Simulation and Experimental Methods for Media Transport System : Part I, Three-Dimensional Sheet Modeling Using Relative Coordinate

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This research presents a three-dimensional modeling technique for a flexible sheet. A relative coordinate formulation is used to represent the kinematics of the sheet. The three-dimensional flexible sheet is modeled by multi-rigid bodies interconnected by out-of-plane joints and plate force elements. A parent node is designated as a master body and is connected to the ground by a floating joint to cover the rigid motion of the flexible sheet in space. Since the in-plane deformation of a sheet such as a paper and a film is relatively small, compared to out-of-plane deformation, only the out-of-plane deformation is accounted for in this research. The recursive formulation has been adopted to solve the equations of motion efficiently. An example is presented to show the validity of the proposed method.

Key Words : Media Transport System, Three-Dimensional Flexible Sheet Model, Floating Joint, Out-Of-Plane Joint, Plate Force, Nodal Body, Parent Node, Child Node, Relative Coordinate, Recursive Formula

1. Introduction

The media transport systems, such as printers, copiers, fax, ATMs, cameras, film develop machines, etc. have been widely used and rapidly developed. The media feeding mechanism for paper, film, money, and cloth is an important key technology for the design and development of the media transport systems. A commercial program of RecurDyn/MTT2D (Cho and Choi, 2001) has been widely used to roughly understand the sheet at an early stage of their design. However, its analysis scope was limited to the two-dimensional space. As a result, many design engineers of such machines have depended on their engineering intuition when they face with three-dimensional problems such as spinning due to unbalanced nip force, different weight or misalignment of drive-driven roller, and design of a guide with a three-dimensional shape. Since the conventional method is truly inefficient, it is absolutely required to develop a computer simulation tool, which analyses the paper feeding and separation process in the three-dimensional space in order to shorten the design time, to reduce the design cost, and to improve the machine performance.

Cho and Choi (2002) developed a computational modeling technique for a two-dimensional film feeding mechanism. The flexible sheet is divided into several thin rigid bodies which are

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connected by revolute joints and rotational spring dampers. Diehl (1995 ; http://www.me.psu.edu/research/bension.html) presented the local static mechanics of an electrometric nip system for a media transport system. The nonlinear finite element method and experimental measurement techniques are used to investigate the large deformable rollers. Several phenomena, such as sheet skewing, of a nip feeding system were well described. Ashida (2000) suggested the computer modeling techniques for the design and analysis of film feeding mechanisms. The primitive dynamic analysis of two-dimensional film-feeding models was presented by using commercial computer program. The paper feeding mechanism with a frictional pad system was investigated by Yanabe (http://www.yanabelab.nagaokaut.ac.jp) by using commercial nonlinear FEA program. It showed the local separation phenomenon between papers and roller, and was proved to be very good agreement with experimental measurements. Shin (http://www.engext.okstate.edu/info/WWW-WHRC.htm ; 1991) developed a web simulation and design tool using roll tensions. He showed that the tension control of each segment is a key design factor for a web system.

Bae has proposed a recursive formulation using the relative coordinates in (Bae et al., 1999). The equations of motion were derived in a compact matrix form by using the velocity transformation method. The actual computation was carried out by using the recursive formulas developed for each joint. Real-time simulation of a vehicle system has been carried out by the recursive method in (Bae et al., 2000). The Jacobian matrix was updated once in a while during time marching of the numerical integration. The recursive method was extended to the flexible body dynamics of constrained mechanical systems in (Bae et al., 2001). A virtual body concept was employed to relieve the implementation burden of the flexible body dynamics coding. A compliant track link model was developed for tracked vehicles in (Ryu et al., 2000). A minimum set of the equations of motion was obtained by the recursive method. Concept of the configuration design variable with the recursive formulation was introduced in (Kim et al., 2001).

This research presents a three-dimensional modeling technique for a flexible sheet. The three-dimensional flexible sheet is modeled by multi-rigid bodies interconnected by the out-of-plane joints and plate force elements. The relative coordinate formulation is used to represent the kinematics of the sheet and to solve the equations of motion efficiently. A parent node is designated as a master body and is connected to the ground by a floating joint to cover the large rigid motion of the flexible sheet in space. Since the in-plane deformation of the flexible sheet such as a paper and a film is relatively small, compared to out-of-plane deformation, only out-of plane deformation is accounted for in this research. The flexible sheet has been modeled as lumped masses connected by out-of-plane joints with plate force elements. The set of equation of motion is minimized by using the relative coordinate and is obtained by the recursive method. The recursive formulas for out-of-plane and floating joints are derived. A numerical example is presented to demonstrate the validity of the proposed method. The proposed three-dimensional flexible sheet model will make it possible to simulate the jamming problem, depending on sheet size, weight, stiffness, temperature, humidity, sheet velocity due to misalignment of drive-driven roller sets, and roller velocities due to gap and wear.

2. Three-Dimensional Flexivle Sheet

2.1 Kinematics definitions

A flexible sheet in the three-dimensional space is shown in Figure 1. The X-Y-Z coordinate system is the inertial reference frame and the x′-y′-z′ primed coordinate systems are the nodal reference frames. The orientation and position of the node reference frame are denoted by $\mathbf{A}$ and $\mathbf{r}$, respectively. The x-axis and z-axis of the node reference frame are defined along longitudinal and lateral directions, respectively. And the y-axis can be defined by right hand rule. As shown in Figure 1, t, L and D are the thickness, length, and depth of the sheet, respectively.

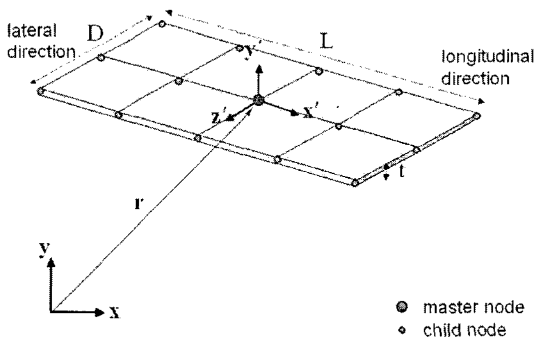

Fig. 1 Three-dimensional flexible sheet

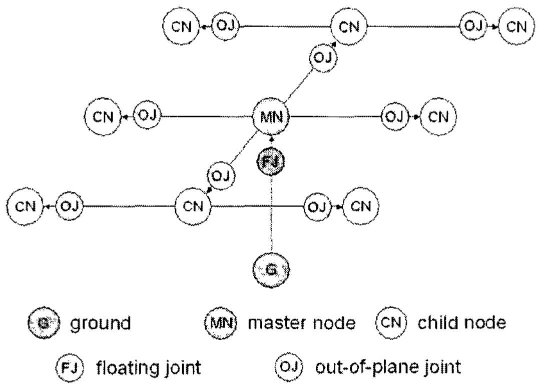

Fig. 2 Sequential connections of joints

The master node which is initially positioned at the center of the sheet is connected to the ground by a floating joint which has 6 degrees of freedom with respect to the ground. The adjacent nodal bodies are connected to the master node by the out-of-plane joints as shown in Figure 2. The out-of-plane joint has 3 degrees of freedom. One is the translational motion along the primed y-axis and the others are the two rotational motions along the primed x-axis and z-axis. The rest of bodies are connected to their inboard bodies by the out-of-plane joints.

The objective of using the out-of-joints is twofold. First, the motion of a nodal body has three degrees of freedom instead of six. Therefore, the number of generalized coordinates is halved, which reduces the solution time. Second, the in-plane deformation is significantly smaller than the out-of-plane deformation. Therefore, frequency of the in-plane motion is significantly higher than this of the out-of-plane motion.

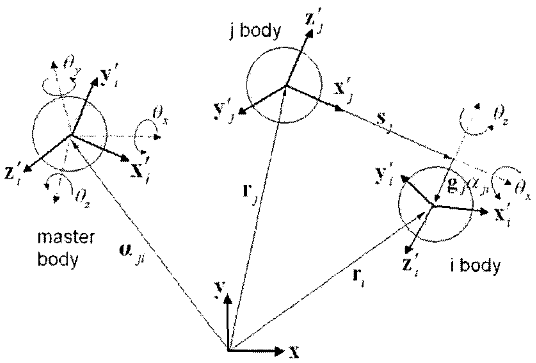

Fig. 3 Floating and out-of-plane joints

When an explicit method is used for numerical integration, such high frequency enforces the stepsize to be very small, which makes solution time very long. Therefore, the generalized coordinates defined for the sheet will remove the high frequency from the resulting equations of motion.

As shown in Figure 3, the orientation and position of the master nodal body are defined as follows.

$$\mathbf{A}_{\mathrm{i}}=\mathbf{A}_{\mathrm{x}}\mathbf{A}_{\mathrm{y}}\mathbf{A}_{\mathrm{z}} \tag{1}$$

$$\mathbf{A}_{\mathrm{x}}=\begin{bmatrix}1 & 0 & 0\\ 0 & c\theta_x & -s\theta_x\\ 0 & s\theta_x & c\theta_x\end{bmatrix},\ \mathbf{A}_{\mathrm{y}}=\begin{bmatrix}c\theta_y & 0 & s\theta_y\\ 0 & 1 & 0\\ -s\theta_y & 0 & c\theta_y\end{bmatrix}$$
$$\mathbf{A}_{\mathrm{z}}=\begin{bmatrix}c\theta_z & -s\theta_z & 0\\ s\theta_z & c\theta_z & 0\\ 0 & 0 & 1\end{bmatrix} \tag{2}$$

$$\mathbf{r}_{\mathrm{i}}=\boldsymbol{\alpha}_{\mathrm{ji}} \tag{3}$$

where the Z-Y-X Euler angles of θ_{x}, θ_{y}, θ_{z} and relative displacement of $\boldsymbol{\alpha}_{\mathrm{ji}}$ are the relative coordinates defined for master nodal body. The orientation and position of adjacent nodal body i are determined from the following equations.

$$\mathbf{A}_{\mathrm{i}}=\mathbf{A}_{\mathrm{j}}\mathbf{A}_{\mathrm{z}}\mathbf{A}_{\mathrm{x}} \tag{4}$$

$$\mathbf{r}_{\mathrm{i}}=\mathbf{r}_{\mathrm{j}}+\mathbf{A}_{\mathrm{j}}\mathbf{s}_{\mathrm{j}}'+\mathbf{g}_{\mathrm{j}}\alpha_{\mathrm{ji}} \tag{5}$$

where $\mathbf{A}_{\mathrm{j}}$ and $\mathbf{r}_{\mathrm{j}}$ are the orientation and position of inboard nodal body j. The vector of $\mathbf{s}_{\mathrm{j}}'$ is an initial position of nodal body i with respect to its base nodal body and the direction vector of $\mathbf{g}_{\mathrm{j}}$ is the y-axis of the base nodal body. The orientation matrices of $\mathbf{A}_{\mathrm{x}}$ and $\mathbf{A}_{\mathrm{z}}$ are can be computed by using two angles of θ_{x} and θ_{z} from

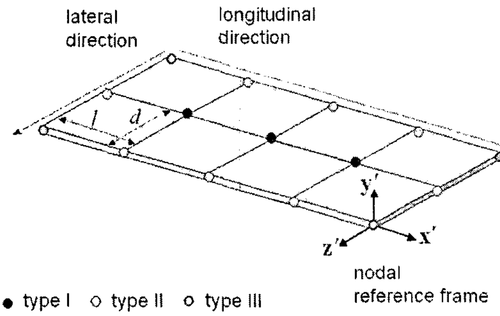

Fig. 4 Nodal mass and moment of inertia

Table 1 Mass and moment of inertia

Type II	Type III
$\frac{m}{2}$	$\frac{m}{4}$
$I_{xx}=\frac{1}{24}m(t^2+d^2)$	$I_{xx}=\frac{1}{48}m(t^2+d^2)$
$I_{yy}=\frac{1}{24}m(l^2+d^2)$	$I_{yy}=\frac{1}{48}m(l^2+d^2)$
$I_{zz}=\frac{1}{24}m(t^2+l^2)$	$I_{zz}=\frac{1}{48}m(t^2+l^2)$

Eq. (2). The angles of θ_x and θ_z and the displacement of α_{j1} are the relative coordinates defined for child nodal bodies and the coordinates are directly integrated.

2.2 Nodal mass and moment of inertia

The sheet is divided into a number of nodal bodies with mass. As shown in Figure 4, the mass and inertia moment of each internal nodal body of type I can be defined as follow.

$$m=\frac{M}{n_1n_2-n_1-n_2+1} \quad (6)$$

$$M=\rho^*t^*L^*D \quad (7)$$

where, M and ρ are total mass and density of sheet, respectively. The n_1 and n_2 are the number of nodal bodies in the longitudinal and lateral directions, respectively.

Also, the inertia moment of these bodies can be defined as follow.

$$I_{xx}=\frac{1}{12}m(t^2+d^2) \quad (8)$$

$$I_{yy}=\frac{1}{12}m(l^2+d^2) \quad (9)$$

$$I_{zz}=\frac{1}{12}m(t^2+l^2) \quad (10)$$

where, d and l are the length and depth of each plate element, respectively. The mass and moment of inertia of nodes on four edges and at four corners are half and quarter of those of the internal nodes as sown in Table 1, respectively.

2.3 Plate force

Four nodal bodies are interconnected by a plate force element. Total Lagrangian methods (Bathe, 1996) can be used to compute the plate force. The absolute positions and orientations of the four nodal bodies are interpolated to compute the strain energy of the plate element. Therefore, the plate force of $\mathbf{f}_{plate}$ can be implicitly written in terms of the positions and orientations of the four nodal bodies as

$$\mathbf{f}_{plate}=f(\mathbf{r}_n, \mathbf{A}_n, \dot{\mathbf{r}}_n, \mathbf{w}_n) \quad (11)$$

where, $\mathbf{r}_n$, $\mathbf{A}_n$, $\dot{\mathbf{r}}_n$ and $\mathbf{w}_n$ are the positions, orientations, velocities and angular velocities of four nodal bodies which belong to each plate force element.

3. Recursive Formulas

The velocity transformation matrix of a joint can be derived from the velocity relationship between the Cartesian coordinate and the relative coordinate. In general, the recursive formulas to recover the Cartesian velocity can be written as follows.

$$\mathbf{Y}_1=\begin{Bmatrix}\dot{\mathbf{r}}_1\\ \mathbf{w}_1\end{Bmatrix}=\mathbf{B}_{j11}\mathbf{Y}_j+\mathbf{B}_{j12}\dot{\mathbf{q}}_{j1} \quad (12)$$

where, $\mathbf{B}_{j11}$ and $\mathbf{B}_{j12}$ are velocity transformation matrices and the relative velocity of $\dot{\mathbf{q}}_{j1}$ is directly integrated. The angular and translational velocities of the master nodal body are obtained by the time derivative of Eqs. (1) and (3) as follow.

$$\begin{Bmatrix}\dot{\mathbf{r}}_1\\ \mathbf{w}_1\end{Bmatrix}=\begin{Bmatrix}\dot{\alpha}_{j1}\\ \begin{Bmatrix}1\\0\\0\end{Bmatrix}\dot{\theta}_x+\mathbf{A}_x\begin{Bmatrix}0\\1\\0\end{Bmatrix}\dot{\theta}_y+\mathbf{A}_x\mathbf{A}_y\begin{Bmatrix}0\\0\\1\end{Bmatrix}\dot{\theta}_z\end{Bmatrix} \quad (13)$$

where, $\mathbf{A}_x\mathbf{A}_y = \mathbf{A}_i\mathbf{A}_z^T$. From Eq. (13), the velocity transformation matrices of Eq. (12) can be obtained as follows.

$$\mathbf{B}_{ji1} = \mathbf{0}$$
$$\mathbf{B}_{ji2} = \begin{bmatrix} \mathbf{I} & 0 \\ 0 & \mathbf{f}' \ \mathbf{A}_x\mathbf{g}' \ \mathbf{A}_i\mathbf{h}' \end{bmatrix} \tag{14}$$

where, $\mathbf{f}' = \{1, 0, 0\}^T$, $\mathbf{g}' = \{0, 1, 0\}^T$ and $\mathbf{h}' = \{0, 0, 1\}^T$. The angular and translational velocities of the child nodal body are obtained by the time derivative of Eqs. (4) and (5) as follow.

$$\begin{Bmatrix} \dot{\mathbf{r}}_i \\ \mathbf{w}_i \end{Bmatrix} = \begin{Bmatrix} \dot{\mathbf{r}}_j - \mathbf{A}_j(\tilde{\mathbf{s}}_j' + \tilde{\mathbf{g}}'\alpha_{ji})\mathbf{A}_j^T\mathbf{w}_j + \mathbf{A}_j\mathbf{g}'\dot{\alpha}_{ji} \\ \mathbf{w}_j + \mathbf{A}_j \begin{Bmatrix} 0 \\ 0 \\ 1 \end{Bmatrix} \dot{\theta}_z + \mathbf{A}_j\mathbf{A}_z \begin{Bmatrix} 1 \\ 0 \\ 0 \end{Bmatrix} \dot{\theta}_x \end{Bmatrix} \tag{15}$$

where, $\mathbf{A}_j\mathbf{A}_z = \mathbf{A}_i\mathbf{A}_x^T$. From Eq. (15), the velocity transformation matrices of Eq. (12) can be obtained as follows.

$$\mathbf{B}_{ji1} = \begin{bmatrix} \mathbf{I} & -\mathbf{A}_j(\tilde{\mathbf{s}}_j' + \tilde{\mathbf{g}}'\alpha_{ji})\mathbf{A}_j^T \\ 0 & \mathbf{I} \end{bmatrix}$$
$$\mathbf{B}_{ji} = \begin{bmatrix} \mathbf{A}_j\mathbf{g}' & 0 & 0 \\ 0 & \mathbf{A}_j\mathbf{h}' & \mathbf{A}_i\mathbf{f}' \end{bmatrix} \tag{16}$$

The accelerations of master and child nodal bodies can be generally obtained by differentiating Eq. (12).

$$\dot{\mathbf{Y}}_i = \begin{Bmatrix} \ddot{\mathbf{r}}_i \\ \dot{\mathbf{w}}_i \end{Bmatrix} = \dot{\mathbf{B}}_{ji1}\mathbf{Y}_j + \dot{\mathbf{B}}_{ji2}\dot{\mathbf{q}}_{ji} + \mathbf{B}_{ji1}\dot{\mathbf{Y}}_j + \mathbf{B}_{ji2}\ddot{\mathbf{q}}_{ji} \tag{17}$$

where, $\dot{\mathbf{B}}_{ji1}$ and $\dot{\mathbf{B}}_{ji2}$ can be obtained by differentiating Eqs. (14) and (16). The relative acceleration of $\ddot{\mathbf{q}}_{ji}$ is directly integrated.

4. Numerical Intergration

The equations of motion for constrained systems have been obtained in (Bae et al., 1999) as follows.

$$\mathbf{F} = \mathbf{B}^T(\mathbf{M}\dot{\mathbf{Y}} + \boldsymbol{\Phi}_z^T\boldsymbol{\lambda} - \mathbf{Q}) = \mathbf{0} \tag{18}$$

where, $\boldsymbol{\lambda}$ is the Lagrange multiplier vector for cut joints (Wittenburg, 1977) in $\mathbf{R}^m$ and $\boldsymbol{\Phi}$ represents the position level constraint vector in $\mathbf{R}^m$. The mass matrix $\mathbf{M}$ and the force vector $\mathbf{Q}$ are defined as follow.

$$\mathbf{M} = \mathrm{diag}(\mathbf{M}_1, \mathbf{M}_2, \cdots, \mathbf{M}_{nbd}) \tag{19}$$

$$\mathbf{Q} = (\mathbf{Q}_1^T, \mathbf{Q}_2^T, \cdots, \mathbf{Q}_{nbd}^T) \tag{20}$$

The equations of motion in Eq. (18), the constraints, and their derivatives comprise a system of over-determined differential algebraic equations (ODAEs).

An explicit numerical integration algorithm for the ODAEs is given in (Yen et al., 1990) as follows.

$$\mathbf{H}(\mathbf{x}) = \begin{bmatrix} \mathbf{F}(\mathbf{q}, \mathbf{v}, \dot{\mathbf{v}}, \lambda) \\ \boldsymbol{\Phi}_q\dot{\mathbf{v}} - \gamma \\ \boldsymbol{\Phi}_q\mathbf{v} - \nu \\ \boldsymbol{\Phi}(\mathbf{q}) \\ \mathbf{U}^T(\mathbf{v} - \xi_1) \\ \mathbf{U}^T(\mathbf{q} - \xi_2) \end{bmatrix} \tag{21}$$

where, $\xi_1 \equiv \frac{1}{b_0}\sum_{i=1}^{k} b_i\mathbf{v}_{n-i}$ and $\xi_2 \equiv \frac{1}{b_0}\sum_{i=1}^{k} b_i\mathbf{q}_{n-i}$ in which k is the order of integration and $\mathbf{b}_i$'s are integration formula coefficients. The unknown variables of $\mathbf{x} = [\mathbf{q}^T, \mathbf{v}^T, \dot{\mathbf{v}}^T, \lambda^T]^T$ and the columns of $\mathbf{U} \equiv \mathbf{R}^{nr\times(nr-m)}$ constitute bases for the parameter space of the position level constraints. $\mathbf{U}$ is chosen as $\begin{bmatrix} \boldsymbol{\Phi}_q \\ \mathbf{U} \end{bmatrix}$, the inverse of which exists. Therefore, the parameter space spanned by the columns of $\mathbf{U}$ and the subspace spanned by the columns of $\boldsymbol{\Phi}_q^T$ constitute the entire $\mathbf{R}^{nr}$ space.

Equation (21) can be rewritten in a decoupled form for the position, velocity, and acceleration. Firstly, the position is obtained by solving the 6th equation of the position integration formula and the 4th equation of the position level constraint as

$$\begin{bmatrix} \boldsymbol{\Phi}_q \\ \mathbf{U}^T \end{bmatrix} \Delta\mathbf{q} = -\begin{bmatrix} \boldsymbol{\Phi} \\ \mathbf{U}^T(\mathbf{q} - \xi_2) \end{bmatrix} \tag{22}$$

$$\mathbf{q}^{i+1} = \mathbf{q}^i + \Delta\mathbf{q} \tag{23}$$

Secondly, the 5th equation of the velocity integration formula and the 3rd equation of the velocity level constraint constitute a system of linear equations and Newton chord method is used to solve them for the $\mathbf{v}$. This method has been employed to avoid regeneration and LU-decomposition processes of the coefficient matrix at every time steps in (Bae et al., 2000). Thus, Newton chord method for these equations is

$$\begin{bmatrix} \Phi_q \\ U^T \end{bmatrix} \Delta v = -\begin{bmatrix} \dot{\Phi} \\ U^T(v-\zeta_1) \end{bmatrix} \quad (24)$$

$$v^{i+1} = v^i + \Delta v \quad (25)$$

Whenever v^{i+1} is computed, the Cartesian velocity of Y is recursively computed by using Eq. (12). Finally, the 1st equation of the motion and the 2nd equation of the acceleration level constraint constitute a system of linear equations. Newton chord method for these equations is

$$\begin{bmatrix} B^T M B & \Phi_q^T \\ \Phi_q & 0 \end{bmatrix} \begin{bmatrix} \Delta \dot{v} \\ \Delta \lambda \end{bmatrix} = -\begin{bmatrix} F \\ \ddot{\Phi} \end{bmatrix} \quad (26)$$

$$\begin{bmatrix} \dot{v} \\ \lambda \end{bmatrix}^{i+1} = \begin{bmatrix} \dot{v} \\ \lambda \end{bmatrix}^{i} + \begin{bmatrix} \Delta \dot{v} \\ \Delta \lambda \end{bmatrix} \quad (27)$$

where, F from Eq. (18) is recursively computed in (Bae et al., 1999).

5. Numerical Example

A fixed and free paper shown in Figure 5 is solved to demonstrate the validity of the proposed method.

The size of paper is 300mm by 210 mm. The Young's modulus and Poisson's ratio are 2000 N/mm and 0.2, respectively. The density is 7.7e-7 kg and the thickness of sheet is 0.1 mm. The sheet has a large deformation in the gravitation. The numerical example that has five mesh types is solved to validate a solution in a commercial FEA program of ANSYS in which the nonlinear static analysis was carried out because of the severe bending. As shown in Table 2 and Figure 6, the maximum deformations in the vertical direction for each mesh type are compared. As a result, the difference between two results is about 0.535 (mm).

As shown in Figure 7, the dynamics analysis in this proposed method is carried out to find the equilibrium position.

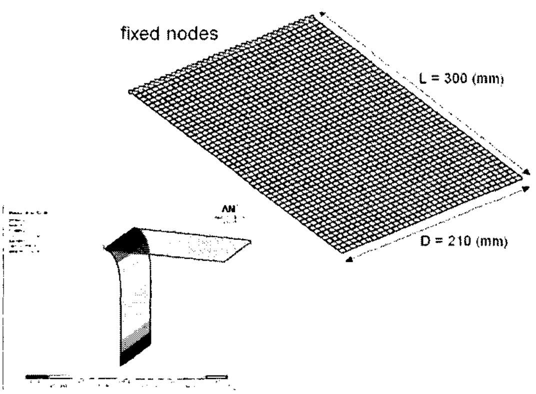

Fig. 5 Examples of fixed-free sheet

Table 2 Maximum vertical deformations

Mesh	No. of Node	Boundary Condition I	
		Proposed (mm)	ANSYS (mm)
1	7 by 5	−285.486	−284.77
2	15 by 11	−282.377	−282.45
3	33 by 23	−281.706	−282.01
4	51 by 35	−281.498	−281.97
5	61 by 41	−281.415	−281.95

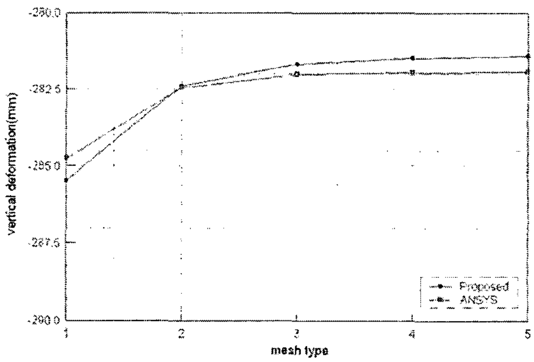

Fig. 6 The comparison of vertical deformations

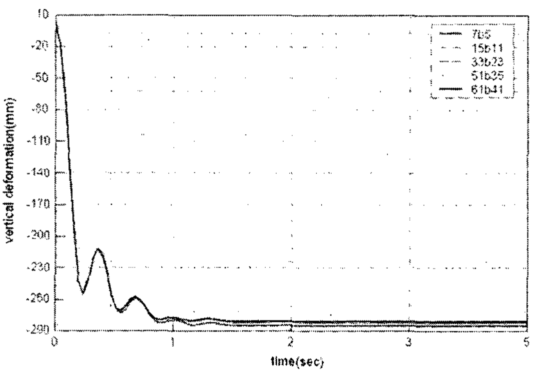

Fig. 7 Vertical deformations in time domain

6. Conclusion

The modeling techniques and dynamics analysis of three-dimensional flexible sheet is investigated in this paper. The flexible sheet is divided by into many bodies interconnected with the plate force elements. The in-plane deformations are constrained and the out-of-plane motion of the plate element is allowed. The plate force is computed based on the strain energy equation. The relative coordinates have been implemented to represent the deformation between nodes. To solve the generalized coordinates efficiently, the recursive formulations of the joints have been proposed. The results obtained from the proposed methods are in good agreement with those from a finite element commercial program. The media transport system manufactures have relied on trial error techniques for the design of their core mechanisms; however the proposed method by employing multibody dynamics in this paper can reduce many difficulties at an early design stage.

References

Ashida, T., 2000, "The Meeting Material of The Japan Society for Precision Engineering," Japan.

Bae, D. S., Han, J. M. and Yoo, H. H., 1999, "A Generalized Recursive Formulation for Constrained Mechanical System Dynamics," *Mech. Struct. And Machines*, Vol. 27, No. 3, pp. 293~315.

Bae, D. S., Han, J. M., Choi, J. H. and Yang, S. M., 2001, "A Generalized Recursive Formulation for Constrained Flexible Multibody Dynamics," *International Journal for Numerical Methods in Engineering*, Vol. 50, pp. 1841~1859.

Bae, D. S., Lee, J. K, Cho, H. J. and Yae, H., 2000, "An Explicit Integration Method for Real-time Simulation of Multibody Vehicle Models," *Computer Methods in Applied Mechanics and Engineering*, Vol. 187, pp. 337~350.

Bathe, K. J., 1996, "Finite Element Procedure," Prentice-Hall, Inc.

Cho, H. J. and Choi, J. H., 2001, "2DMTT development specification," Technical report, FunctionBay Inc.

Cho, H. J., Bae, D. S., Choi, J. H. and Suzuki, T., 2002, "Dynamics Analysis and Contact Modeling for Two-Dimensional Media Transport System," *Proceedings of the ASME Design Engineering Technical Conferences*, DETC2003/VIB48338, p. 605.

Diehl, T., 1995, "Two Dimensional and Three Dimensional Analysis of Nonlinear Nip Mechanics with Hyper Elastic Material Formulation," Ph.D. Thesis, University of Rochester, Rochester, New York.

Kim, H. W., Bae, D. S. and Choi, K. K., 2001, "Configuration Design Sensitivity Analysis of Dynamics for Constrained Mechanical Systems," *Computer Methods in Applied Mechanics and Engineering*, Vol. 190, pp. 5271~5282.

Ryu, H. S., Bae, D. S., Choi, J. H. and Shabana, A. A., 2000, "A Compliant Track Link Model for High-speed, High-mobility Tracked Vehicles," *International Journal for Numerical Methods in Engineering*, Vol. 48, pp. 1481~1502.

Shin, K. H., 1991, "Distributed Control of Tension in Multi-Span Web Transport Systems," Ph.D. Thesis, University of Oklahoma State.

Wittenburg, J., 1977, "Dynamics of Systems of Rigid Bodies," B.G. Teubner, Stuttgart.

Yen, J., Haug, E. J. and Potra, F. A., 1990, "Numerical Method for Constrained Equations of Motion in Mechanical Systems Dynamics," Technical Report R-92, Center for Simulation and Design Optimization, Department of Mechanical Engineering, and Department of Mathematics, University of Iowa, Iowa City, Iowa.

http://www.engext.okstate.edu/info/WWW-WHRC.htm

http://www.me.psu.edu/research/bension.html

http://www.yanabelab.nagaokaut.ac.jp