynamic output feedback", *Int. J. Bifurcation and Chaos*, vol. 7, no. 3, 671-679, 1997.
 - [17] J.J. Slotine & W.Li, "Applied Nonlinear Control", Prentice-Hall, NJ, 1991.
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