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Coordinate Transformation Parameter Estimation for Korean Seas and Islands

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Abstract

According to revisions of survey law taking effect on January 1, 2003, the Korean geodetic datum has been changed from a local geodetic to a world geodetic system. In this study, the datum transformation parameters especially for the maritime geographical data are determined. From database constructed through MGIS, a total of 492 coordinate pairs were selected and used in the parameter determination after outlier testing. Based on the parameter estimation, the Molodensky model is selected for datum transformation. For higher accuracy, Application of network optimization and a least squares collocation with Gaussian model has resulted in the accuracy better than 15 cm in coordinate transformation.

Keywords : *Datum Transformation, Station Optimization, Least Square Collocation*

1. Introduction

To keep pace with international trends and to establish the spatial data infrastructure which is worldwide compatible, The Korean survey law was revised to change the national datum to the World Geodetic Datum on January 1, 2003. To this end, the constructed spatial data needs to be converted to data based on the new datum and this is done by constructing a transformation platform.

Land data conversion to the new datum was initiated relatively early, comparing with that of the ocean data throughout the project of Geodesy 2002 managed by National Geographic Information Institute (NGII). Using 107 first order common points which have coordinates both in the Bessel ellipsoid based old Tokyo Datum and the GRS 80 based new datum, the transformation parameters are determined by using the Molodensky-Badekas algorithm (Yoon, 2003). In addition, the distortion which still remains after the similarity transformation due to the different measurement methods and network adjustment, is modeled and corrected using a least squares collocation; therefore, a final accuracy of the transformation better than 20 cm is achieved.

In October 2003, the National Oceanographic Research Institute (NORI) initiated a project on the determination of the datum transformation parameters for ocean data. Since the data used in the determination of the transformation parameters for land data is mainly distributed in land, NORI decided to determine the independent transformation parameters so that all ocean GIS data can be transformed by these new parameters.

In this paper, the procedure and the results of the Korean ocean datum transformation are presented. With a description of the data used in this research, the detailed step-by-step procedures to achieve high accuracy in the transformation are described. This is followed by the presentation of the developed datum transformation tools and the analyzed, results.

2. Data Acquisition

The data used in this study had been acquired from 1997 to 2002 through the campaign for the determi-

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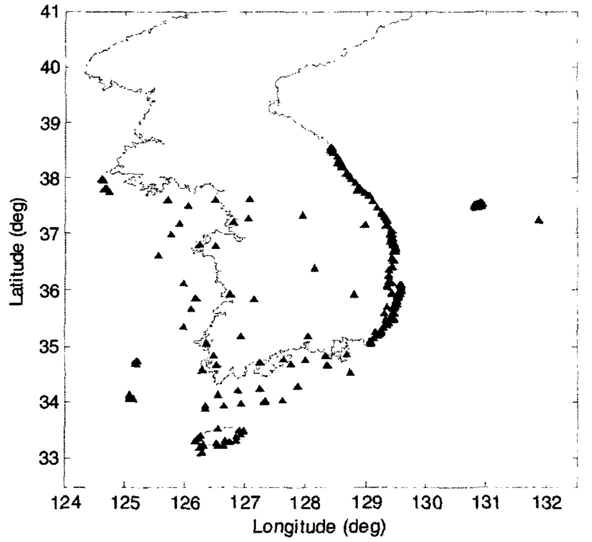

Fig. 1. Stations available for the datum transformation.

nation of a Korean territorial baseline issued by NORI. The majority of data is distributed on the shore or Islands (see Fig. 1). A total of 493 points were observed by GPS static surveying technique at 30 second recording interval for 24 hours, among which 12 and 18 control points located on land and shore area are included. The measured data was processed by GPSurvey software in baseline determination mode by fixing the control points. This was followed by the network adjustment using TrimNet Plus to achieve higher accuracy. In this procedure, the data that shows precision of the adjusted coordinates larger than 15 cm were considered to be outliers and removed, and the adjustment was reperformed to finalize the precise coordinates for 315 points. Note that the data is significantly dense in the area of the East Sea and southern Islands. This nonuniform distribution potentially causes some biases in the estimated transformation parameters. Details on the effect of the stations nonuniform distribution is explained later on.

3. General Procedure for the Transformation Parameter Estimation

In general, the transformation of the coordinates from one datum to another is performed in following two steps. First, the parameters of similarity transformation models are estimated. Secondly, the distortion modeling and correction for the residuals after applying the similarity transformation is applied. In the first step, the well known models such as Bursa-Wolf, Molodensky-Badekas and Veis models are usually selected and the parameters are estimated by a least square adjustment (Rapp, 1993). In the second step, a least square collocation is applied to correct the remaining distortions after the similarity transformation. In this study, however, one more step is applied namely the station optimization. If the stations used in the parameter estimation are distributed irregularly, it is highly possible to obtain the biased estimates. Therefore, a network optimization theory is applied to regularize the station distribution. In the following sections, the results without and with optimization procedure is described so that the difference could be noticed.

4. Transformation Parameter Estimation

Table 1 shows the estimated parameters with standard deviations for the popular seven parameter models using the 315 stations of data described previously. The Bursa-Wolf model has larger standard deviations of the estimates than the other two models because the origin of the rotation in the Bursa-Wolf model is at the center of the earth while the data used are only from Korea, which is significantly biased in a global sense. The other two models, however, use a local origin as the rotational origin, thus rotational parameters are relatively well estimated. Although the superiority between the Molodensky-Badekas and Veis models

Table 1. Estimated parameters with standard deviations for three similarity transformations.

Model		Tx (m)	Ty (m)	Tx (m)	Rx (arcsec)	Ry (arcsec)	Rz (arcsec)	Scale (ppm)
BW	Par.	-121.24	471.20	654.80	-1.62	2.03	1.92	8.17
	S.D.	0.99	0.75	0.87	0.03	0.03	0.03	0.10
MB	Par.	-146.84	504.34	685.66	-1.62	2.03	1.92	8.17
	S.D.	0.02	0.02	0.02	0.03	0.03	0.03	0.10
Veis	Par.	-146.84	504.34	685.66	-0.01	-0.01	-3.23	8.17
	S.D.	0.02	0.02	0.02	0.03	0.03	0.02	0.10

Table 2. Residuals of the Molodensky-Badekas transformation for all 315 stations.

	△X (m)	△Y (m)	△Z (m)
Mean	0.00	-0.04	0.05
Std. Dev.	0.52	0.44	0.42

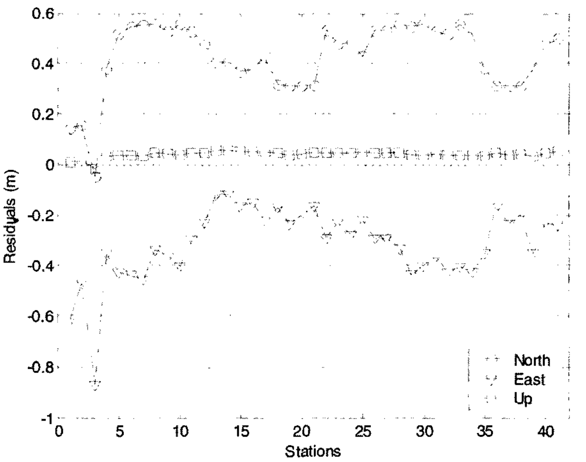

Fig. 2. Residuals on stations of Jeju Island after applying Molodensky-Badekas transformation with estimates from all available data.

was not clearly found, the former is selected as the datum transformation model for Korean ocean spatial data. The rationale for this selection is to use the same transformation model as the land data conversion and relative simplicity in concept of the model.

Applying the Molodensky-Badekas transformation with the estimated parameters to the 315 stations, coordinate biases less than 10 cm with standard deviations less than 55 cm are achieved as shown in Table 2.

When analyzing the residuals in regions, significant biases are found in the transformation results. For example, in the Jeju Island region, it can be recognized that the residuals are considerably biased in the direction of east and north (Fig. 2). As mentioned before, the reason for this bias was considered to be the fact of nonequally distributed data used in the parameter estimation. Therefore, the station distribution needs to be optimized for mitigating this effect by the network optimization algorithm, which comes in the next section.

5. Station Optimization

For the optimization of the station distribution, the theory of network optimization can be implemented. In general, optimization is considered to maximize the accuracy while maintaining the reliability and cost at certain levels. According to Grafarend (1972), the conditions of the optimized network are the homogeneity and isotropy of the errors at each station. A variance-covariance matrix which is homogeneous and isotropic in an ideal network (so- called "criterion matrix") possesses the Taylor-Karman structure (TK-structure).

The Taylor-Karman structure defines the auto- and cross-covariance between two points in the network. Following Grafarend (1972), the general expression for the TK-structured criterion matrix between $P_i(x_i, y_i, z_i)$ and $P_j(x_j, y_j, z_j)$ is given by

$$\sigma_0^2 C_{ij} := \begin{bmatrix} \Sigma_m(s) & 0 & 0 \\ 0 & \Sigma_m(s) & 0 \\ 0 & 0 & \Sigma_m(s) \end{bmatrix} + [\Sigma_l(s) - \Sigma_m(s)] \cdot \frac{1}{s^2} \begin{bmatrix} (x_i-x_j)^2 & (x_i-x_j)(y_i-y_j) & (x_i-x_j)(z_i-z_j) \\ (x_i-x_j)(y_i-y_j) & (y_i-y_j)^2 & (y_i-y_j)(z_i-z_j) \\ (x_i-x_j)(z_i-z_j) & (y_i-y_j)(z_i-z_j) & (z_i-z_j)^2 \end{bmatrix} \tag{1}$$

where $\Sigma_l(s)$ and $\Sigma_m(s)$ represent the longitudinal and cross-covariance functions, respectively, and σ_0^2 is the unit-free variance component. C denotes the ideal cofactor matrix of the estimated network point coordinates.

Once the criterion matrix is computed, the weights of each measurement between stations can be estimated from the condition of minimizing the difference between the cofactor matrix $Q_{\hat{\xi}}$ of the estimated point coordinates $\hat{\xi}$ and an ideal criterion matrix (e.g., with TK-structure) C :

$$\left\| Q_{\hat{\xi}} - C \right\| = \min . \tag{2}$$

After several steps of manipulation in the least squares sense, one can derive the normal equation for the weights:

$$(ACA^T * ACA^T)\underline{\hat{p}} = vecdiag(ACA^T) , \tag{3}$$

where A represents the design matrix of the measurement, $vecdiag$ is the operator of diagonal element after vectorization, and the symbol * defines the Hadamard product of matrices with equal size, namely:

$$\underset{k\times l}{G} * \underset{k\times l}{H} = [g_{ij} \cdot h_{ij}]_{k\times l} . \tag{4}$$

Table 3. Estimated parameters of the Molodensky-Badekas model using optimized stations of data

Parameters	Tx (m)	Ty (m)	Tx (m)	Rx (arcsec)	Ry (arcsec)	Rz (arcsec)	scale (ppm)
Estimates	-145.36	504.49	686.80	-1.57	1.97	1.87	8.62
Std. Dev.	0.05	0.06	0.06	0.09	0.08	0.10	0.33

Table 4. Residuals of the transformation using estimates calculated from optimized stations data only

	△X (m)	△Y (m)	△Z (m)
Mean	0.1009	0.0759	0.0284
Std. Dev.	0.5323	0.4752	0.4426

For detailed implementation of the station optimization, please see Bae (2005).

Based on this network optimization theory, one can sequentially select the stations which satisfy the conditions of the homogeneous and the isotropic weight. Then, the error ellipses calculated from the weight show more or less equal size and circle shape. After applying the procedures, a total 42 stations out of 315 stations were selected as illustrated in Figure 3. It is important to note that the stations are almost equally distributed and the error ellipses more or less have equal size of circular shape.

Tables 3 and 4 show the estimated transformation parameters of the Molodensky-Badekas model with the standard deviations using the data from the optimized 42 stations and residuals for all 315 stations, respectively. Comparing these with the corresponding Tables 1 and 2, one can notice that the overall precision and residuals are slightly poorer because of the small number of stations used in the adjustment. However, the residuals in the regions show that the biases are considerably reduced as seen in the example of Jeju Island (see Fig. 3 and 4). Based on this uniformity and residual analysis, it can be decided to select the Molodensky-Badekas model with the parameters in Table 3 as the datum transformation method for Korean ocean spatial data.

6. Distortion Correction

If there are distortions in old coordinates for some reason s such as different surveying and adjustment methods, the similarity transformation would still leave significant residuals as in Fig. 5 and 6. To correct the distortion, it has to be modeled and corrected with some mathematical methods such as multiple regression, least squares collocation, and surface fitting (Collier, 2002). Considering that the distortion can be complex and highly variable, a least squares collocation (LSC) method has been applied to the coordinates produced from the similarity transformation to predict the distortions on regularized grids.

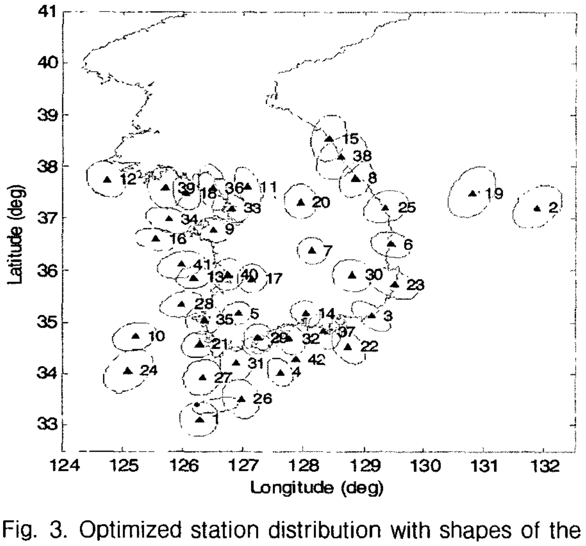

Fig. 3. Optimized station distribution with shapes of the error ellipses.

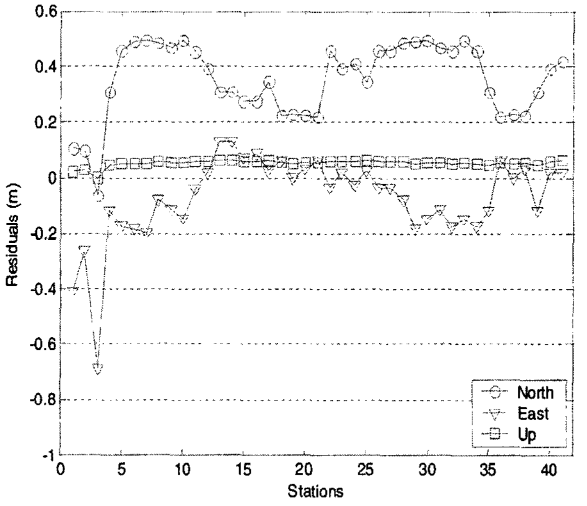

Fig. 4. Residuals on stations of Jeju Island after applying the Molodensky-Badekas transformation with estimates from optimized 42 stations of data.

In this study, the well-known Gaussian function is selected as analytical function for the autocorrelation:

$$C(\|h\|) = C_0 e^{-A^2h^2}, \tag{5}$$

where C is the covariance which depends only on the radial distance h between two points, A is the reducing factor, and C_0 is the variance.

As usual, the parameters in Eq. (5) are estimated using the empirical autocorrelation constructed from data bins with an interval of 0.075 degree with the data lag for the correlation of five degrees. Based on the empirical autocorrelation calculated from data, the reducing factors and variances are estimated using least squares adjustment (Table 5). Using the estimated parameters of the Gaussian function, the distortion at a certain point can be calculated using LSC. For details on the theory of LSC (see Moritz, 1980).

The distortions are predicted and saved in 1'×1' grids for latitudes of 33° to 39° and longitudes of 124° to 132°. Once the distortions are saved in a grid form, the distortion at an arbitrary point can be determined by using bilinear interpolation.

To evaluate the accuracy of the distortion determination, 165 points which have not been used for the calculation of transformation/distortions are selected and the seven parameter transformation and distortion correction are applied (Table 6). The results show that the coordinate transformation was successfully performed

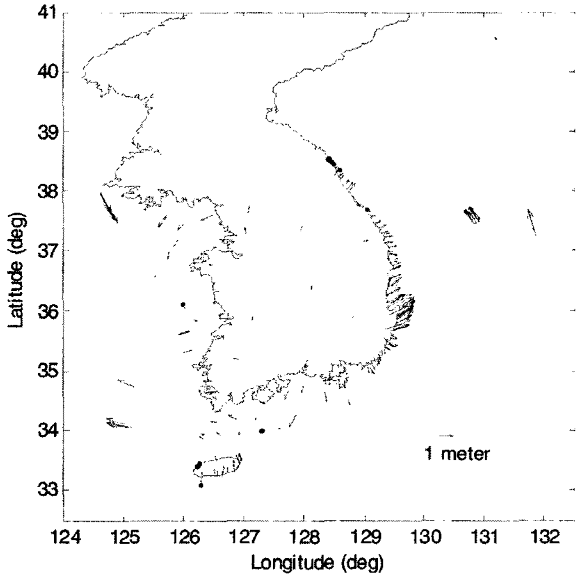

Fig. 5. The residuals of horizontal positions after the similarity transformation.

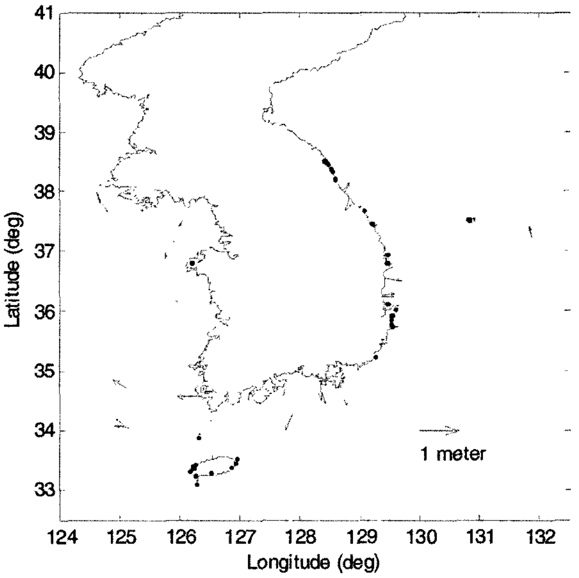

Fig. 7. The residuals of horizontal positions after the distortion correction.

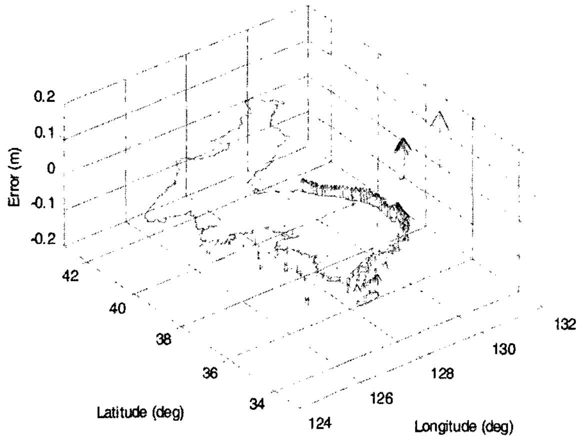

Fig. 6. The residuals of heights after the similarity transformation.

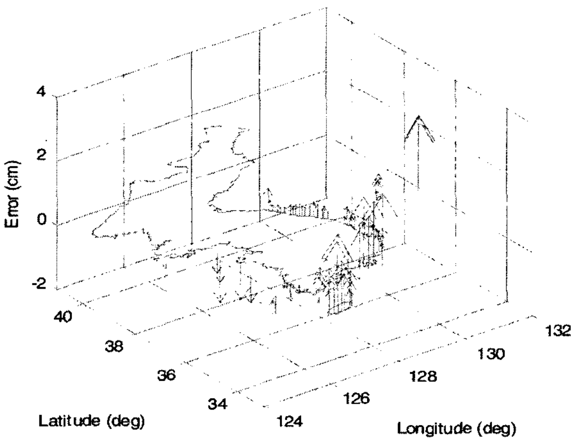

Fig. 8. The residuals of horizontal positions after the distortion correction.

Table 5. Estimated variance and reducing factor using empirical autocorrelation

	Latitude	Longitude	Height
C_0 (deg^2)	2.477×10^{-11}	4.673×10^{-11}	1.195×10^{-3}
A (1/deg)	4.154	2.385	5.669

Table 6. Statistics of transformation check with 165 check points

	Latitude	Longitude	Height
Mean (cm)	0.13	-1.02	0.44
Std. Dev. (cm)	11.39	12.86	0.64

with biases less than 1.5 cm in the 3 dimensional position and standard deviations less than 15 cm. Overall trends shown in Figures 7 and 8 also confirm the successful transformation.

7. Conclusion

For the datum transformation of Korean ocean spatial data, estimation of the parameters in three similarity transformation methods (i.e., the Bursa-Wolf, Molodensky-Badekas, and Veis models) are performed and analyzed. Based on the analyses and comparisons of those models, the Molodensky-Badekas model was selected since it shows better precision on the estimates than Bursa-Wolf, and it is conceptually simpler than Veis model.

To achieve high accuracy in the datum transformation, the station distribution was optimized using network optimization theory. It was found that the station optimization considerably contributes to eliminate the biases in the transformation achieving biases less than 10cm with standard deviation less than 55 cm.

Further improvement on the transformation was achieved by modeling and predicting the distortions with least squares collocation. The predicted distortions are saved in 1'×1' grids for all three components so that the distortion at an arbitrary point can be calculated with simple interpolation. The accuracy of the transformation was checked with 165 checkpoints and an overall accuracy better than 15 cm was achieved.

It is considered that this study could be a good example of conducting a datum transformation showing intensive data screening, station optimization, and distortion correction, so that it can be referenced by any national coordinate transformation task.

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