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A Modeling of Impact Dynamics and its Application to Impact Force Prediction

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In this paper, the contact force between two colliding bodies is modeled by using Hertz's force-displacement law and nonlinear damping function. In order to verify the appropriateness of the proposed contact force model, the drop type impact test is carried out for different impact velocities and different materials of the impacting body, such as rubber, plastic and steel. In the drop type impact experiment, six photo interrupters in series close to the collision location are installed to measure the velocity before impact more accurately. The characteristics of contact force model are investigated through experiments. The parameters of the contact force model are estimated using the optimization technique. Finally the estimated parameters are used to predict the impact force between two colliding bodies in opening action of the magnetic contactor, a kind of switch mechanism for switching electric circuits.

Key Words : Contact Force Model, Impact Force Prediction, Hertz's Force-Displacement Law, Drop Type Impact Test

1. Introduction

Impact is the most common type of dynamic loading conditions that produce the impulsive force and gives rise to the transient responses of the mechanical systems, which is very important because it dominates the dynamic behaviors of systems after impact. Therefore, in order to correctly simulate and design these systems, the impact phenomena must be analyzed. Many researchers have studied various methods for predicting the impact response (Salah, 2000 ; Rothbart, 1985). A simple method for impact analysis is to use Newton's law assuming that impact occurs instantaneously. This method is based on the conservation of momentum and energy (Raymond, 1991 ; Rosa, 1999). Although this method is relatively efficient, the disadvantage is not to predict the response of the system during impact. Another method overcoming such a disadvantage is to use the contact force model for representing forces arising from impact. The contact force is modeled by a nonlinear spring based on the Hertz law and a proper damper in parallel (Goldsmith, 1960 ; Hunt, 1975 ; Lankarani, 1990 ; Shivaswamy, 1997). In this method, the responses during impact can be predicted because the contact forces act in a continuous manner.

In this paper, the impact dynamics between two colliding bodies is analyzed by the contact force model using Hertz's force-displacement law and nonlinear damping function. In order to verify the appropriateness of the proposed contact force model, the drop type impact test is carried out for

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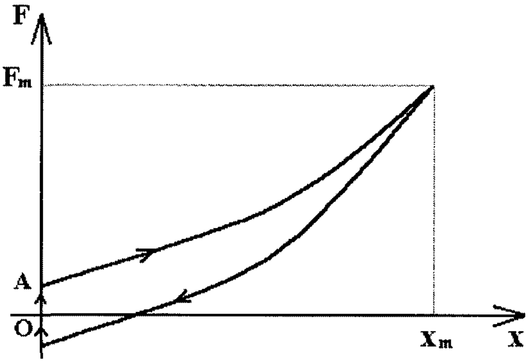
different impact velocities and different materials of the impacting body, such as rubber, plastic and steel. In the drop type impact experiment, six photo interrupters in series close to the collision location are installed to measure the velocity before impact more accurately. The contact force model is investigated through many experiments. Its parameters are estimated using the optimization technique. Finally, we apply the force model to the prediction of impact force between two colliding bodies in opening action of the magnetic contactor while the parameters is estimated from the drop test.

2. Contact Force Model

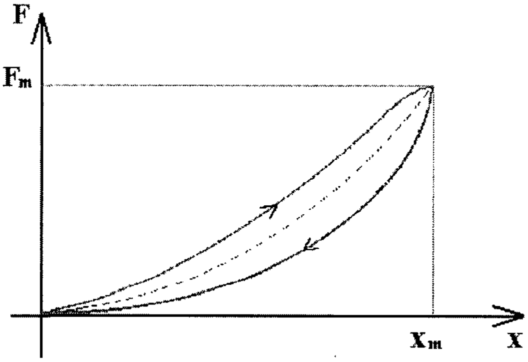
In the contact force models, the simplest one is the Kelvin-Voigt model, which contains a parallel linear spring-damper element. However, Hunt and Crossely (1975) showed that the linear model does not represent the physical nature of energy transfer process. Instead, they used the nonlinear spring element by the Hertz force-displacement law (Timoshenko, 1970). Therefore, such a contact force model is related to the model with parallel nonlinear spring and linear damper elements. The contact force is given by the following equation.

$$F = kx^p + c\dot{x} \tag{1}$$

In Equation (1), x is the penetration depth, k depicts the spring constant which can be obtained from Hertz law, the power p depends on the surface geometry of the impacting bodies and c means damping coefficient. The discrepancy of this model is that the force model has discontinuity at the moment of impact. Just before impact, the contact force is apparently zero. However, the damping force is applied instantaneously due to initial velocity, as shown in Figure 1(a).



(a) Linear damper



(b) Hysteresis damper

Fig. 1 Contact force model with different dampers

Table 1 Parameter estimation results

Parameters	Initial value	Estimated value
$k[\mathrm{N/m^p}]$	1.85E8	3.51E10
p	1.96	2.79
$\mu_d[\mathrm{N \cdot s/m^{p+1}}]$	8.77E7	2.70E10

The contact force model to overcome above these shortcomings was proposed by Lankarani and Nikravesh (1990). This contact force model can be expressed by the following equation.

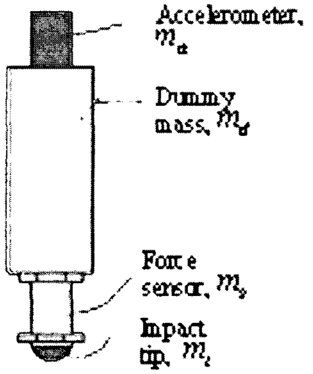
$$F = kx^p + \mu_d x^p \dot{x} \tag{2}$$

where the parameter μ_d is called the hysteric damping coefficient. The force-displacement relation of this model is shown in Figure 1(b). The above Equation (2) is considered as the contact force model in this paper.

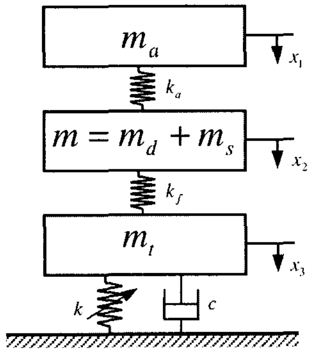
3. Dynamics of Two Colliding Bodies

The force sensor and accelerometer are attached to the impact body as shown in Figure 2(a). Figure 2(b) shows the mathematical model between two colliding bodies.

In Figure 2(b), m_a means the mass on the piezo-material in accelerometer, m is the mass



(a) Impacting body



(b) Mathematical model

Fig. 2 Structure of impacting body and mathematical model

including masses of impact body and force transducer body. Also, m_t represents tip mass attached to the force transducer, which can be regarded as impacting part. k_a and k_f are spring constants of piezo-materials in the accelerometer and force transducer respectively. The dynamic equations of considered system are as follows :

$$m_a\ddot{x}_1 + k_a(x_1 - x_2) = 0 \tag{3}$$

$$m\ddot{x}_2 - k_a(x_1 - x_2) + k_f(x_2 - x_3) = 0 \tag{4}$$

$$m_t\ddot{x}_3 - k_f(x_2 - x_3) + kx_3^p + \mu_d x_3^p \dot{x}_3 = 0 \tag{5}$$

If k_a and k_f are much more than k (k_a, $k_f \gg k$), the above equations are reduced as follows :

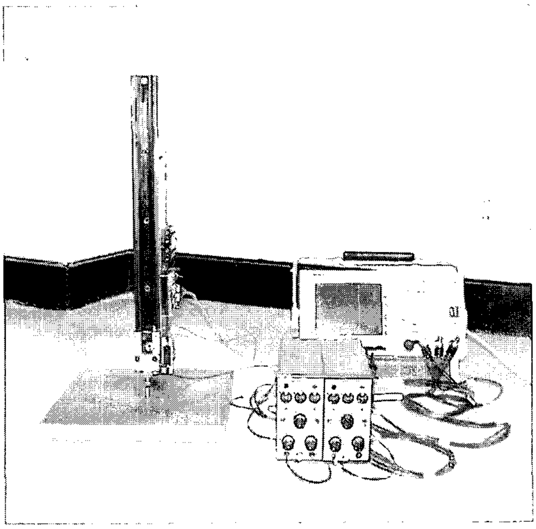
$$M\ddot{x}_3 + kx_3^p + \mu_d x_3^p \dot{x}_3 = 0 \quad (\because\ M = m_a + m + m_t) \tag{6}$$

4. Experimental Setup and Results

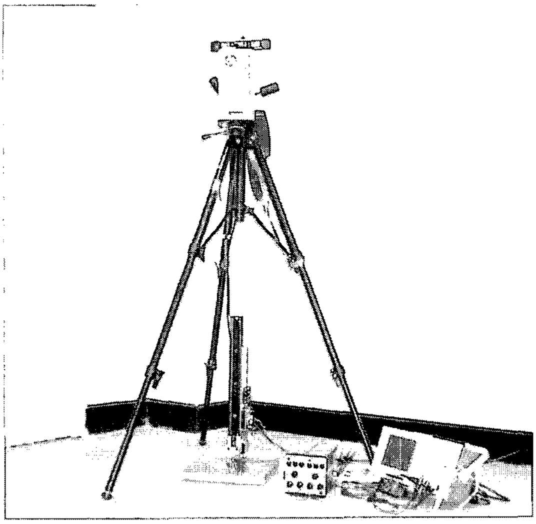
4.1 Experimental setup

Figure 3(a) shows the experimental setup consisting of the force sensor, the accelerometer, and the photo interrupters. The impact body is dropped along the rail with linear bearing. The impacting surface is steel plate and the impacting tip can be changed with various materials, such as rubber, plastic, and steel. In order to change the impact velocity, the drop height is adjusted.

In the impact experimental setup, six photo interrupters in series close to the collision location are installed to measure the velocity before impact more accurately. Here, the interval between the interrupters is about 6 mm. The laser velocity meter is used to validate and calibrate the



(a) Only photo-interrupter (general setup)



(b) Photo-interrupter and laser velocity meter

Fig. 3 Experimental setup

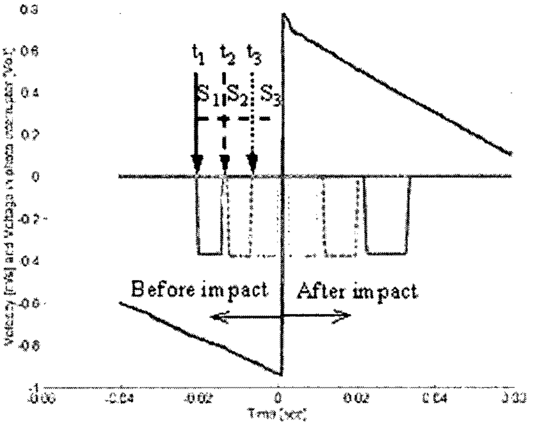


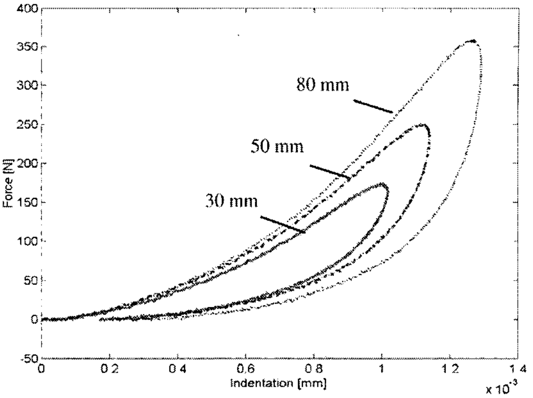
Fig. 4 Velocity curve and voltage output of photo interrupter at 50 mm drop

calculation method of the impact velocity using photo interrupters as shown in Figure 3(b). However, the velocity meter is only used below 60 mm drop because the measurable velocity is up to 1 m/sec.

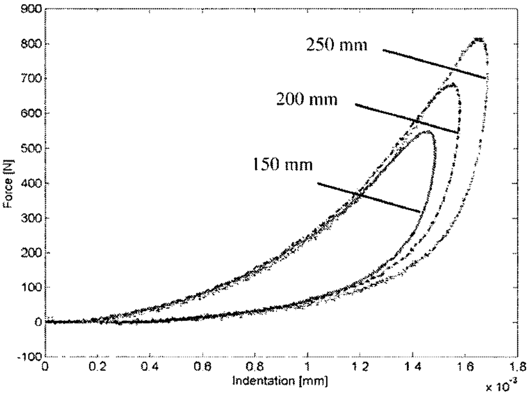
Figure 4 shows the velocity curve and the voltage outputs of three interrupters of six photo interrupters at 50 mm drop. Here, the three interrupters are near to the collision location than three others. Three instant time when the impacting body passes through photo interrupters, the interval between the interrupters, and the distance between last interrupter and collision location are used to calculate the velocity before impact. The calculated velocity from the interrupters is similar to the measured velocity from the laser meter with an allowable error.

4.2 Results in drop tests

Figure 5(a) and (b) show the force-indentation curves for rubber tip with various drop heights. These figures show that the contact force model in the drop of rubber tip is the model with the hysteric damping. Also, it is found that the damping energy, the closed-loop area of the curve, increases if the drop height is high. However, from a number of experiments, we found that the hysteric model cannot be applied in the cases of plastic tip (higher drop) and steel tip. These cases seem to be the plastic deformation model, which is more complex than the hysteric model.



(a) 30, 50, 80 mm drop



(b) 150, 200, 250 mm drop

Fig. 5 Force-indentation curves

4.3 Estimation of impact parameters

In this section, we illustrate both estimation scheme and optimization method for unknown parameter estimation of the contact force model as shown in Figure 6. By adjusting the parameters of the model to make the mismatched output errors between the plant and the model converge to zero, the unknown parameters can be determined according to this scheme (Borgan, 1982). Here, the plant means the force-indentation curve obtained from the drop experiment and the model means the curve calculated from the impact dynamics of Equation (6) using given parameters.

The difficulties in computing the gradients of the objective function, which is a weighted square of the output errors, exist. Therefore, the modified simplex method is selected to overcome the

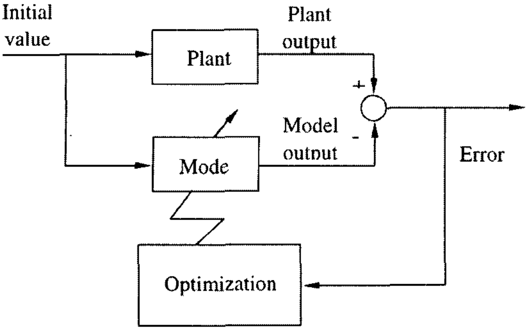


Fig. 6 Parameter estimation scheme

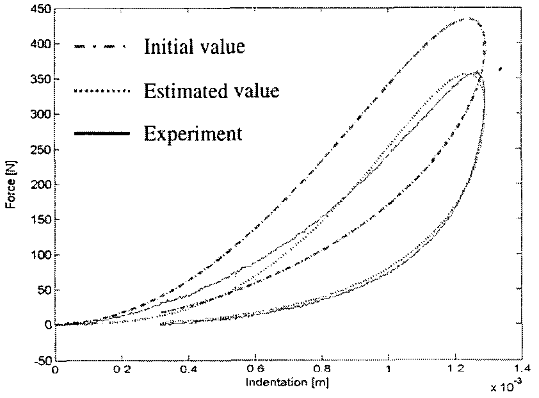


Fig. 7 Initial and final estimation

difficulties. Nelder (1965) modified the simplex method to speed up the rate of convergence and increase the chances for finding a global optimum. This method has proven to be applicable for minimizing any mathematical function with multiple variables.

The modified simplex method is applied to estimate the impact parameters of the contact force model. The result of parameter estimation is shown in Table 1.

Figure 7 shows the force-indentation curves by initial parameters and initial parameters. It can be found that the unknown parameters are successfully estimated. Figure 8 shows the variation of objective function during iteration of optimization process. The response of the model has a good agreement with that of the experiment at the 50th iteration. If a contact force model has more complex parameters than the hysteric model, the estimation scheme could be applied better.

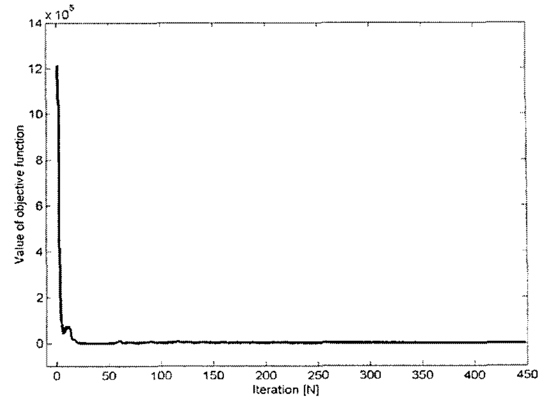
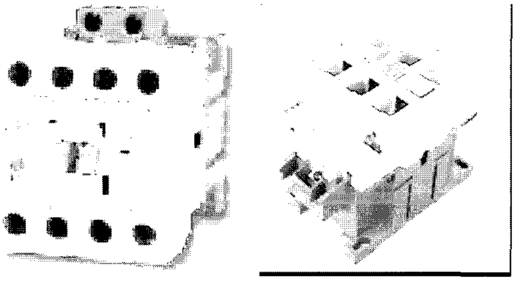


Fig. 8 Variation of objective function

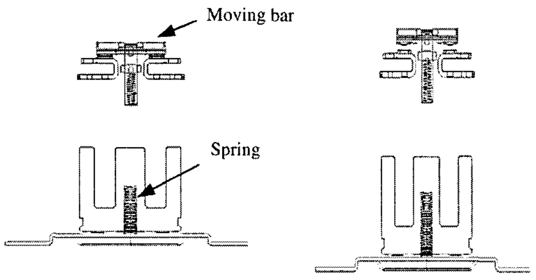
5. Applications of Contact Force Model to Impact for Prediction

Circuit breakers use spring-actuated linkage, which completes switching action within 10 or 20 milliseconds. Therefore, a lot of impacts between components in switch mechanisms occur during closing and opening operation. Such an impact damages to linkage and other component, and so reduces the lifetime of breaker. For good design of circuit breaker, an impact force is predicted and decreased to make the switching operation reliable and stable. In this paper, we apply the force model to the prediction of impact force between two colliding bodies in opening action of the magnetic contactor.

A magnetic contactor is an electro-magnetically operated switch that serves to open and close high-energy electric circuits as shown in Figure 9. In the opening of the contactor, the moving bar with moving electric contact collides with the outer case to be stopped at the end of the opening stroke. Some part of the kinetic energy of the moving bar disappears with energy loss during the impact and the remainder is transmitted to the outer case and moving bar. After the impact, if the amount of the rebounded moving bar is high, the arc between electric contacts occurs because the contacts approach together again. Therefore, such a rebound is the most important characteristics that have much influence on electrical performance in the opening responses. Also, because the



(a) Front view (b) Interior view



(c) On state (d) Off state

Fig. 9 Operation of magnetic contactor

closing and opening action frequently occurs, the impact between the moving bar and the outer case reduces the mechanical lifetime of the contactor. In this paper, in order to design the structure of the outer case impacted by the moving bar and to reduce the rebound, we firstly predict and analyze the impact force in opening action of the magnetic contactor.

The dynamics of the magnetic contactor is analyzed by using a multi-body dynamic program, ADAMS. In order to obtain the impact parameters used in the contact force model between colliding bodies in the opening of the contactor, the drop test at a height of 5 mm using impacting body of 509 gram is carried out. The experimental condition of the drop test is the same condition as the real product (shape, material, impulse).

Figure 10 shows the force-indentation curves at 5 drop test. From Figure 10, it is found that there is no damping during the impact in case of the contactor. Therefore, the impact parameters of the contact force model estimated from Figure 10 are as follows: $k=2270$ N/m, $p=1$, $\mu \cong 0$.

The estimated parameters are directly used in the contact force model of the dynamic model.

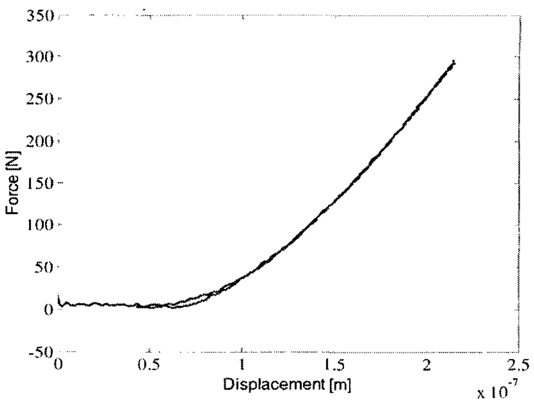


Fig. 10 Force-indentation curves at 5 mm drop test

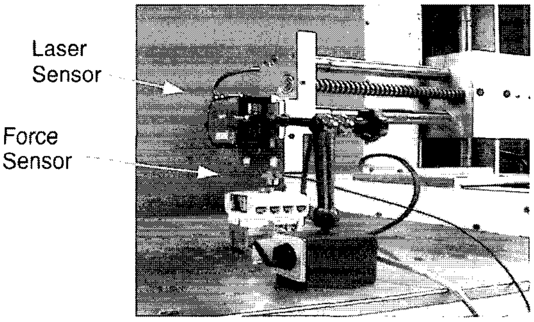


Fig. 11 Impact force measurement in opening of magnetic contactor

Figure 11 shows the experimental setup for measuring the impact force occurred between moving bar and outer case in opening of the magnetic contactor. Also, the displacement of moving bar is measured using the laser sensor. Here, because it is difficult to measure directly the impact force between the moving bar and outer case, the moving bar is impacted with the force sensor tip with the same material and shape as the outer case.

Two colliding bodies of moving bar and outer case is a contact between two locally plane surface and the contact area has a rectangular shape. Therefore, when two plane surfaces are in contact, the contact force law is linear, which is logical since there is no geometric nonlinearity due to curvature. The general equation of the contact force is as follows.

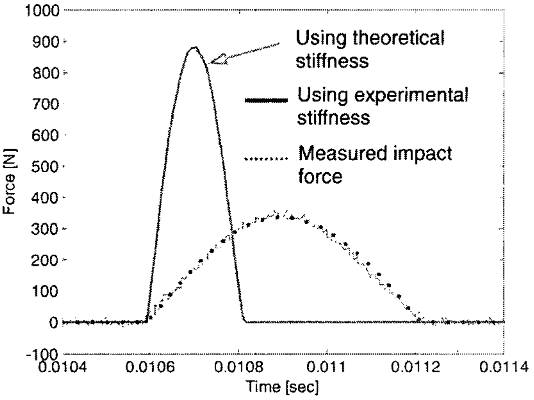


Fig. 12 Comparisons of experiment and simulation in impact force

$$F = kx^p,\ p = 1$$

$$k = \frac{2\sqrt{ab}}{m(h_1 + h_2)},\ h_i = \frac{1 - v_i^2}{E_i} \qquad (7)$$

where the area of the contact surface is and the coefficient is obtained for different ratios of the rectangle sides(Timoshenko, 1970).

From Equation 7, the calculated stiffness of the contact force model is $k = 19300$ N/mm. Because the calculated stiffness is much greater than the stiffness obtained from the drop test, it is expected that the impact force by the calculated stiffness is considerably very high.

Figure 12 shows the comparisons of the measured impact force from the force sensor and the simulated forces by theoretical and experimentally estimated stiffness. These results reveal that the impact parameters of the contact force model must be obtained from the experiment to predict the impact force between colliding bodies.

6. Conclusions

The contact force model has been derived from the experimental data, and we find out that this model gives a good approximation to predict the impact responses of two colliding bodies. In order to measure the velocity before impact in the drop experiment, six photo interrupters in series close to the collision location are installed and calibrated by using the laser velocity meter. The parameters of the contact force model are estimated using the optimization technique. The contact force model obtained by drop test is applied to predict the impact force between the moving bar and the outer case in the magnetic contactor. Furthermore, the impact force by the theoretically calculated stiffness is much greater than that by the stiffness estimated from the drop type impact test.

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