

Improvement of the Steering Feel of an Electric Power Steering System by Torque Map Modification

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This paper discusses a dc motor equipped electric power steering (EPS) system and demonstrates its advantages over a typical hydraulic power steering (HPS) system. The tire-road interaction torque at the steering tires is calculated using the 2 d.o.f. bicycle model, in other words by using a single-track model, which was verified with the J-turn test of a real vehicle. Because the detail parameters of a steering system are not easily acquired, a simple system is modeled here. In previous EPS systems, the assisting torque for the measured driving torque is developed as a boost curve similar to that of the HPS system. To improve steering stiffness and return-ability of the steering system, a third-order polynomial as a torque map is introduced and modified within the preferred driving torques researched by Bertollini. Using the torque map modification sufficiently improves the EPS system.

Key Words : Electric Power Steering, Assist Torque, Steering Feel, On-Center Handling, Return-Ability, Steering Stiffness

Nomenclature

α : Side slip angle at C.G. (center of gravity) [rad]

γ : Yaw rate [rad/s]

δ : Steering angle [rad]

θ : Steering wheel angle [rad]

θ_m : EPS motor angle [rad]

i_a : EPS motor input current [A]

a_y : Lateral acceleration at C.G. [m/s²]

v : Side slip velocity at C.G. [m/s]

u : Longitudinal velocity [m/s]

1. Introduction

Recently the automotive industry has focused on improving vehicle performance, safety and convenience for drivers. Steering assist systems play an important role in each area. During low-speed maneuvers, power-assisted steering systems reduce the amount of effort needed by the driver for steering. However, it gives a hard feel to steering to drivers during high speed maneuvers. The conventional hydraulic power steering (HPS) system, which is made up of an engine-

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driven hydraulic pump and a hydraulic actuator, decreases engine efficiency but requires complex hydraulic components. To cope with the deficiencies of HPS, an electric power steering (EPS) system has been vigorously researched. Since the EPS system uses an engine-independent motor without complex hydraulic units, the weight and volume of steering systems can be reduced. Thus, EPS systems achieve better fuel and space economy and maintains the feel of the steering even during quick changes in driving conditions through software. Moreover no harm is done at the environment because no hydraulic fluid is used (Burton, 2003). Generally, there are three types of EPS. Light vehicles have installed a column type where the assist motor connects to the steering column through spur gears and delivers assist torque to the column, for example Twingo of Renault, MGF of Rover, ACTY of Honda, and ALTO/SERVO of Suzuki. For heavy vehicles, the pinion type has been adopted, for example MIRA/Hi-Jet of Daihatsu and MINICA of Mitsubishi. The steer-by-wire type gives the most efficient space, but countermeasure for malfunction should be researched.

Steering feel should be established for fine tuning of steering systems. Adams (1983) researched the feel of power steering and Norman (1984) introduced center handling performance. Bertolini and Hogan (1999) drew up a preference curve as a function of vehicle speed based on the steering effort by various drivers using VTI driving simulators. Rakan and Wang (2001) used the boost curves of assist torque for a given vehicle speed. Based on these objective indices, Camuffo et al. (2002) tuned an EPS to have the same steering feel of an HPS. Using a steer-by-wire EPS, Tong Jin Park et al. (2002) controlled the front wheel motor by PID controller to minimize the error between steering angle and wheel angle. With steering wheel angle and torque sensors attached to a steering column, Burton (2003) calculated assistance torque by summing the high gain related to steering torque and the low gain related to steering position. To improve return-ability, Kurishige et al. (2000) developed a control strategy based on an estimation of

alignment torque generated by tires and road surfaces without sensors. Since steering torque assistance and return-ability are not active at the same time, Kim and Song (2002) separated the two control algorithms where the reference steering torque was determined by the torque map based on vehicle speed and steering wheel position.

In this paper, the 2 degree of freedom (d.o.f.) bicycle model will be used to calculate the tire road interaction torque. Although the steering of a vehicle affects its rolling motion, a description of the rolling system is beyond the scope of this paper since an EPS does not control the active driving angle but the assisting torque. Since it is difficult to acquire detailed parameters of the steering system, the steering system will be modeled similarly to the research of Kurishige et al. (2000). In previous EPS systems (Chabaan and Wang, 2001), assisting torque for the measured driving torque was making a chart as a boost curve like the HPS of Adams's research (1983). However, the boost curve map did not improve steering stiffness or return-ability. To improve steering stiffness and return-ability, we will introduce and modify a third-order polynomial as a torque map. The preferred driving torque range developed by Bertolini and Hogan (1999) will be fitted as a linear polynomial function. This proposed torque map will be verified for the 2 d.o.f. vehicle model with a simple steering system. To determine steering input, the relationship between δ and a_y will be derived here. In conclusion, the EPS system will be improved sufficiently on steering stiffness and return-ability by the proposed torque map.

2. Mathematical Models and Verifications

2.1 Vehicle model

The classical single-track model (Choi et al., 2002) is obtained by lumping the two front wheels into one wheel in the centerline of the vehicle, the same is done with the two rear wheels as shown in Fig. 1. By the vehicle kinematics, a lateral acceleration at C.G. is given by

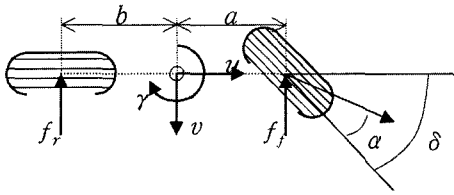


Fig. 1 Coordinate of the single-track model

$$a_y = \dot{v} + u \cdot \gamma \tag{1}$$

Since tires can be modeled as linear within $|a_y| \leq 0.3 \text{ g}$, lateral force at the front tire is obtained as

$$F_f = C_f \cdot a \tag{2}$$

And the side slip angle for δ is presented by

$$\alpha = \delta - \frac{v + a \cdot \gamma}{u} \tag{3}$$

The vehicle model is described as

$$\begin{bmatrix} \dot{v} \\ \dot{\gamma} \end{bmatrix} = \begin{bmatrix} \frac{2(C_f + C_r)}{mu} & -u \\ \frac{2(aC_f - bC_r)}{I_{zz}u} & \frac{2(a^2C_f + b^2C_r)}{I_{zz}u} \end{bmatrix} \begin{bmatrix} v \\ \gamma \end{bmatrix} + \begin{bmatrix} \frac{2C_f}{m} \\ \frac{2aC_f}{I_{zz}} \end{bmatrix} \delta \tag{4}$$

where

- a : distance from C.G. to front axle
- b : distance from C.G. to rear axle
- l : wheel base (i.e. $l = a + b$)
- C_f/C_r : front/rear tire cornering stiffnesses
- I_{zz} : yaw moment of inertia of vehicle
- m : vehicle total mass

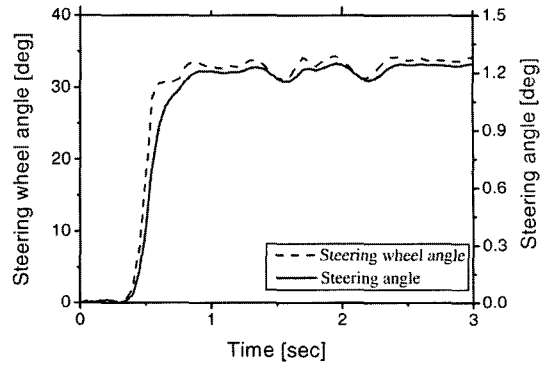
From the IFAC benchmark example (Ackermann and Darenberg, 1990), the steering angle is limited to $|\delta| \leq 40 \text{ deg}$ and the steering angle rate is limited to deg/s . The relationship between δ and θ , is assumed to be dominated by the first order delay and the multiplication of the gear ratio between the steering angle and the steering wheel angle. This formula is as follows :

$$\delta(s) = \frac{n}{Ts + 1} \theta(s) \tag{5}$$

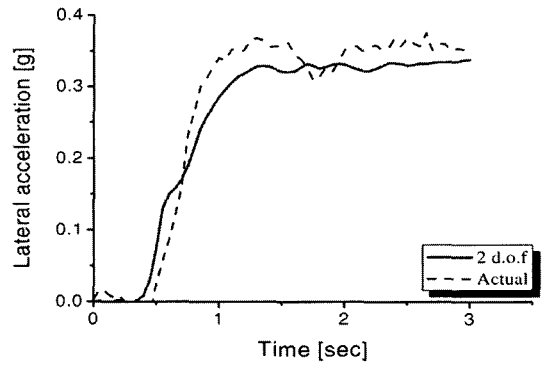
where

- n : gear ratio between δ and θ
- T : delay time between δ and θ

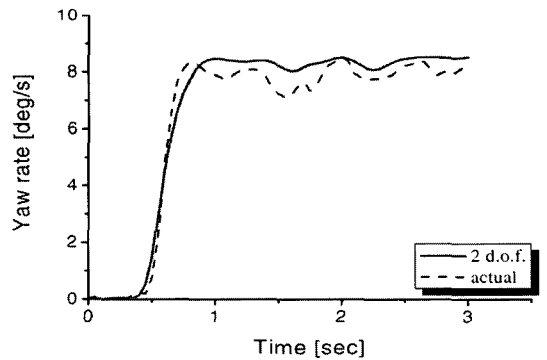
Usually a J-turn test is fulfilled to verify the steering performance such as rollover or riding performance. In simulation, θ is stepped up to



(a) Steering wheel angle θ and steering angle δ



(b) Lateral acceleration a_y



(c) Yaw rate γ

Fig. 2 J-turn test for the model verification

34° within 0.2 sec with $u = 22 \text{ m/s}$. Figure 2 shows the J-turn results for the single-track model and the actual vehicle. Figure 2(a) shows the angle input of steering wheel and the delayed steering angle. The lateral acceleration and the yaw rate are settled to around 3.4 m/s^2 and 8.3 deg/s as shown in Figs. 2(b) and (c). The integrals of time multiplied by the absolute magnitude of error (ITAE) for the lateral accelerati-

on and the yaw rate are 0.29 m/s² and 0.42 deg/s, where the nonlinearity of the tire may be the main factor for the errors. The single-track model has the characteristics nearly similar to the model of actual vehicle.

2.2 Steering system

The masses, inertias, dampings, and stiffnesses of rack-bar, tie rod, and tire had been entirely or partly considered in the previous researches by Park et al.(2002), Kurishige et al.(2000), Kim and Song (2002). In the researches by Bertollini and Hogan (1999) and Camuffo et al.(2002), the tuning or design of the EPS controllers were depended on simulators. So the steering model was not described. As this system is modeled in detail, we can describe its responses more actually. However, to find out all of the parameters precisely is not a simple task. In this paper, a steering column is simply modeled with the mass moment of inertia, while damping and stiffness are ignored. The steering column is obtained as

$$J\ddot{\theta} = \tau_a + \tau_d - \tau_t \tag{6}$$

where

- J : inertia of steering column
- τ_a : assist torque
- τ_d : driving torque
- τ_t : load torque (i.e. $\tau_t = \tau_l + \tau_f$)
- τ_t : tire-road interaction torque
- τ_f : friction torque

EPS motor generating τ_a can be modeled as a dc motor as follows :

$$\begin{aligned} J_m\ddot{\theta}_m + B_m\dot{\theta}_m &= \tau_m - \frac{1}{N} \tau_a \\ &= k_T \cdot i_a - \frac{1}{N} \tau_a \end{aligned} \tag{7}$$

where

- J_m : inertia of EPS motor
- B_m : damping of EPS motor
- τ_m : EPS motor torque
- k_T : EPS motor constant
- N : gear ratio of EPS motor

Thus, the steering system is obtained as

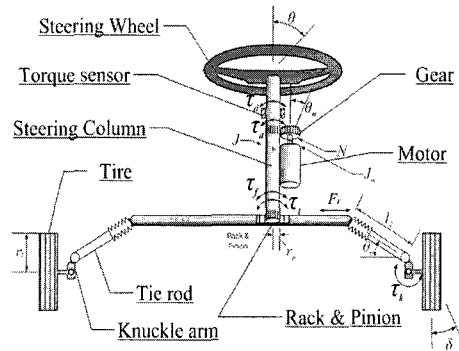


Fig. 3 Constitution of steering system equipped an EPS

$$\begin{aligned} (J + J_m N^2) \ddot{\theta} + B_m N^2 \dot{\theta} \\ = k_T N \cdot i_a + \tau_d - \tau_t - \tau_f \end{aligned} \tag{8}$$

The obtained steering system is shown in Fig. 3.

To find the tire-road interaction torque, the kingpin torque, τ_k , (Gillespie, 2002) is obtained as

$$\tau_k = \tau_v + \tau_L + \tau_A \tag{9}$$

where

- τ_A : aligning torque
- τ_L : lateral torque
- τ_v : vertical torque

Because of the kingpin offset angle and a lateral inclination angle, the vertical force on the tire produces the vertical torque. When the kingpin offset angle and the lateral inclination angle are small, a torque generated by the vertical force can be approximated by

$$\tau_v = -F_z d \sin \lambda \cdot \sin \delta \tag{10}$$

where

- F_z : vertical force
- d : kingpin offset angle
- λ : lateral inclination angle

Since the lateral force produces a torque through the longitudinal offset resulting from the caster angle, a torque generated by the lateral force is given by

$$\tau_L = F_y r_t \tan v \tag{11}$$

where

- F_y : lateral force acting at the tire center
- r_t : tire radius

ν : caster angle

The lateral force is developed by a tire at a point behind the tire center. So the aligning torque is written as

$$\tau_A = \rho F_y \cos \sqrt{\lambda^2 + \nu^2} \quad (12)$$

where ρ is pneumatic trail distance. Considering the length from kingpin to rack-bar, the rack-bar force, F_r , is written by

$$F_r = \frac{\tau_k}{\{l_0 \cos(\theta_0 - \delta) + l_0 \cos(\theta_0 + \delta)\}} \quad (13)$$

where l_0 and θ_0 are the length and the angle of tie-rod. The ground reactions, τ_t , on the tire are described by

$$\tau_t = F_r \cdot r_p \quad (14)$$

where r_p is the radius of pinion. The friction

torque in eq. (8) is defined as follows :

$$\tau_f = F_f \cdot \text{sgn}(\dot{\theta}) \quad (15)$$

where F_f is friction gain. More complex frictions in the steering system are ignored here.

By the results of slalom test by INS with a GPS Multi-Antenna System (Hong et al., 2003), the simple steering model is verified. Considering the driver safety, the vehicle speed and the steering wheel angle are limited. In the experimentation, θ and τ_d are measured by encoder and torque sensor attached on steering column. F_y is calculated by the 2 d.o.f. vehicle model. Figure 4 shows the plots of θ versus τ_d . As the vehicle speed is increased, the side slip angle are increased. Therefore, higher speed needs more driving torque. The gap between experimentation and simulation is due to modeling uncertainties or measuring errors.

3. EPS Control by Modification of Torque Map

Typical hydraulic power steering systems are controlled by the reference torque map. Many researchers including Adams (1983) have adopted a boost curve as the map, where the assisting level is related to τ_d and u . Based on the previous models of a vehicle and a steering system, Fig. 5 shows the block diagram of the EPS control system, where is measured by a hole-sensor on transmission and the driving torque is measured by a torque sensor attached to the steering column between the steering wheel and assisting motor.

The steering feel in vehicle handling has been characterized generally by “on center handling” (Norman, 1984), which is determined by driving a vehicle on a normal highway under low lateral acceleration without wind or road disturbances. The steering feel indices are defined by the relationships among θ , τ_d , and a_y . These parameters are calculated or measured under $u=100$ km/h, where the steering wheel is a 0.2 Hz sinusoidal wave. The lateral acceleration of a vehicle is dominantly related to the magnitude of the steering angle. When the lateral velocity and the yaw rate in eq. (4) have settled (i.e. $\dot{v}=\dot{\gamma}=0$),

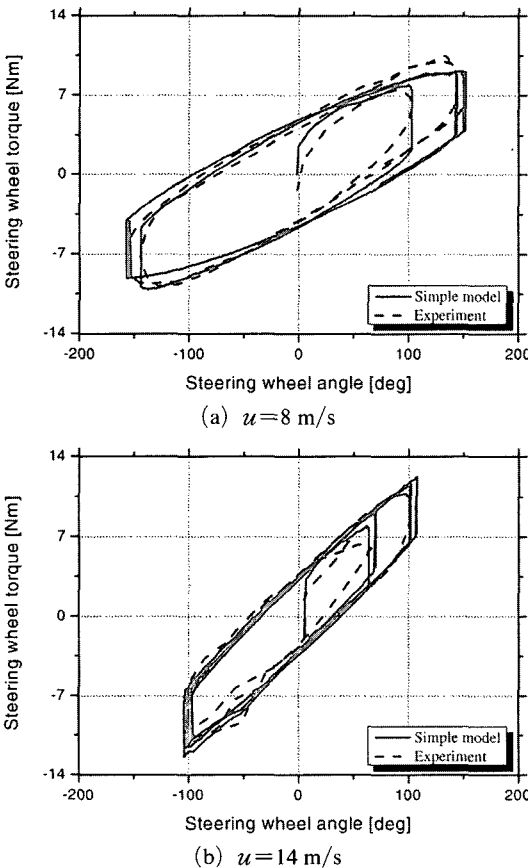


Fig. 4 Steering wheel angle θ vs. steering wheel torque τ_d

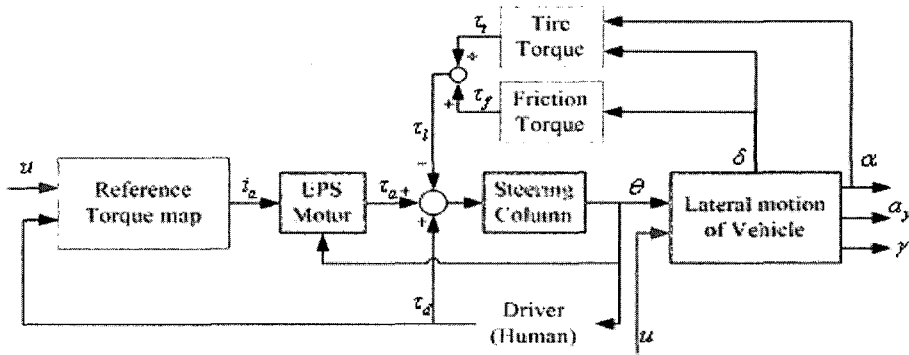


Fig. 5 EPS control system including an reference torque map

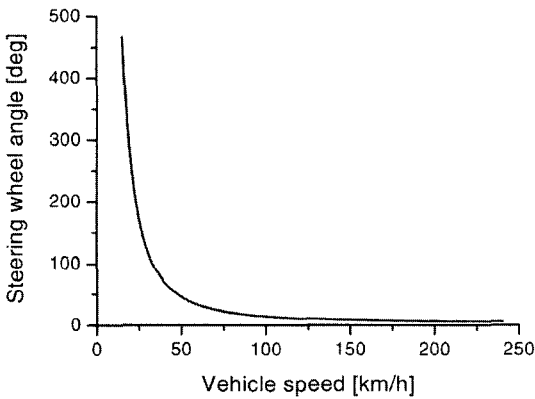


Fig. 6 Steering wheel angles θ to be $a_y=0.2\text{ g}$

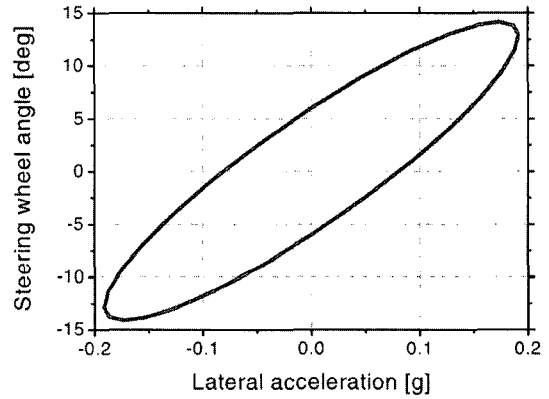


Fig. 7 Lateral acceleration a_y vs. steering wheel angle θ

the steering angle for the given a_y can be described as a function of vehicle speed u by

$$\delta = \left\{ \frac{1 - K(l-2)}{u^2} - K \cdot m \right\} a_y \quad (16)$$

where

$$K = \frac{aC_f - bC_r}{2C_f C_r (a+b)} \quad (17)$$

Figure 6 shows the steering wheel angle for $a_y=0.2\text{ g}$ (acceleration of gravity) by eq. (16).

Figures 7~9 show the results of “on center handling” without power assistance. The plot of θ versus a_y as shown in Fig. 7 has three steering indices: steering sensitivity at $a_y=0.1\text{ g}$, minimum steering sensitivity, and steering hysteresis. The ratio of steering sensitivity at $a_y=0.1\text{ g}$ and minimum steering sensitivity shows the influence of steering compliance and the hysteresis is related to the lag of yaw rate with steering input. In Fig. 7, the steering sensitivity at 0.1 g is

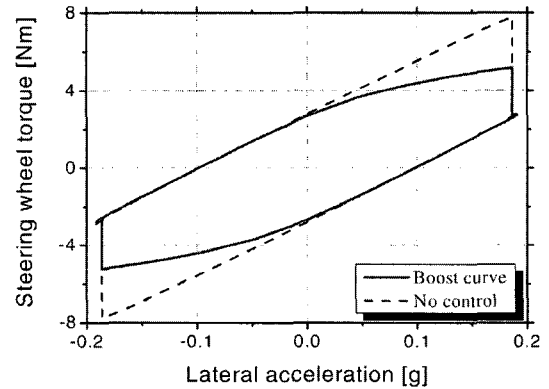


Fig. 8 Lateral acceleration a_y vs. steering wheel torque τ_a by the boost curve map

2.2238 g/100 deg; the minimum steering sensitivity is 1.5825 g/100 deg; and the steering hysteresis is 11.3121 deg. The ratio of sensitivity is about 71.6% which is that of a compact front power

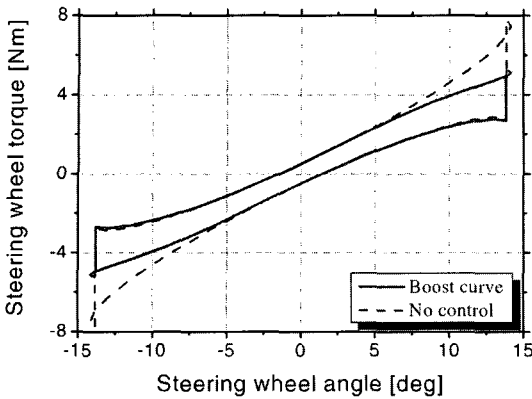


Fig. 9 Steering wheel angle θ vs. steering wheel torque τ_d by the boost curve map

vehicle according to Norman’s research (1984). In the plot of τ_d versus α_y as shown in Fig. 8, steering indices such as lateral acceleration at 0 Nm, steering wheel torques and steering wheel torque gradients at 0 g and 0.1 g are indicated. At the point of releasing a steering wheel after step steering, τ_d will be 0 Nm and lateral acceleration causes the steering wheel return to center. Thus, the lateral acceleration at 0 Nm is an indication of return-ability. Steering wheel torque at 0 g is mainly dependent on Coulomb’s friction in the steering system. The steering torque gradient at 0 g is influenced by the kingpin axis torque gradient and the overall steering ratio. The steering wheel torque at 0.1 g is a measure of steering effort. The steering torque gradient at 0.1 g is a measure of road feel just off straight ahead. With typical power steering systems, both torque and torque gradient at 0.1 g are significantly reduced. The results of a steering system by nature and an EPS by a boost curve shows that lateral acceleration at 0 Nm and steering wheel torque at 0 g are similar, and steering wheel torque and steering wheel gradients at 0 g and 0.1 g are decreased. Drivers of an EPS equipped vehicle are able to reduce the steering effort and improve road feel, but return-ability is not improved by the boost curve alone. In the plot of τ_d versus θ as shown in Fig. 9, return-ability and steering stiffness are analyzed by steering wheel torque at 0 deg and steering torque gradient at 0 deg. These indices can’t be improved by the boost curve at all.

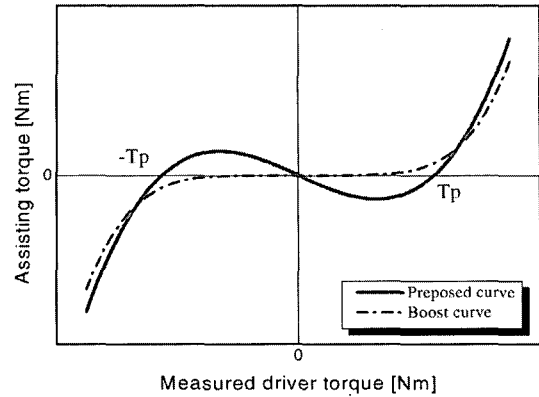


Fig. 10 Assisting torque τ_a for measured driver torque τ_d

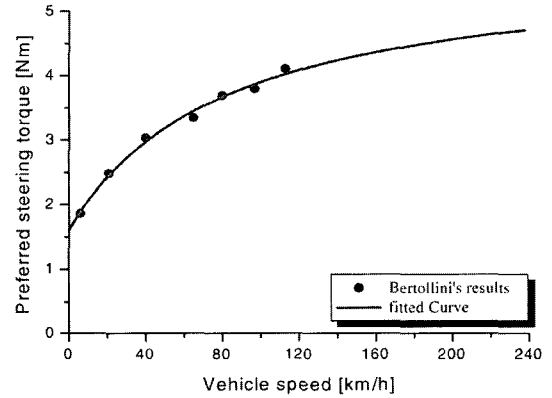


Fig. 11 The preferred steering torque τ_p by Bertollini

Therefore it is necessary to change the reference torque map from the typical boost curve.

Since the assisting torque and the returning torque are not simultaneously required, a cubic curve is introduced as a reference torque map as follows ;

$$\tau_a = k_a \cdot \tau_d (\tau_d + T_p) (\tau_d - T_p) \quad (18)$$

where k_a is the tuning gain considering the maximum torque of motor or steering feel and T_p is a preference torque depended on u . The cubic curve map as shown in Fig. 10 generates a returning torque when driving torque is smaller than T_p . Bertollini and Hogan (1999) developed the preference torques as shown in Fig. 11 using the VTI driving simulator. Here the preference torques are fitted as a rational polynomial curve as follows

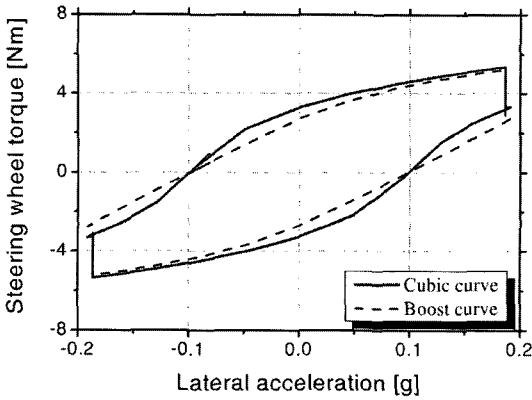


Fig. 12 Lateral acceleration a_y vs. steering wheel torque τ_d by the proposed cubic curve map

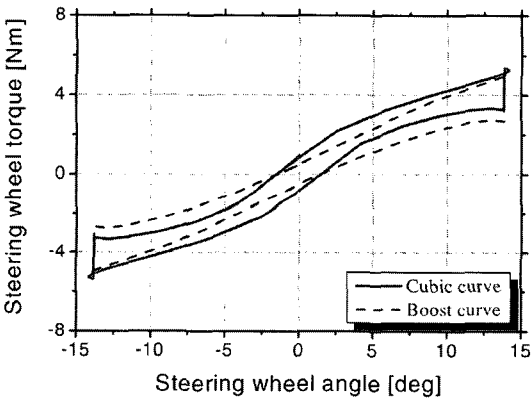


Fig. 13 Steering wheel angle θ vs. steering wheel torque τ_d by the proposed cubic curve map

$$T_p = \frac{5.78u + 131.5}{u + 82.09} \quad (19)$$

where the root mean square (RMS) error of fitting is 0.0847 Nm.

Figures 12 and 13 show the results of the proposed cubic curve map. In Fig. 12, lateral acceleration at 0 Nm is unchanged, steering wheel torques at 0 g and 0.1 g have increased slightly, and the steering wheel torque gradient at 0 g and 0.1 g have decreased. Because the steering wheel torque gradient at 0 deg is increased, the steering stiffness improves. But the return-ability deteriorates as shown in Fig. 13. Therefore the cubic curve map should be modified to improve the return-ability.

According to the states of the driving torque,

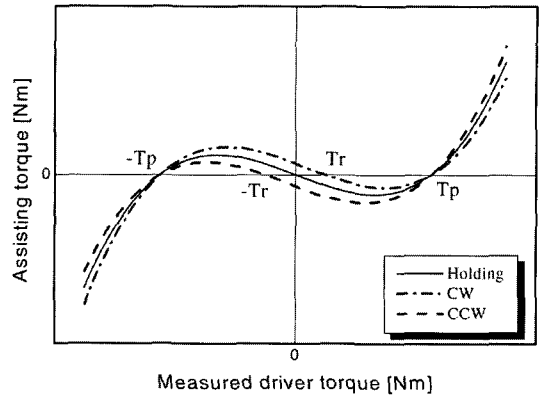


Fig. 14 Assisting torque curves to improve the return-ability

the cubic curve is modified as shown in Fig. 14. When the driving torque is increasing (i.e. $\dot{\tau}_d > 0$) in spite of $\tau_d = 0$ Nm, an EPS motor generates the assisting torque up to T_r . This manner also applies to the reverse steering. As a result, the reference torque map by the modified cubic curve is defined by

$$\tau_a = \begin{cases} k_a \cdot \tau_d (\tau_d + T_p) (\tau_d - T_p), & \dot{\tau}_d = 0 \text{ (holding)} \\ k_a (\tau - T_r) (\tau + T_p) (\tau - T_p), & \dot{\tau}_d > 0 \text{ (CW)} \\ k_a (\tau + T_r) (\tau + T_p) (\tau - T_p), & \dot{\tau}_d < 0 \text{ (CCW)} \end{cases} \quad (20)$$

where T_r is the tuning gain, which should be further studied. In Figs. 15 and 16, lateral acceleration at 0 Nm and steering wheel torque at 0 deg are reduced, so return-ability is improved. Simultaneously torque gradients are increased. All of results are summarized in the Table 1. Indices in the plot of θ versus a_y have no concern with power steering systems. Lateral accelerations at 0 Nm related with return-ability are alike. By using power steering systems, steering effort and road feel are decreased. Although some indices in the plot of τ_d versus a_y become a little worse by the modified curve, the indices may be improved by fine tuning k_a and T_r . The modification improves steering wheel torque and gradient at 0 deg remarkably. The steering wheel torque (0.1682 Nm) at 0 deg by the modified curve is smaller than the half of torque (0.5027 Nm) by the boost curve. The steering wheel torque gradient (0.6554 Nm/deg) at 0 deg is larger than 0.3590 Nm/deg.

Table 1 The results of steering feel indices

	Steering indices	Meaning	No control	Boost curve	Proposed curve	Modified curve
a_y vs. θ	Steering sensitivity at 0.1 g [g/100 deg]	steering compliance	2.2238			
	Minimum steering sensitivity [g/100 deg]	steering compliance	1.5825			
	Steering hysteresis [deg]	phase lag	11.3121			
a_y vs. τ_d	Lateral acceleration at 0 Nm [g]	return-ability	0.0981	0.0982	0.0989	0.0839
	Steering wheel torque at 0 g [Nm]	coulomb friction	2.7921	2.6943	3.2754	3.1904
	Steering wheel torque at 0.1 g [Nm]	steering effort	5.5509	4.4141	4.6085	4.6734
	Steering wheel torque gradient at 0 g [Nm/g]	steering ratio & kingpin	27.8484	20.6239	15.3623	17.4083
	Steering wheel torque gradient at 0.1g [Nm/g]	road feel	26.9798	11.2782	9.9777	10.6909
θ vs. τ_d	Steering wheel torque at 0° [Nm]	return-ability	0.5006	0.5027	0.8281	0.1682
	Steering wheel torque gradient at 0° [Nm/deg]	steering stiffness	0.3603	0.3590	0.5304	0.6554

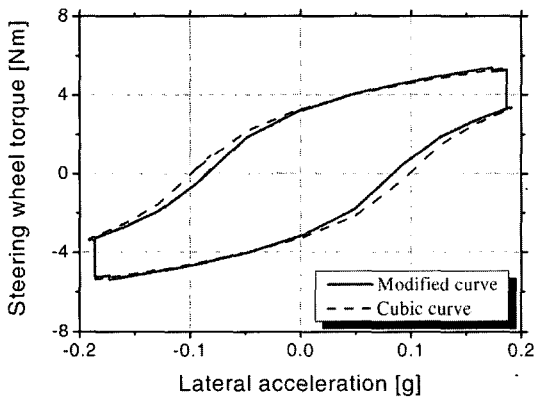


Fig. 15 Lateral acceleration a_y vs. steering wheel torque τ_d by the modified cubic curve map

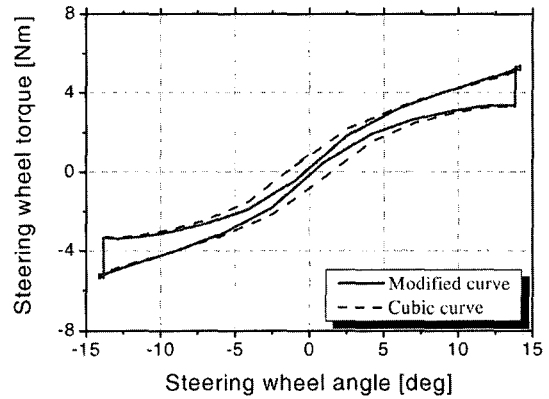


Fig. 16 Steering wheel angle θ vs. steering wheel torque τ_d by the modified cubic curve map

4. Conclusions

This paper discussed a dc motor equipped electric power steering (EPS) system. The tire-road interaction torque at the steering tires was calculated using a 2 d.o.f. bicycle model. The results of J-turn test for the model verification showed

8% lateral acceleration error and 5% yaw rate error. A simple steering system was modeled and tested for the sinusoidal steering inputs (slalom test), the simple steering model had errors caused by uncertainties such as uncertain parameters, friction, and backlash. Hence, the steering model and the 2 d.o.f. bicycle model could replace with an actual vehicle. To improve steering stiffness

and return-ability of the steering system, a third-order polynomial as a torque map was introduced and modified within the preferred driving torques researched by Bertollini and Hogan. The preferred driving torque was fitted as a rational polynomial function here. The relationship between δ and a_y was derived in order to determine steering input. The proposed and modified torque maps were simulated using the 2 d.o.f. vehicle model with a simple steering system. By using the modified torque map, the steering wheel torque at 0 deg as the index of return-ability was 0.1682 Nm and the steering wheel torque gradient at 0 deg as the index of steering stiffness was 0.6554 Nm/deg. After all, the return-ability and steering stiffness of EPS system were improved sufficiently.

Acknowledgments

This research was financially supported by Korea Science and Engineering Foundation (KOSEF) through the Engineering Research Center for Net Shape and Die Manufacturing at Pusan National University and by Pusan National University Research Grant.

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