Walking Pattern Generation employing DAE Integration Method

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A stable walking pattern generation method for a biped robot is presented in this paper. In general, the ZMP (zero moment point) equations, which are expressed as differential equations, are solved to obtain a stable walking pattern. However, the number of differential equations is less than that of unknown coordinates in the ZMP equations. It is impossible to integrate the ZMP equations directly since one or more constraint equations are involved in the ZMP equations. To overcome this difficulty, DAE (differential and algebraic equation) solution method is employed. The proposed method has enough flexibility for various kinematic structures. Walking simulation for a virtual biped robot is performed to demonstrate the effectiveness and validity of the proposed method. The method can be applied to the biped robot for stable walking pattern generation.

Key Words : Biped Robot, Stable Walking Pattern, ZMP (Zero Moment Point) Equation, DAE (Differential and Algebraic Equation)

1. Introduction

In the future, humans will coexist with robots in the same society. The biped robot has almost the same mechanisms as a human and is suitable for moving in an environment. Recently many studies have been devoted to not only the development of the biped robot but also the generation of human-like walking motion.

Many studies have been focused on walking pattern synthesis. Zerrugh and Radcliffe (1979) investigated the walking pattern for a biped robot by recording human kinematic data. McGeer (1990) described a natural walking pattern generated by passive interaction of gravity and inertia on a downhill slope. To extend the minimum-energy walking method to level ground and uphill slopes, Channon et al. (1992), Rostami and Bessonnet (1998), and Roussel et al. (1998) proposed methods for gait generation by minimizing the cost function of energy consumption. Silva and Machado (1999), investigated the required actuator power and energy by adjusting walking parameters. Since a biped robot tends to tip over easily, it is necessary to take stability into account when determining a walking pattern. Many researchers have been working on the schemes to stabilize the walking of biped robots. Kajita and Tani (1991) proposed the inverted pendulum mode to generate trajectory. A more complicated but more accurate method was proposed, based upon the ZMP equation, which describes rela-

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tionship between the joint motions and force applied at the ground (Park and Rhee, 1998). Zheng and Shen (1990) proposed a method of gait synthesis for static stability. Chevallereau et al. (1998) discussed dynamic stability when specifying a low energy reference trajectory. To ensure the dynamic stability of a biped robot, Takanishi et al. (1985), Shih et al. (1990), Hirai et al. (1998), and Dasgupta and Nakamura (1999) proposed methods of walking pattern synthesis based on ZMP (Vukobratovic and Juricic, 1969). Huang et al. (1999) proposed a method to plan a walking pattern consisting of a foot trajectory and a hip trajectory. They formulated the constraints of a foot trajectory and generate foot and hip trajectories with a high stability margin. Basically, they tried to design a desired ZMP trajectory first, then derive the hip motion or torso motion required to achieve that ZMP trajectory. The conventional methods require the iterative calculations to obtain the stable walking pattern and therefore, they cannot be applied for robots with various kinematic structure.

This paper presents a new method for the stable walking pattern generation. ZMP equation and constraints are solved simultaneously to avoid iterative calculations such as FFT method. The proposed method has enough flexibility for various kinematic structures. To verify the effectiveness and validity of the proposed method, stable walking pattern for a biped robot is generated and walking simulation is performed for the given walking pattern.

2. ZMP Equation

The ZMP is defined as the point on the ground about which the sum of all the moment of active forces is equal to zero. If the ZMP is inside the contact polygon between the foot and the ground, the biped robot is stable. This contact polygon is called the stable region. Figure 1 shows the reference frames and ZMP. In the figure, XYZ is global reference frame and $x_i y_i z_i$ is body fixed reference frame on the i-th body. $\mathbf{r}_i$ is a position vector to the origin of $x_i y_i z_i$ and $\mathbf{r}_{ZMP}$ is a position vector to the p_{ZMP}.

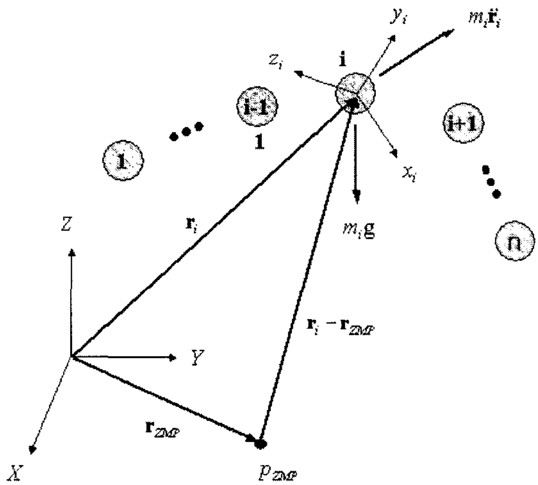

Fig. 1 The reference frame and the ZMP

Suppose that the ground applies a pure force to the biped robot at a location p_{ZMP}, called the ZMP. Then the dynamics of the biped robot becomes (Park and Rhee, 1998).

$$\sum_{i=1}^{n} m_i(\tilde{\mathbf{r}}_i - \tilde{\mathbf{r}}_{ZMP})(\ddot{\mathbf{r}}_i + \mathbf{g}) = 0 \quad (1)$$

In the above equation, n is the number of bodies and the tilde over a vector denotes a skew symmetric matrix of the vector. Considering the dynamics in the Z-direction only under the assumption of no external moments, Eq. (1) can be expressed as

$$X_{ZMP} = \frac{\sum_{i=1}^{n} m_i(\ddot{y}_i + g_y)x_i - \sum_{i=1}^{n} m_i(\ddot{x}_i + g_x)y_i}{\sum_{i=1}^{n} m_i(\ddot{y}_i + g_y)} \quad (2)$$

or

$$\mathbf{a}^T\dot{\mathbf{Y}} = \sum_{i=1}^{n}\mathbf{a}_i^T\dot{\mathbf{Y}}_i = r_X$$

where

$$\dot{\mathbf{Y}}_i = [\ddot{x}_i \ \ddot{y}_i \ \dot{\omega}_{iz}]^T \quad (3)$$

$$\mathbf{a}_i = [m_i y_i \ m_i(X_{ZMP} - x_i) \ 0]^T \quad (4)$$

$$r_X = \sum_{i=1}^{n} m_i g_z (X_{ZMP} - x_i) + \sum_{i=1}^{n} m_i g_x y_i \quad (5)$$

Figure 2 shows a 2 dimensional biped robot to be considered here. The bodies are connected by several kinematic joint as shown in the figure. Since kinematic joint imposes a certain conditions

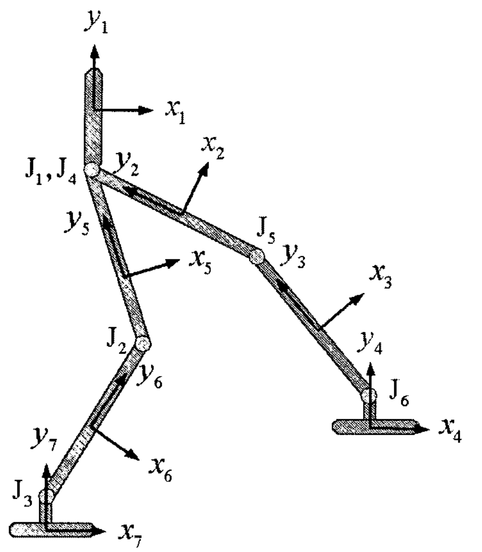

Fig. 2 Biped robot with 6 DOF

on the relative motion between the two adjacent bodies, it becomes clear that all the Cartesian coordinates are not independent.

3. DAE Integration Method

Constraints which kinematic joints impose can be expressed as (Nikravesh, 1988)

$$\Phi^K(\mathbf{Z})=\mathbf{0} \tag{6}$$

where

$$\mathbf{Z}\equiv[\mathbf{Z}_1^T\ \mathbf{Z}_2^T \cdots \mathbf{Z}_n^T]^T \tag{7}$$

$$\mathbf{Z}_i=[x_i\ y_i\ \theta_i]^T \tag{8}$$

A prescribed motion for one or more generalized coordinates can be defined to move the biped robot as in desired pattern. This motion is called as driving constraint. For example, the driving constraint for x coordinate of the 4-th body can be expressed as

$$\Phi^M=x_4-f(t) \tag{9}$$

where $f(t)$ is prescribed motion which is the function of time.

For constrained mechanical systems, constraints of dimension m, their time derivatives, and second derivatives are expressed as follows

$$\Phi=\begin{bmatrix}\Phi^K\\ \Phi^M\end{bmatrix}=\mathbf{0} \tag{10}$$

$$\dot{\Phi}=\Phi_Z\mathbf{Y}-\Phi_t=\mathbf{0} \tag{11}$$

$$\ddot{\Phi}=\Phi_Z\dot{\mathbf{Y}}-\gamma=\mathbf{0} \tag{12}$$

where

$$\gamma=-(\Phi_Z\mathbf{Y})_Z\mathbf{Y}-2\Phi_{Zt}\mathbf{Y}-\Phi_{tt} \tag{13}$$

Generalized coordinates partitioning method (Haug and Yen, 1992; abbreviated hereafter GCP method) is employed to solve ZMP equation in Eq. (2). GCP method is widely used to solve equations of motion of constrained multibody systems.

The generalized coordinates $\mathbf{Z}$ of dimension n are partitioned into independent coordinates $\mathbf{Z}_i$ of dimension $n-m$ and dependent coordinates $\mathbf{Z}_d$ of dimension m. So the generalized coordinates can be written as

$$\mathbf{Z}=[\mathbf{Z}_i^T\ \mathbf{Z}_d^T]^T \tag{14}$$

Now, if $\mathbf{Z}_i$ are given, $\mathbf{Z}_d$ can be obtained by the iterative procedure that employs the following matrix equation.

$$\begin{bmatrix}\Phi_{Z_d} & \Phi_{Z_i}\\ \mathbf{0} & \mathbf{I}\end{bmatrix}\Delta\mathbf{Z}=-\begin{bmatrix}\Phi\\ \mathbf{0}\end{bmatrix} \tag{15}$$

Using the above equation, the improved solution for the next iteration can be obtained as

$$\mathbf{Z}=\mathbf{Z}+\Delta\mathbf{Z} \tag{16}$$

Using Eqs. (15) and (16), the iteration continues until the solution variance remains within specified allowable error tolerance. The procedure so far mentioned is usually called the position analysis. With the position analysis solution found, the velocity analysis solution can be obtained by solving the following velocity constraint equations.

$$\begin{bmatrix}\Phi_{Z_d} & \Phi_{Z_i}\\ \mathbf{0} & \mathbf{I}\end{bmatrix}\mathbf{Y}=\begin{bmatrix}-\Phi_t\\ \mathbf{Y}_i\end{bmatrix} \tag{17}$$

The solution of Eq. (17) can be obtained without the iterative procedure which is used in the position analysis. With the position and velocity analysis solutions, Eqs. (2)-(12) are combined in the following matrix form

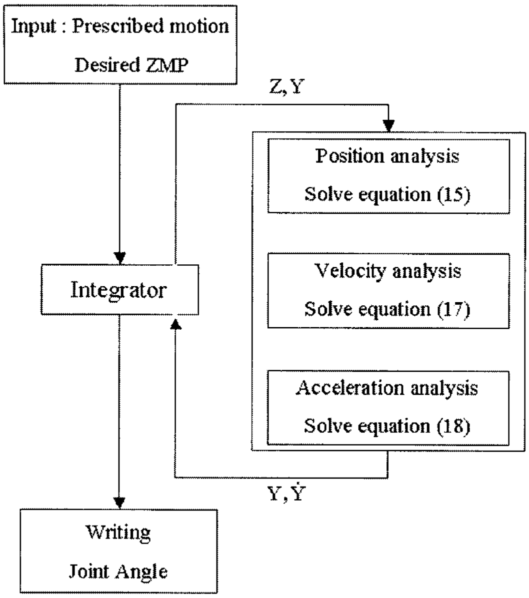

Fig. 3 Flowchart for stable walking pattern generation

$$\begin{bmatrix} \mathbf{a} \\ \Phi_Z \end{bmatrix} \dot{\mathbf{Y}} = \begin{bmatrix} r_X \\ \gamma \end{bmatrix} \tag{18}$$

Figure 3 shows the flowchart for stable walking pattern generation.

4. Initial Condition for Initial Standing State

In the initial standing state, it is necessary to set velocity and acceleration values as zeroes. Zero initial velocity can be imposed through the initial values for the numerical integration. However, zero acceleration cannot satisfy the ZMP equation for a given ZMP. In this section, numerical method to impose zero acceleration is presented.

Acceleration is set to zero in Eq. (2). Then, the equation can be re-written as

$$f_{ZMP} = -X_{ZMP}\sum_{i=1}^{n} m_i g_y + \sum_{i=1}^{n} m_i g_y x_i - \sum_{i=1}^{n} m_i g_x y_i = 0 \tag{19}$$

To obtain initial condition with zero velocity and acceleration, Eqs. (10)-(19) should be solved simultaneously. Combining these equations gives the following equation.

$$\mathbf{R} = \begin{bmatrix} f_{ZMP} \\ \Phi \end{bmatrix} \tag{20}$$

where $\mathbf{R}$ is a residual vector. To solve the above equation, Newton-Raphson method can be used

$$\begin{aligned} \mathbf{R}_Z \Delta \mathbf{Z} &= -\mathbf{R} \\ \mathbf{Z} &= \mathbf{Z} + \Delta \mathbf{Z} \end{aligned} \tag{21}$$

where

$$\mathbf{R}_Z = \begin{bmatrix} (f_{ZMP})_Z \\ \Phi_Z \end{bmatrix} \tag{22}$$

$$(f_{ZMP})_{Z_i} = [m_i g_y \ \ -m_i g_x \ \ 0] \tag{23}$$

5. Numerical Example

Numerical simulation was performed for the biped robot shown in Fig. 2. DE integrator (Shampine, 1975) was used for Numerical integration. The biped robot used in the simulation is 1.05 m tall and weight 36 kg. Mass, mass moment of inertia, and length of each body are presented in Table 1. x_1 for the trunk is selected as the independent coordinate. Figure 4 shows the

Table 1 Length and inertia properties of the biped robot

	Length (m)	Mass (kg)	Inertia (kg·m^2)
body 1	0.2	10	10^{-4}
body 2	0.4	5	10^{-4}
body 3	0.4	5	10^{-4}
body 4	0.05	3	10^{-4}
body 5	0.4	5	10^{-4}
body 6	0.4	5	10^{-4}
body 7	0.05	3	10^{-4}

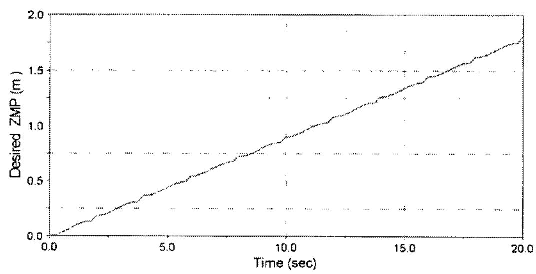

Fig. 4 Desired ZMP trajectory

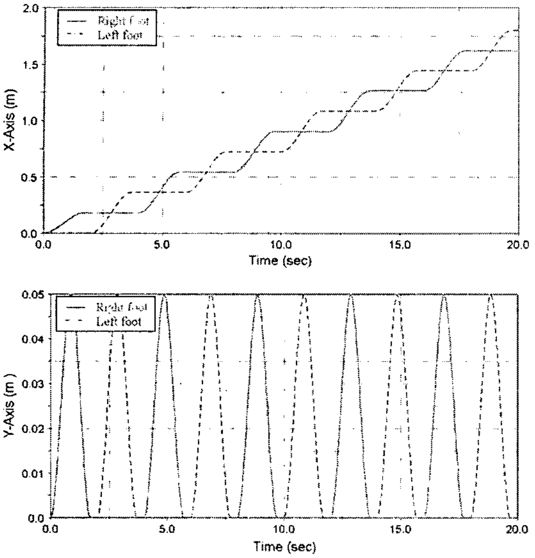

Fig. 5 Prescribed motions

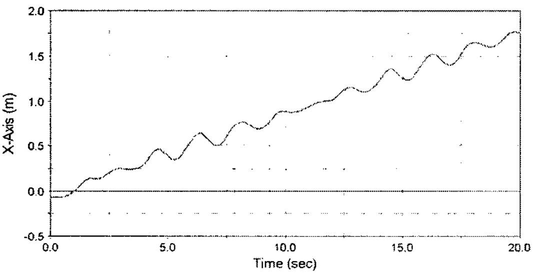

Fig. 6 x_1 of the hip

trajectory of prescribed desired ZMP. Figure 5 shows the prescribed motions of the hip and feet in direction of X and Y. Height of the hip was fixed at $y_1=0.843$ m, pitch angles of the hip and feet were fixed at $\theta_1=\theta_4=\theta_7=0.0$ rad, and step length was fixed at 0.36 m. Figure 6 shows the analysis result which is the x_1 of the hip to satisfy the ZMP equation in a given desired ZMP. To verify the effectiveness of the proposed method, the model of biped robot was constructed on commercial program ADAMS. In the model, contact between the ground and feet were considered. Trajectories of joints, which are obtained from the proposed method, were imposed to the relative joint motions on ADAMS. Figure 7 shows that the robot walks stably without tipping over. From the analysis results, it is known that the

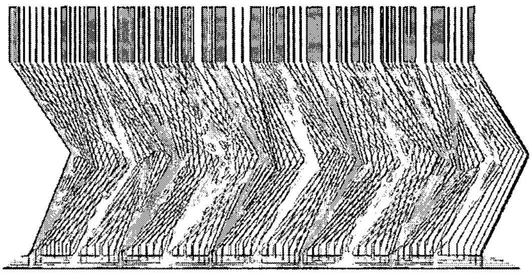

Fig. 7 Stick diagram of the biped walking

proposed method can be applied to the stable walking pattern generation effectively.

6. Conclusion

In this paper, a new method for the stable walking pattern generation is proposed. DAE solution method is employed to solve ZMP equation. That is, ZMP equation and constraints are solved simultaneously. The proposed method does not require the iterative calculations such as FFT method. This method has enough flexibility to be applied for robots with various kinematic structure and a lot of DOF and for various types of motion. Stable walking pattern for the two dimensional biped robot is generated and walking simulation for a virtual biped robot is performed to demonstrate the effectiveness and validity of the proposed method.

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Appendix : Constraints for a Revolute Joint

Figure A1 shows the revolute joint which connect adjacent bodies i and j. In the figure, a point p is common to the bodies. Constraint equations for revolute joint (2D) can be written as

$$\Phi^K = \mathbf{r}_j + \mathbf{A}_j \mathbf{s}'_j - \mathbf{r}_i - \mathbf{A}_i \mathbf{s}'_i \quad \text{(A1)}$$

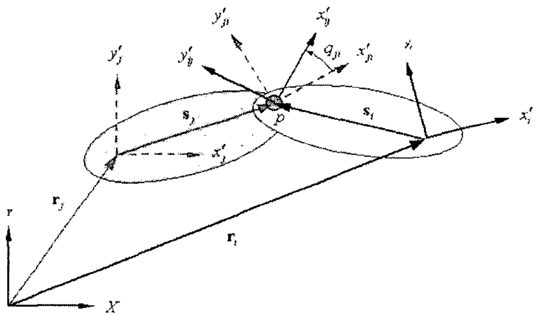

Fig. A1 Revolute joint between body j and i

where

$$\mathbf{A}_i=\begin{bmatrix}\cos(\theta_i) & -\sin(\theta_i)\\ \sin(\theta_i) & \cos(\theta_i)\end{bmatrix} \quad (A2)$$

Time derivative of Eq. (A1) gives the following velocity constraints.

$$\dot{\Phi}^K=\Phi^K_{Z_i}\mathbf{Y}_i+\Phi^K_{Z_j}\mathbf{Y}_j \quad (A3)$$

where

$$\Phi^K_{Z_i}=[-\mathbf{I}\ \ -\mathbf{D}_i\mathbf{s}'] \quad (A4)$$

$$\Phi^K_{Z_j}=[\mathbf{I}\ \ \mathbf{D}_j\mathbf{s}'_j] \quad (A5)$$

$$\mathbf{D}_i=\begin{bmatrix}-\sin(\theta_i) & -\cos(\theta_i)\\ \cos(\theta_i) & -\sin(\theta_i)\end{bmatrix} \quad (A6)$$

Similarly, acceleration constraints are obtained by taking time differentiation of Eq. (A3).

$$\ddot{\Phi}^K=\Phi^K_{Z_i}\dot{\mathbf{Y}}_i+\Phi^K_{Z_j}\dot{\mathbf{Y}}_j-\boldsymbol{\gamma}^K \quad (A7)$$

where

$$\boldsymbol{\gamma}^K=\mathbf{A}_j\mathbf{s}'_j\omega_j^2-\mathbf{A}_i\mathbf{s}'_i\omega_i^2 \quad (A8)$$