Model-based Reference Trajectory Generation for Tip-based Learning Controller

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The non-minimum phase characteristic of a flexible manipulator makes tracking control of its tip difficult. The level of the tip tracking performance of a flexible manipulator is significantly affected by the characteristics of the tip reference trajectory as well as the characteristics of the flexible manipulator system. This paper addresses the question of how to best specify a reference trajectory for the tip of a flexible manipulator to follow in order to achieve the objectives of reducing: tip tracking error, residual tip vibration, and the required actuation effort at the manipulator joint. A novel method of tip-based learning controller for the flexible manipulator system is proposed in the paper, where a model of the flexible manipulator system with a command shaping filter is used to generate a smooth and realizable tip reference trajectory for a tip-based learning controller.

Key Words: Command Shaping, Tip Control, Flexible Link, Learning Control

1. Introduction

The common difficulty reported when using various control schemes to achieve fast yet accurate tip positioning of a flexible manipulator is the presence of the motion-induced residual vibrations in the manipulator. In a typical configuration the manipulator is actuated at the end of the link opposite the tip (i.e. at the "base" or joint end) and what results - in the case where accurate positioning of the tip is of interest - is a non-collocated control problem. The base and tip motions in a flexible manipulator are typically out of phase and may oppose one another: given a desired reference trajectory for the tip to follow, the base motion required to achieve it will generally not follow that same reference trajectory, especially in the case of high-speed, high-acceleration reference motions. To achieve more accurate and more direct position control of the tip where the most of the useful work is done a tip-based-control approach seems more feasible than a joint-based-control approach. A tip-based-controller obviously requires a reference for the tip to be regulated based upon and the question is what to use. For the tip tracking problem, many researchers focused on the development of control method design assuming given random smooth reference input. The level of the tip tracking performance of a flexible manipulator is significantly affected by the characteristics of the tip reference trajectory as well as the characteristics of the flexible manipulator system. Manipulator reference trajectories are typically designed to optimize the dynamics capabilities of the actuator at the base, in which case their use as the tip reference trajectory might be physically unrealizable. Given a periodic desired reference trajectory to

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(Manuscript **Received** November 29, 2004; **Revised** December 15, 2004)

be tracked by the tip, the controller in (Hu, 2000) used repetitive learning control to determine the corresponding base motion required. Though effective in the objective of attenuating tip tracking error, the method required a relatively large, sometimes prohibitive, control effort at the joint actuator (needed in order to cancel out vibrations at the manipulator tip) and was also relatively ineffective at suppressing residual tip vibration. By properly designing the tip reference trajectory, it is possible to overcome these two undesirable effects. This paper addresses the question of how to best specify a reference trajectory for the tip of a flexible manipulator and proposes a novel configuration of tip-controller that utilizes a model-based tip reference trajectory generation method in order to achieve the objectives of reducing tip tracking error, reducing residual tip vibration, and reducing the required actuation effort (or motion) at the base. In our new tip-based controller, a model of the flexible manipulator system is used to generate a smooth and realizable tip reference trajectory which is to be tracked by the tip of the actual system. With a properly designed command shaping filter (Singer, 1990) implemented in the model the tip response of the model is free of residual vibration. Then a multirate repetitive learning controller (MRLC) is used to ensure that the tip response of the actual system follows the smooth and vibration-free tip response of the model. In the following sections of the paper, the dynamics of the flexible manipulator system is described first and the description of the model-based reference trajectory generation method using a command shaping filter follows. Then follows the proposed flexible manipulator tip controller based on repetitive learning control and the use of the tip reference trajectory generated from the model. Simulation results utilizing the proposed tip-based controller are subsequently presented and discussed.

2. Dynamics of Flexible Manipulator

The flexible manipulator system of our interest in this paper is shown Fig. 1. It is assumed that the system possesses a single dominant elastic mode. Such systems may be modeled as a system of two masses coupled by a linear spring and a linear damper acting in parallel, as shown in Fig. 2. The mass m_1 is located at the actuator (which applies to it a control force u) and represents the base of the manipulator. The mass m_2 represents the manipulator tip. The flexibility of the manipulator is modeled with the spring (k) and damper (b, usually a relatively small quantity). The displacement of the base mass and the tip mass, each measured from a fixed position for which the spring is unstretched, are denoted by x_{base} and x_{tip}. Their respective velocities are $\dot{x}_{base}$ and $\dot{x}_{tip}$, where $\cdot$ denotes differentiation with respect to time, t. The control force u is used to drive the base mass in such a way that results in the tip mass precisely tracking a desired reference trajectory. As a result, for later use in the control method formulation, as well as in subsequent numerical simulations, it is necessary to determine the (linear) dynamics between u and x_{tip}. The following two transfer functions are computed for this purpose :

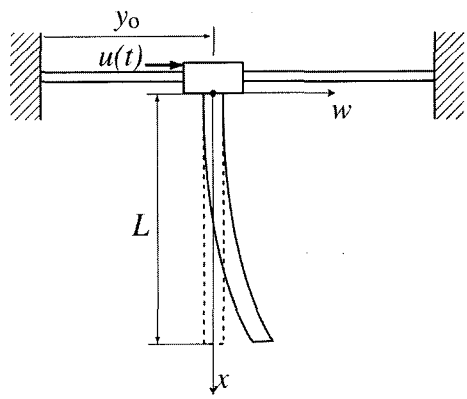


Fig. 1 Gantry type flexible manipulator system

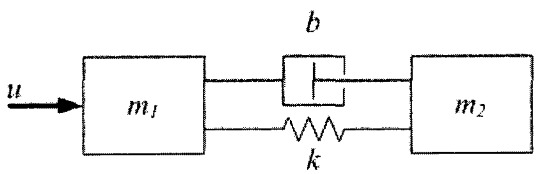


Fig. 2 A mass-spring-damper model of flexible link

$$G_1(s) = X_{base}(s)/U(s) \quad (1)$$

$$G_2(s) = X_{tip}(s)/X_{base}(s) \quad (2)$$

where s is the Laplace variable. The equations of motion in state-space form for the two-mass system shown in Fig. 2 are given by

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{b}u \quad (3)$$

where $\mathbf{x} = [x_{base}\ x_{tip}\ \dot{x}_{base}\ \dot{x}_{tip}]^T$, $\mathbf{b} = [0\ 0\ 1/m_1\ 0]^T$, and

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -k/m_1 & k/m_1 & -b/m_1 & b/m_1 \\ k/m_2 & -k/m_2 & b/m_2 & -b/m_2 \end{bmatrix} \quad (4)$$

Taking the input to be u and the output to be x_{base}, it is found that the transfer function $G_1(s) = X_{base}(s)/U(s)$ is given by

$$G_1(s) = \frac{s^2 + (b/m_2)s + k/m_2}{m_1(s^4 + (b/m_1 + b/m_2)s^3 + (k/m_1 + k/m_2)s^2)} \quad (5)$$

The second transfer function is obtained by considering the base mass position as the input and the tip mass position as the output. The resulting equation of motion is given by

$$m_2\ddot{x}_{tip} = k(x_{base} - x_{tip}) + b(\dot{x}_{base} - \dot{x}_{tip}) \quad (6)$$

The corresponding transfer $G_2(s)$ is

$$G_2(s) = \frac{(b/m_2)s + k/m_2}{s^2 + (b/m_2)s + k/m_2} \quad (7)$$

The control formulation and numerical simulations appearing in the following sections of this paper will take place in the discrete-time domain, since the future plan is to implement the developed control method digitally on an experimental test bed manipulator. Therefore, discrete-time equivalents of the transfer functions and $G_1(s)$ and $G_2(s)$ will actually be used.

3. Control Formulation

3.1 Controller configuration

The discrete-time block diagram representing the conventional joint-based control system for the flexible manipulator is shown Fig. 3, where transfer functions $G_1(z)$ and $G_2(z)$ represent an actual flexible manipulator system whose transfer functions are calculated in the previous section. A standard PD feedback controller, $D(z)$, is used to apply the required control force to the base (joint) mass. Conventionally $D(z)$ is modeled as a PD controller with an added one-step time delay:

$$D(z) = (K_p + K_d(1 - z^{-1})/T)z^{-1} \quad (8)$$

where T is the discrete-time sample period and K_p and K_d are constant control gains. The discrete-time block diagram representing the control system proposed in this paper is shown in Fig. 4. The system is comprised of the flexible manipulator tip controller and the tip reference trajectory generation part including the model of the manipulator. In Fig. 4, $\hat{G}_1(z)$ and $\hat{G}_2$ represent a model of $G_1(z)$ and $G_2(z)$. $C(z)$ represents a command shaping filter. Command shaping, a feedforward approach, seeks to reduce tip vibrations by reshaping the desired trajectory to be tracked by the joint in order to produce a new trajectory that will not excite the resonances of the flexible manipulator. Thus $\hat{x}_{tip}$, the calculated tip response of a model system with a well-designed filter $C(z)$, should show no residual vibration. To apply more direct regulation on the motion of the tip, we suggest the use of $\hat{x}_{tip}$ as a reference trajectory for the tip-control. We know that $\hat{x}_{tip}$ is nearly guaranteed to be

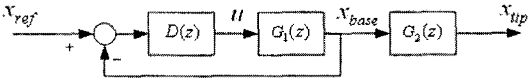


Fig. 3 Block diagram of conventional joint control system

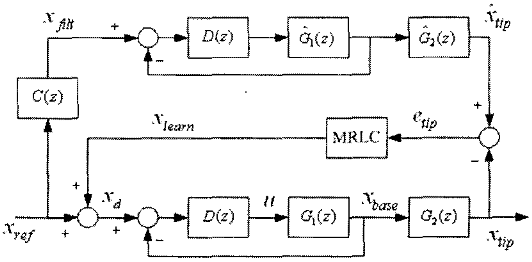


Fig. 4 Block diagram of proposed control system

physically realizable in an actual system. The particular tip controller we will use is a multi-rate repetitive learning controller (MRLC), which is to modify the motion trajectory the base mass of the actual system is commanded to follow, x_d. A repetitive learning controller estimates, through practice, the control input required to theoretically achieve perfect tracking of a periodic desired trajectory. The output of a tip-based repetitive learning controller x_{learn} in the block diagram is determined using the tip tracking error, e_{tip} as input and is used to form a re-shaped reference input x_d for the joint controller to follow. The gradual modification of the periodic reference trajectory obtained by adding a progressively-refined x_{learn} term to it forms an x_d that, when rigorously followed by the base mass, leads to e_{tip} approaching zero. In Fig. 4, a PD controller has input $e_{base}=x_d-x_{base}$ and output u that actuates the joint. Details of command shaping and MRLC are demonstrated in the following sub-sections.

3.2 Command shaping filter

A command shaper reshapes the desired input to a flexible system such that the resonances of the elastic system modes are not excited. It takes the form of a finite impulse response (FIR) filter, with filter parameters determined by the resonant frequencies and the damping ratios of the undesired elastic modes of the flexible system.

The control system in this paper uses a particular command shaping technique called the optimal arbitrary time-delay filter (OATF). For single elastic mode cancellation, the three-term OATF is given by the following equation:

$$c(t)=\frac{1}{M}\{\delta(t)-2\cos(\omega_d T_d)e^{-\varsigma\omega_n T_d}\delta(t-T_d)+e^{-2\varsigma\omega_n T_d}\delta(t-2T_d)\} \quad (9)$$

where T_d is the time delay, δ is the unit impulse function centered at $t=0$, ω_n is the first natural frequency of the flexible system, ς is the corresponding damping ratio, ω_d is the corresponding damped natural frequency, and $M=1-2\cos(\omega_d T_d)e^{-\varsigma\omega_n T_d}+e^{-2\varsigma\omega_n T_d}$. It cancels the poles of the flexible system with filter zeros (Magee, 1998). To prevent potential problems that may be caused by large-magnitude impulses (for example, high required accelerations or actuator saturation), a positive impulse constraint that allows only positive coefficients in the filter can be applied in designing a command shaper (Singhose, 1999). The proposed control scheme in this paper uses a reference trajectory that is filtered by a positive impulse command shaping filter as the reference signal for the manipulator tip controller.

3.3 Multirate repetitive learning controller

The various components of the MRLC are shown in Fig. 5. Because the MRLC is designed to operate at a slower rate (slower by an integer multiple, m) than the rest of the control system, signals leading into and leaving it must be, respectively, down-sampled and p-sampled. In Fig. 5, $A(z)$ is a linear weighted averaging filter, $I(z)$ is a linear interpolation (up-sampling) filter, and $\{\}^*$ denotes down-sampling (Rhim, 2001; Sadegh et al., 2002). The so-called Q-filter, $Q^*(z)$, and the constant learning gain, K_L are to be designed such that necessary and sufficient MRLC stability conditions are satisfied. Refer to (Sadegh, 2002) for details. The time-delay z^{-g} in the block diagram serves to make $Q^*(z)z^{-g}$ causal, and therefore implementable, in the event that $Q^*(z)$ is improper (i.e., has relative degree less than zero): g is defined as $g=-1\times\min\{0, \textit{relative degree of } Q^*(z)\}$.

The term $R^*(z)$ is the N-step delay positive feedback loop given by

$$R^*(z)=\frac{1}{z^{N-1}} \quad (10)$$

where integer N is equal to a value such that the product NmT equals the period of the desired reference trajectory x_{ref}. (For example, for a trajectory of period equal to 1 sec, T=0.001 s,

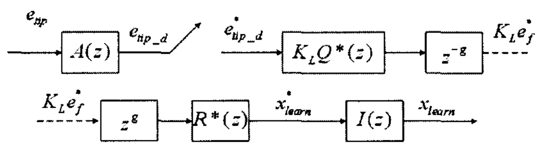


Fig. 5 MRLC components

and $m=5$, the resulting $N=200$.) The expression in Eq. (10) may be equivalently expressed as

$$R^*(z)=\sum_{i=0}^{N-1}\frac{c_i}{z-a_i} \tag{11}$$

where the c_i are undetermined coefficient and

$$a_i=\exp\left(\frac{2\pi j}{N}i\right) \tag{12}$$

are pole locations. a_0 is the fundamental frequency and the N-1 integer multiples of the fundamental frequency are the harmonic frequencies. The compensating time-advance term z^g, which cancels z^g is "absorbed" into the update term $R^*(z)$. As a result, the output of the MRLC, before final up-sampling, is given by

$$x^*_{learn}(K)=x^*_{learn}(K-N)+K_L e^*_f(K-N+g) \tag{13}$$

where K is used to denote the down-sampled sample index. From Eq. (13), the MRLC formulates its current output by adding an update term $K_L e^*_f(K-N+g)$ to the value its output had one full period ago ($x^*_{learn}(K-N)$).

4. Simulation Results

To demonstrate the improved behavior of tip positioning using the model-based tip reference generation approach, a series of numerical simulations have been performed with the simplified model of a flexible manipulator system possessing a single dominant elastic mode shown in Fig. 2 and described in section 2. Section 4.1 describes the system parameters used in the simulations and section 4.2 presents the simulation results.

4.1 Configuration

The system parameters used in the numerical simulations are: base mass $m_1=8$ kg, tip mass $m_2=2$ kg, damping ratio $\varsigma=b(2m_2\omega_n)=0.005$, and natural frequency of the elastic mode $\omega_n=\sqrt{k/m_2}=62.96$ rad/s. (Note that the spring constant k and the damping b may be determined from the above system parameters.) The numerical simulations take place in the discrete-time domain and the sample period used is 0.001 sec. Discrete-time equivalents of the plant transfer functions $G_1(s)$ and $G_2(s)$ are obtained assuming a zero-order hold function on their inputs. The control system considered has four components: a model of the flexible manipulator, an OATF command shaping filter designed for the model of the flexible manipulator, a PD feedback controller, and a MRLC. In the simulation 5% mismatch in the frequencies of the model and the actual system is enforced ($\hat{\omega}_n=1.05\omega_n$). Details of the up-sampling and down-sampling operations used in MRLC design appear in (Sadegh et al., 2002), along with the two necessary and sufficient conditions for asymptotic stability. The stability conditions for the MRLC are satisfied by proper design of the Q-filter $Q^*(z)$ and the learning gain K_L. Using a down-sampling ratio of $m=5$, an acceptable Q-filter for the flexible manipulator system model considered may be designed as the following zero-phase error tracking controller (Tomizuka, 1987):

$$Q^*(z)=\frac{\sum_{i=0}^{11} g_i z^i}{\sum_{j=1}^{8} h_j z^j} \tag{14}$$

with $g_{11}=1$, $g_{10}=-2.0032$, $g_9=-0.4426$, $g_8=3.1138$, $g_7=-0.8413$, $g_6=-1.6827$, $g_5=0.6574$, $g_4=0.3575$, $g_3=-0.1141$, $g_2=-0.01452$, $g_1=-1.0094\times10^{-4}$, $g_0=-9.5941\times10^{-8}$, $h_8=0.2642$, $h_7=-0.3489$, $h_6=-0.1004$, $h_5=0.2601$, $h_4=-0.01197$, $h_3=-0.03133$, $h_2=-0.001021$, and $h_1=-5.0359\times10^{-6}$. Observe that the Q-filter defined by Eq. (14) is non-causal, with relative degree equal to (-3). Therefore, we require the time advance g in Eq. (13) to be equal to 3. Along with the above Q-filter, selection of $K_L=0.0001$ may be shown to meet both the MRLC stability conditions. To avoid potential problems that may be introduced by a negative impulse command shaping filter, positive impulse command shaping filters have been used in the simulations. Included in the paper are only simulation results for a positive impulse command shaping filter with a time delay of $T_d=0.025$ sec. Different positive impulse filters have been observed to exhibit very similar responses. Lastly, to simulate a more realistic control environment, noise has been modeled at the base actuator and at the tip position measurement.

4.2 Results

The periodic joint reference trajectories used in the numerical simulations appear in Fig. 6. Shown are both the unfiltered reference trajectory x_{ref} and the corresponding filtered trajectory x_{filt}. x_{ref} is an offset sinusoid with frequency 5π rad/sec alternating with zero-velocity segments. The amplitude of displacement is 0.050 m and the period is 1 sec. Fig. 7 shows the tip responses of the actual system and the model. In plot (a), the dotted line represents the tip response in the model and the effect of the filtered input is evident. The filter is able to successfully suppress the residual tip vibrations in the model. The resulting tip mass motion in the model is relatively vibration-free. Plots (b) and (c) show the transition of the tip response in the actual system and the tip tracking error defined as the difference between the tip response of the model and the tip response of the actual system decreases. Fig. 8 shows the output of the MRLC, x_{learn} and the modified joint reference input x_d. x_{learn} is calculated based on the periodic tip tracking error and grows to drive the tip tracking error to zero. x_d which is the sum of x_{ref} and x_{learn} is fed to the joint controller for the actual system to regulate the motion of the base. The modification of the commanded base motion is for the purpose of getting the tip mass to move as desired. The base is able to influence the motion of the tip through the action-reaction forces they exert on one another through their flexible coupling. Fig. 9 shows the transition of the difference between x_d and x_{filt}. As the learning controller learns the joint reference trajectory that does not excite the vibration of the flexible link, the difference between the filtered reference x_{filt} and the learned joint reference input x_d moves toward zero. That is x_d converges to x_{filt}. Fig. 10 shows the control effort exerted by the PD controller on the base mass and the magnitude of the maximum required force is in a realistic range.

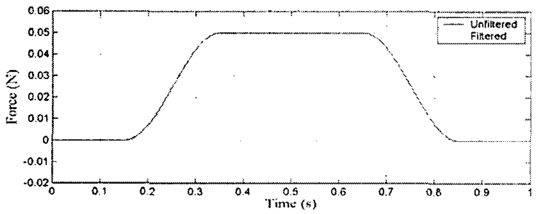
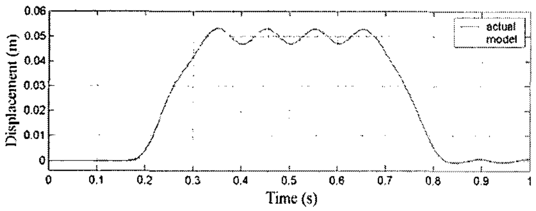
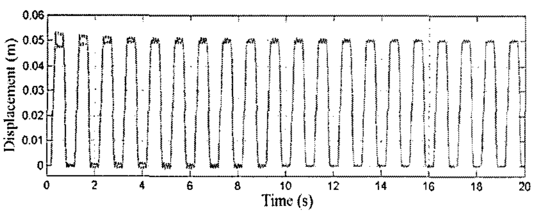


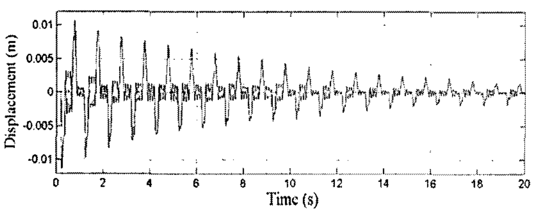
Fig. 6 Filtered and unfiltered joint reference trajectories



(a) Tip position

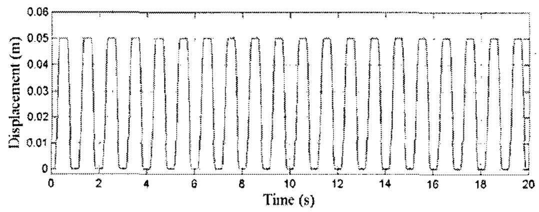


(b) Tip position

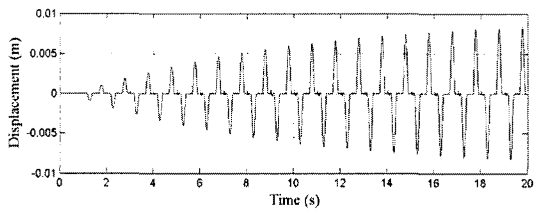


(c) Tip error

Fig. 7 Tip position and tip tracking error



(a) MRLC output



(b) Learned joint reference input

Fig. 8 MRLC output and learned joint reference input

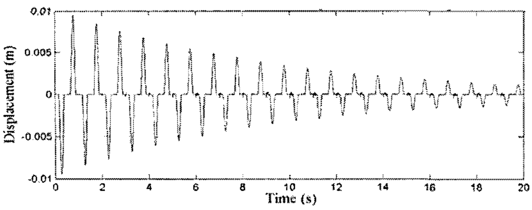


Fig. 9 Difference between filtered joint input and learned joint input

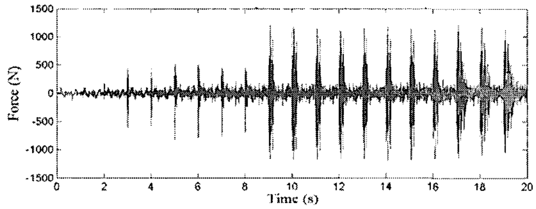


Fig. 10 Control effort

5. Conclusions

The use of the tip response calculated from a model as the reference trajectory to be tracked by the tip of a flexible manipulator is proposed in this paper. The work was placed in the larger context of how to best design a tip reference trajectory for the non-collocated control system, i.e., a flexible manipulator where the actuator is located at the joint. With the use of a command shaping filter, the model produces a tip response free of residual vibration. The vibration-free tip response of the model is smooth and is nearly guaranteed to be physically realizable in an actual system. Then a multirate repetitive learning controller (MRLC) is used as the tip controller and it is to modify the motion trajectory the base mass of the actual system is commanded to follow. The MRLC estimates the control input required to decrease the tip tracking error which is defined as the difference between the tip response in the model and the actual tip response. It is numerically shown that the proposed control configuration generates a realizable residual-vibration-free reference trajectory for the tip control of the manipulator system and it can eventually drive the tip tracking error to the noise level with realistic control effort magnitude. Also it is shown that as the MRLC practices learning process the modified joint reference input for the actual system converges to the filtered joint reference input in the model.

Acknowledgment

This research was supported by the Kyung Hee University Research Fund in 2004.

References

Hu, A.-P. and N. Sadegh, 2000, "Non-Colocated Control of a Flexible-Link Manipulator Tip Using a Multirate Repetitive Learning Controller," *The 7th Mechatronics Forum International Conference*, Atlanta, GA.

Magee, D. P. and Book, W. J., 1998, "Optimal Filtering to Minimize Elastic Behavior in Serial Link Manipulators," *Proceedings of the American Control Conference*, Philadelphia, PA, pp. 2637~2642.

Rhim, S., Hu, A.-P., Sadegh, N. and Book, W. J., 2001, "Combining a Multirate Repetitive Learning Controller with Command Shaping for Improved Flexible Manipulator Control," *ASME Journal of Dynamic Systems, Measurement, and Control*, Vol. 123, pp. 385~390.

Sadegh, N., Hu, A.-P. and James, C., 2002, "Synthesis, Stability Analysis, and Experimental Implementation of a Multirate Repetitive Learning Controller," *ASME Journal of Dynamic Systems, Measurement, and Control*, Vol. 124, pp. 668~674.

Singer, N. C. and Seering, W. P., 1990, "Preshaping Command Inputs to Reduce System Vibration," *ASME Journal of Dynamic Systems, Measurement, and Control*, Vol. 112, pp. 76~82.

Singer, N. C., Singhose, W. and Seering, W., 1999, "Comparison of Filtering Methods for Reducing Residual Vibrations," *European Journal of Control*, Vol. 5, pp. 211~218.

Tomizuka, M., 1987, "Zero-Phase Error Tracking Algorithm for Digital Control," *ASME Journal of Dynamic Systems, Measurement, and Control*, Vol. 109, pp. 65~68.