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Study on the Frame Structure Modeling of the Beam Element Formulated by Absolute Nodal Coordinate Approach

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Accurate seismic analyses of large deformable moving structures are still unsolved problems in the field of earthquake engineering. In order to analyze these problems, the nonlinear finite element method formulated by the absolute nodal coordinate approach is noticed. Because, this formulation has several advantages over the standard procedures on mass matrix, elastic forces and damping forces in the case of large displacement problems. But, it has not been fully studied to build frame structure models by using beam elements in the absolute nodal coordinate formulation. In this paper, we propose the connecting method of the beam elements formulated by the absolute nodal coordinate. The coordinate transformation matrix of this element is introduced into the frame structure. This beam element has the characteristic that the mass matrix and bending stiffness matrix are constant even if in the case of large displacement problems, and this characteristic is being kept after the transformation. In order to verify the proposed method, we show the numerical simulation results of frame structures for a vibration problem and a large displacement problem.

Key Words : Finite Element Method, Beam, Absolute Nodal Coordinate Approach, Frame Structure, Seismic Analysis

1. Introduction

There are many reports on the methods to formulate flexible beam element which undergoes large displacement and large rotation (De Veubeke, 1976 ; Huston, 1981 ; Huston, 1991 ; Simo, and Vu-Quoc, 1986 ; Iura and Atluri, 1995 ; Honke et al., 1998). Recently, the absolute nodal coordinate formulation was proposed by Shabana et. al. (1996 ; 1998).

This formulation employs nodal slopes instead of employing infinitesimal or finite rotations at the beam nodes which are used in the traditional formulations. The authors proposed a new formulation for elastic force calculation which is not dependent on the element coordinate.

From this, the bending stiffness matrix of a beam element whichout using the rotational matrix has been obtained for a large displacement and rotation problem, and consequently a time constant Rayleigh damping was introduced in the structural analysis (Takahashi and Shimizu,

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1999 ; Takahashi et al., 2002). There have been many studies for developing flexible elements by making use of the absolute nodal coordinate approach. This approach is not completed but is still developing. For this reason, there are many studies on theoretical formulation but a few studies for applying the absolute nodal coordinate formulation to practical structural problems. As one of the application examples, Terumichi models a rail in railway vehicle problems by the absolute nodal coordinate formulation and obtained satisfactory results (Ikuta et al., 1999). But he only uses the straight beam model connected by many straight beam elements. There are no such other examples in the two dimensional frame structures so far. Since the absolute nodal coordinate formulation uses the slopes as nodal coordinates instead of using the angles which are used in the standard linear finite element (FE) formulation, the standard coordinate transformation matrix for the linear FE formulation can not be used when element matrices are assembled. The frame structure can be modeled by the DAE by using the constraint equation of node connection, but it may be convenient for us to model the structure, by a new coordinate transformation matrix for the absolute nodal coordinate approach. There are no such reports on the study of frame structure modeling and analysis by using the beam element formulated by the absolute nodal coordinate formulation. There are few experimental verifications on the structures formulated by the absolute nodal coordinate formulation by means of actual scale structural models.

This paper describes a formulation of frame structure by means of the absolute nodal coordinate beam element for practical problems. A method to connect beam elements is proposed by assembling the superposition of the element matrices with a new developed coordinate transformation matrix for the absolute nodal coordinate. Equations of motion of the frame structure can be described by ordinary differential equations with constant mass and bending stiffness matrices in large displacement problems. Since the formulation can treat large displacement and large rotation problems under infinitesimal deformation, this may be applied for a moving flexible structures which change their shape with time. To verify the usefulness of the proposed formulation, simulations have been conducted. The results are compared with the results of traditional FE analysis and seismic experiment.

2. Formulation for Beam Element by Absolute Nodal Coordinate Approach

We define the global coordinate system O-XY, and consider the uniform slender beam in this coordinate system as shown in Fig. 1. The beam has length l, cross sectional area A, mass density ρ. The global position vector $\mathbf{r}$ of an arbitary point p on the neutral axis of the beam can be written as

$$\mathbf{r}=\mathbf{S}\mathbf{e} \tag{1}$$

$\mathbf{e}$ is the vector of nodal coordinates

$$\mathbf{e}=[e_1\ e_2\ e_3\ e_4\ e_5\ e_6\ e_7\ e_8]^T \tag{2}$$

where e_1 and e_2 are the translational coordinates at the node at A, e_5 and e_6 are the translational coordinates at the node at B, e_3 and e_4 are the spatial derivatives of the displacements of the node at A defined in the XY coordinate system, and e_7 and e_8 are the spatial derivatives of the displacements of the node at B defined in the XY coordinate system.

We assume Bernoulli-Euler theory for the beam. The shape function $\mathbf{S}$ is written as

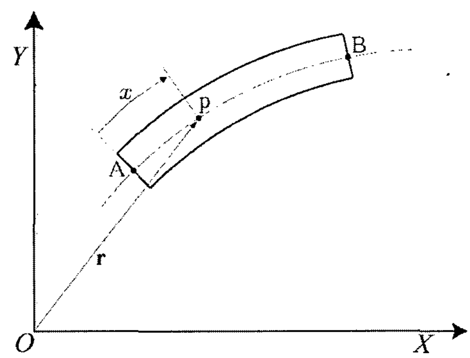

Fig. 1 Deformation of beam

$$\mathbf{S}=\begin{bmatrix}\mathbf{S}_1\\ \mathbf{S}_2\end{bmatrix}=\begin{bmatrix} 1-3\xi^2+2\xi^3 & 0 \\ 0 & 1-3\xi^2+2\xi^3 \\ 1(\xi-2\xi^2+\xi^3) & 0 \\ 0 & 1(\xi-2\xi^2+\xi^3) \\ 3\xi^2-2\xi^3 & 0 \\ 0 & 3\xi^2-2\xi^3 \\ l(\xi^3-\xi^2) & 0 \\ 0 & l(\xi^3-\xi^2) \end{bmatrix}^T \tag{3}$$

where x is a distance from point A to p, and $\xi=x/l$.

The kinetic energy of the beam is defined as

$$T=\frac{1}{2}\int_0^l \rho\dot{\mathbf{r}}^T\dot{\mathbf{r}}\,dx \tag{4}$$

where $\dot{\mathbf{r}}$ is the global velocity vector of an arbitrary point p on the beam element defined as

$$\dot{\mathbf{r}}=\mathbf{S}\dot{\mathbf{e}} \tag{5}$$

Substituting Eq. (5) into Eq. (4), the mass matrix $\mathbf{M}$ can be obtained as follows.

$$\mathbf{M}=A\rho l\begin{bmatrix} \frac{13}{35} & 0 & \frac{11l}{210} & 0 & \frac{9}{70} & 0 & -\frac{13l}{420} & 0 \\ 0 & \frac{13}{35} & 0 & \frac{11l}{210} & 0 & \frac{9}{70} & 0 & -\frac{13l}{420} \\ \frac{11l}{210} & 0 & \frac{l^2}{105} & 0 & \frac{13l}{420} & 0 & -\frac{l^2}{140} & 0 \\ 0 & \frac{11l}{210} & 0 & \frac{l^2}{105} & 0 & \frac{13l}{420} & 0 & -\frac{l^2}{140} \\ \frac{9}{70} & 0 & \frac{13l}{420} & 0 & \frac{13}{35} & 0 & -\frac{11l}{210} & 0 \\ 0 & \frac{9}{70} & 0 & \frac{13l}{420} & 0 & \frac{13}{35} & 0 & -\frac{11l}{210} \\ -\frac{13l}{420} & 0 & -\frac{l^2}{140} & 0 & -\frac{11l}{210} & 0 & \frac{l^2}{105} & 0 \\ 0 & -\frac{13l}{420} & 0 & -\frac{l^2}{140} & 0 & -\frac{11l}{210} & 0 & \frac{l^2}{105} \end{bmatrix} \tag{6}$$

Next we explain the derivation of elastic forces without local element frame. At first, let us consider the elastic forces for the axial deformation of the beam. The strain energy U_l is defined as

$$U_l\frac{1}{2}\int_0^l EA\varepsilon^2\,dx \tag{7}$$

where E is the modulus of elasticity and ε is the strain for the axial deformation. Now, it is assumed that the deformation of the beam is infinitesimal, and the length of the deformed beam is equal to the distance between point A and B in Fig. 1, then the axial strain of the beam can be written as

$$\varepsilon=\frac{l_d-l}{l}=\frac{\sqrt{(e_5-e_1)^2+(e_6-e_2)^2}}{l} \tag{8}$$

where, $l_d(=\sqrt{(e_5-e_1)^2+(e_6-e_2)^2})$ is the current length of the deformed beam and l is the length of the undeformed beam. Substituting from Eq. (8) into Eq. (7), the axial strain energy can be obtain, and the vector of the element generalized elastic forces for the axial deformation can be written as

$$\mathbf{F}_l=\mathbf{K}_l\mathbf{e} \tag{9}$$

where,

$$\mathbf{K}_l=\frac{EA\varepsilon}{l_d}\begin{bmatrix} 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{10}$$

Next, let us consider elastic forces for the infinitesimal bending deformation. The strain energy U_t for the bending deformation is defined as

$$U_t=\frac{1}{2}\int_0^l EI\kappa^2\,dx \tag{11}$$

where I is the second moment of area and κ is the curvature of the deformed beam. The following relationsip is used in order to obtain the strain energy for the bending deformation (Takahashi and Shimizu, 1999)

$$\kappa^2=\left(\left(\frac{\partial^2\mathbf{S}}{\partial x^2}\mathbf{e}\right)^T\left(\frac{\partial^2\mathbf{S}}{\partial x^2}\mathbf{e}\right)\right) \tag{12}$$

Substituting Eq. (12) into Eq. (11), the vector of the element generalized elastic forces for the bending deformation can be lead as

$$\mathbf{F}_t=\left(\frac{\partial U_t}{\partial \mathbf{e}}\right)^T=\mathbf{K}_t\mathbf{e} \tag{13}$$

wher,

$$\mathbf{K}_t=\int_0^l EI\left(\frac{\partial^2\mathbf{S}}{\partial x^2}\right)^T\left(\frac{\partial^2\mathbf{S}}{\partial x^2}\right)dx \tag{14}$$

$$K_t=\frac{EI}{l^3}\begin{bmatrix} 12 & 0 & 6l & 0 & -12 & 0 & 6l & 0 \\ 0 & 12 & 0 & 6l & 0 & -12 & 0 & 6l \\ 6l & 0 & 4l^2 & 0 & -6l & 0 & 2l^2 & 0 \\ 0 & 6l & 0 & 4l^2 & 0 & -6l & 0 & 2l^2 \\ -12 & 0 & -6l & 0 & 12 & 0 & -6l & 0 \\ 0 & -12 & 0 & -6l & 0 & 12 & 0 & -6l \\ 6l & 0 & 2l^2 & 0 & -6l & 0 & 4l^2 & 0 \\ 0 & 6l & 0 & 2l^2 & 0 & -6l & 0 & 4l^2 \end{bmatrix} \tag{15}$$

The bending stiffness matrix $\mathbf{K}_t$ is symmetric constant matrix. Finally, the vector of the element generalized elastic forces can be obtained as

$$\mathbf{F}_e=\mathbf{F}_l+\mathbf{F}_t \tag{16}$$

The equation of motion of the absolute nodal coordinate formulation with Rayleigh damping effect (Takahashi et al., 2002) which is limited to bending modes is written as

$$\mathbf{M}\ddot{\mathbf{e}}+\mathbf{F}_d+\mathbf{F}_e=\mathbf{Q} \tag{17}$$

where,

$$\mathbf{F}_d=(\alpha\mathbf{M}+\beta\mathbf{K}_t)\dot{\mathbf{e}} \tag{18}$$

3. Frame Structure Model

3.1 Joint at straight part

Let us consider the connection method of beam elements at the straight part as shown in Fig. 2. In this case, the connection point P and Q have same nodal coordinates, then it is possible to combine these elements to each other without the coordinate transformation matrix. The equations of motion of the two elements can be written as

$$\mathbf{M}^{(1)}\ddot{\mathbf{e}}^{(1)}=\mathbf{Q}^{(1)}-(\mathbf{K}_l^{(1)}+\mathbf{K}_t^{(1)})\mathbf{e}^{(1)} \tag{19}$$

$$\mathbf{M}^{(2)}\ddot{\mathbf{e}}^{(2)}=\mathbf{Q}^{(2)}-(\mathbf{K}_l^{(2)}+\mathbf{K}_t^{(2)})\mathbf{e}^{(2)} \tag{20}$$

where, the suffix $^{(i)}$ means the element number. In order to simplify the description of the equation, we described the mass matrix and the stiffness matrix of 8×8 matrix by using 2×2 matrix as follows. And the nodal coodinate vector and

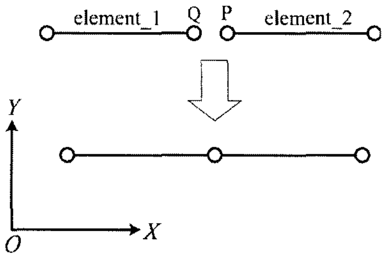

Fig. 2 Joint at straight part

generalized force vector are described as follows by using 2×1 vector.

$$\mathrm{M}^{(i)}=A\rho l\begin{bmatrix} \frac{13}{35} & 0 & \frac{11l}{210} & 0 & \frac{9}{70} & 0 & -\frac{13l}{420} & 0 \\ 0 & \frac{13}{35} & 0 & \frac{11l}{210} & 0 & \frac{9}{70} & 0 & -\frac{13l}{420} \\ \frac{11l}{210} & 0 & \frac{l^2}{105} & 0 & \frac{13l}{420} & 0 & -\frac{l^2}{140} & 0 \\ 0 & \frac{11l}{210} & 0 & \frac{l^2}{105} & 0 & \frac{13l}{420} & 0 & -\frac{l^2}{140} \\ \frac{9}{70} & 0 & \frac{13l}{420} & 0 & \frac{13}{35} & 0 & -\frac{11l}{210} & 0 \\ 0 & \frac{9}{70} & 0 & \frac{13l}{420} & 0 & \frac{13}{35} & 0 & -\frac{11l}{210} \\ -\frac{13l}{420} & 0 & -\frac{l^2}{140} & 0 & -\frac{11l}{210} & 0 & \frac{l^2}{105} & 0 \\ 0 & -\frac{13l}{420} & 0 & -\frac{l^2}{140} & 0 & -\frac{11l}{210} & 0 & \frac{l^2}{105} \end{bmatrix} \tag{21}$$

$$=\begin{bmatrix} \mathrm{M}_{11}^{(i)} & \mathrm{M}_{12}^{(i)} & \mathrm{M}_{13}^{(i)} & \mathrm{M}_{14}^{(i)} \\ \mathrm{M}_{21}^{(i)} & \mathrm{M}_{22}^{(i)} & \mathrm{M}_{23}^{(i)} & \mathrm{M}_{24}^{(i)} \\ \mathrm{M}_{31}^{(i)} & \mathrm{M}_{32}^{(i)} & \mathrm{M}_{33}^{(i)} & \mathrm{M}_{34}^{(i)} \\ \mathrm{M}_{41}^{(i)} & \mathrm{M}_{42}^{(i)} & \mathrm{M}_{43}^{(i)} & \mathrm{M}_{44}^{(i)} \end{bmatrix}$$

$$\mathbf{K}^{(i)}=\mathbf{K}_l^{(i)}+\mathbf{K}_t^{(i)} =\begin{bmatrix} \mathrm{K}_{11}^{(i)} & \mathrm{K}_{12}^{(i)} & \mathrm{K}_{13}^{(i)} & \mathrm{K}_{14}^{(i)} \\ \mathrm{K}_{21}^{(i)} & \mathrm{K}_{22}^{(i)} & \mathrm{K}_{23}^{(i)} & \mathrm{K}_{24}^{(i)} \\ \mathrm{K}_{31}^{(i)} & \mathrm{K}_{32}^{(i)} & \mathrm{K}_{33}^{(i)} & \mathrm{K}_{34}^{(i)} \\ \mathrm{K}_{41}^{(i)} & \mathrm{K}_{42}^{(i)} & \mathrm{K}_{43}^{(i)} & \mathrm{K}_{44}^{(i)} \end{bmatrix} \tag{22}$$

$$\mathbf{e}^{(i)}=[e_1^{(i)}e_2^{(i)}\,|\,e_3^{(i)}e_4^{(i)}\,|\,e_5^{(i)}e_6^{(i)}\,|\,e_7^{(i)}e_8^{(i)}]^T =[\mathbf{e}_{12}^{(i)}\ \mathbf{e}_{34}^{(i)}\ \mathbf{e}_{56}^{(i)}\ \mathbf{e}_{78}^{(i)}]^T \tag{23}$$

$$\mathbf{Q}^{(i)}=[\mathbf{Q}_{12}^{(i)}\ \mathbf{Q}_{34}^{(i)}\ \mathbf{Q}_{56}^{(i)}\ \mathbf{Q}_{78}^{(i)}]^T \tag{24}$$

The equation of motion of connected beams can be written as

$$\bar{\mathbf{M}}\ddot{\bar{\mathbf{e}}}+\bar{\mathbf{K}}\bar{\mathbf{e}}=\bar{\mathbf{Q}} \tag{25}$$

where, the nodal coordinate vector, the mass matrix, the stiffness matrix and generalized force vector can be described as follows.

$$\bar{\mathbf{e}}=\begin{bmatrix}\mathbf{e}_{12}^{(1)}\\ \mathbf{e}_{34}^{(1)}\\ \mathbf{e}_{56}^{(1)}\\ \mathbf{e}_{78}^{(1)}\\ \mathbf{e}_{56}^{(2)}\\ \mathbf{e}_{78}^{(2)}\end{bmatrix}=\begin{bmatrix}\mathbf{e}_{12}^{(1)}\\ \mathbf{e}_{34}^{(1)}\\ \mathbf{e}_{12}^{(2)}\\ \mathbf{e}_{34}^{(2)}\\ \mathbf{e}_{56}^{(2)}\\ \mathbf{e}_{78}^{(2)}\end{bmatrix} \tag{26}$$

$$\bar{\mathbf{M}}=\begin{bmatrix}\mathbf{M}_{11}^{(1)} & \mathbf{M}_{12}^{(1)} & \mathbf{M}_{13}^{(1)} & \mathbf{M}_{14}^{(1)} & 0 & 0\\ \mathbf{M}_{21}^{(1)} & \mathbf{M}_{22}^{(1)} & \mathbf{M}_{23}^{(1)} & \mathbf{M}_{24}^{(1)} & 0 & 0\\ \mathbf{M}_{31}^{(1)} & \mathbf{M}_{32}^{(1)} & \mathbf{M}_{33}^{(1)}+\mathbf{M}_{11}^{(2)} & \mathbf{M}_{34}^{(1)}+\mathbf{M}_{12}^{(2)} & \mathbf{M}_{13}^{(2)} & \mathbf{M}_{14}^{(2)}\\ \mathbf{M}_{41}^{(1)} & \mathbf{M}_{42}^{(1)} & \mathbf{M}_{43}^{(1)}+\mathbf{M}_{21}^{(2)} & \mathbf{M}_{44}^{(1)}+\mathbf{M}_{22}^{(2)} & \mathbf{M}_{23}^{(2)} & \mathbf{M}_{24}^{(2)}\\ 0 & 0 & \mathbf{M}_{31}^{(2)} & \mathbf{M}_{32}^{(2)} & \mathbf{M}_{33}^{(2)} & \mathbf{M}_{34}^{(2)}\\ 0 & 0 & \mathbf{M}_{41}^{(2)} & \mathbf{M}_{42}^{(2)} & \mathbf{M}_{43}^{(2)} & \mathbf{M}_{44}^{(2)}\end{bmatrix} \tag{27}$$

$$\bar{\mathbf{K}}=\begin{bmatrix}\mathbf{K}_{11}^{(1)} & \mathbf{K}_{12}^{(1)} & \mathbf{K}_{13}^{(1)} & \mathbf{K}_{14}^{(1)} & 0 & 0\\ \mathbf{K}_{21}^{(1)} & \mathbf{K}_{22}^{(1)} & \mathbf{K}_{23}^{(1)} & \mathbf{K}_{24}^{(1)} & 0 & 0\\ \mathbf{K}_{31}^{(1)} & \mathbf{K}_{32}^{(1)} & \mathbf{K}_{33}^{(1)}+\mathbf{K}_{11}^{(2)} & \mathbf{K}_{34}^{(1)}+\mathbf{K}_{12}^{(2)} & \mathbf{K}_{13}^{(2)} & \mathbf{K}_{14}^{(2)}\\ \mathbf{K}_{41}^{(1)} & \mathbf{K}_{42}^{(1)} & \mathbf{K}_{43}^{(1)}+\mathbf{K}_{21}^{(2)} & \mathbf{K}_{44}^{(1)}+\mathbf{K}_{22}^{(2)} & \mathbf{K}_{23}^{(2)} & \mathbf{K}_{24}^{(2)}\\ 0 & 0 & \mathbf{K}_{31}^{(2)} & \mathbf{K}_{32}^{(2)} & \mathbf{K}_{33}^{(2)} & \mathbf{K}_{34}^{(2)}\\ 0 & 0 & \mathbf{K}_{41}^{(2)} & \mathbf{K}_{42}^{(2)} & \mathbf{K}_{43}^{(2)} & \mathbf{K}_{44}^{(2)}\end{bmatrix} \tag{28}$$

$$\bar{\mathbf{Q}}=\begin{bmatrix}\mathbf{Q}_{12}^{(1)}\\ \mathbf{Q}_{34}^{(1)}\\ \mathbf{Q}_{56}^{(1)}+\mathbf{Q}_{12}^{(2)}\\ \mathbf{Q}_{78}^{(1)}+\mathbf{Q}_{34}^{(2)}\\ \mathbf{Q}_{56}^{(2)}\\ \mathbf{Q}_{78}^{(2)}\end{bmatrix} \tag{29}$$

3.2 Joint at corner part

Let us consider the connection method of beam elements at the corner part as shown in Fig. 3. In this case, the coordinate transformation matrix is necessary to connect these beam elements. Since the translational coordinates of the point P and point Q are same, the following equation can be obtained.

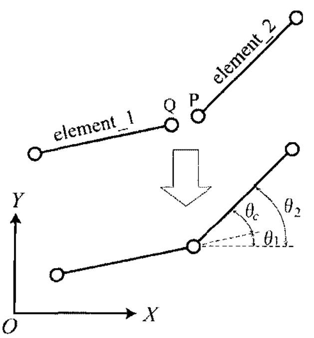

Fig. 3 Joint at corner part

$$\mathbf{e}_{56}^{(1)}=\mathbf{e}_{12}^{(2)} \tag{30}$$

The global slope coordinates of the point P and the point Q are described by using the angle θ and axial strain ε of the connection point as follows.

$$e_7^{(1)}=(1+\varepsilon_Q^{(1)})\cos\theta_1$$

$$e_8^{(1)}=(1+\varepsilon_Q^{(1)})\sin\theta_1$$

$$e_3^{(2)}=(1+\varepsilon_P^{(2)})\cos\theta_2$$

$$e_4^{(2)}=(1+\varepsilon_P^{(2)})\sin\theta_2$$

where, θ_1 and θ_2 means the angle, and $\varepsilon_Q^{(1)}$ and $\varepsilon_P^{(2)}$ mean the axial strain at the point P and the point Q as shown in Fig. 3. Here, we assume that the axial strain is infinitesimal in our study, these equations can be written as

$$e_7^{(1)}=(1+\varepsilon_Q^{(1)})\cos\theta_1=\cos\theta_1$$

$$e_8^{(1)}=(1+\varepsilon_Q^{(1)})\sin\theta_1=\sin\theta_1$$

$$e_3^{(2)}=(1+\varepsilon_P^{(2)})\cos\theta_2=\cos\theta_2$$

$$e_4^{(2)}=(1+\varepsilon_P^{(2)})\sin\theta_2=\sin\theta_2$$

$e_3^{(2)}$ and $e_4^{(2)}$ can be written as follows by using the angle θ_1 and θ_c.

$$\begin{aligned}e_3^{(2)}&=\cos\theta_2=\cos(\theta_1+\theta_c)\\ &=\cos\theta_1\cos\theta_c-\sin\theta_1\sin\theta_c\end{aligned}$$

$$\begin{aligned}e_4^{(2)}&=\sin\theta_2=\sin(\theta_1+\theta_c)\\ &=\sin\theta_1\cos\theta_c-\cos\theta_1\sin\theta_c\end{aligned}$$

where, θ_c means the connection angle of two beam elements shown in Fig. 3, and this is a constant value. Then, coordinates $e_3^{(2)}$ and $e_4^{(2)}$ can be described by the coordinates $e_7^{(1)}$ and $e_8^{(1)}$ and connection angle θ_c as

$$\begin{aligned}\begin{bmatrix}e_3^{(2)}\\ e_4^{(2)}\end{bmatrix}&=\begin{bmatrix}\cos\theta_c & -\sin\theta_c\\ \sin\theta_c & \cos\theta_c\end{bmatrix}\begin{bmatrix}\cos\theta_1\\ \sin\theta_1\end{bmatrix}\\ &=\begin{bmatrix}\cos\theta_c & -\sin\theta_c\\ \sin\theta_c & \cos\theta_c\end{bmatrix}\begin{bmatrix}e_7^{(1)}\\ e_8^{(1)}\end{bmatrix}\end{aligned} \tag{31}$$

Consequently, the coordinate transformation matrix of the beam element_2 to connect with the beam element 1 can be written as

$$\mathbf{A}=\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \cos\theta_c & -\sin\theta_c & 0 & 0 & 0 & 0 \\ 0 & 0 & \sin\theta_c & \cos\theta_c & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (32)$$

Since the angle θ_c is a constant, the coordinate transformation matrix $\mathbf{A}$ of the beam element formulated by the absolute nodal coordinate becames constant matrix even in the large displacement problem.

The equation of motion of two beam elements after the connection can be written as

$$\bar{\mathbf{M}}\ddot{\bar{\mathbf{e}}}+\bar{\mathbf{K}}\bar{\mathbf{e}}=\bar{\mathbf{Q}} \quad (33)$$

where,

$$\bar{\mathbf{e}}=[\mathbf{e}_{12}^{(1)}, \mathbf{e}_{34}^{(1)}, \mathbf{e}_{56}^{(1)}, \mathbf{e}_{78}^{(1)}, \mathbf{e}_{56}^{(2)}, \mathbf{e}_{78}^{(2)}]^T \quad (34)$$

$$\bar{\mathbf{M}}=\begin{bmatrix} \mathbf{M}_{11}^{(1)} & \mathbf{M}_{12}^{(1)} & \mathbf{M}_{13}^{(1)} & \mathbf{M}_{14}^{(1)} & \mathbf{0} & \mathbf{0} \\ \mathbf{M}_{21}^{(1)} & \mathbf{M}_{22}^{(1)} & \mathbf{M}_{23}^{(1)} & \mathbf{M}_{24}^{(1)} & \mathbf{0} & \mathbf{0} \\ \mathbf{M}_{31}^{(1)} & \mathbf{M}_{32}^{(1)} & \mathbf{M}_{33}^{(1)}+\hat{\mathbf{M}}_{11}^{(2)} & \mathbf{M}_{34}^{(1)}+\hat{\mathbf{M}}_{12}^{(2)} & \hat{\mathbf{M}}_{13}^{(2)} & \hat{\mathbf{M}}_{14}^{(2)} \\ \mathbf{M}_{41}^{(1)} & \mathbf{M}_{42}^{(1)} & \mathbf{M}_{43}^{(1)}+\hat{\mathbf{M}}_{21}^{(2)} & \mathbf{M}_{44}^{(1)}+\hat{\mathbf{M}}_{22}^{(2)} & \hat{\mathbf{M}}_{23}^{(2)} & \hat{\mathbf{M}}_{24}^{(2)} \\ \mathbf{0} & \mathbf{0} & \hat{\mathbf{M}}_{31}^{(2)} & \hat{\mathbf{M}}_{32}^{(2)} & \hat{\mathbf{M}}_{33}^{(2)} & \hat{\mathbf{M}}_{34}^{(2)} \\ \mathbf{0} & \mathbf{0} & \hat{\mathbf{M}}_{41}^{(2)} & \hat{\mathbf{M}}_{42}^{(2)} & \hat{\mathbf{M}}_{43}^{(2)} & \hat{\mathbf{M}}_{44}^{(2)} \end{bmatrix} \quad (35)$$

$$\bar{\mathbf{K}}=\begin{bmatrix} \mathbf{K}_{11}^{(1)} & \mathbf{K}_{12}^{(1)} & \mathbf{K}_{13}^{(1)} & \mathbf{K}_{14}^{(1)} & \mathbf{0} & \mathbf{0} \\ \mathbf{K}_{21}^{(1)} & \mathbf{K}_{22}^{(1)} & \mathbf{K}_{23}^{(1)} & \mathbf{K}_{24}^{(1)} & \mathbf{0} & \mathbf{0} \\ \mathbf{K}_{31}^{(1)} & \mathbf{K}_{32}^{(1)} & \mathbf{K}_{33}^{(1)}+\hat{\mathbf{K}}_{11}^{(2)} & \mathbf{K}_{34}^{(1)}+\hat{\mathbf{K}}_{12}^{(2)} & \hat{\mathbf{K}}_{13}^{(2)} & \hat{\mathbf{K}}_{14}^{(2)} \\ \mathbf{K}_{41}^{(1)} & \mathbf{K}_{42}^{(1)} & \mathbf{K}_{43}^{(1)}+\hat{\mathbf{K}}_{21}^{(2)} & \mathbf{K}_{44}^{(1)}+\hat{\mathbf{K}}_{22}^{(2)} & \hat{\mathbf{K}}_{23}^{(2)} & \hat{\mathbf{K}}_{24}^{(2)} \\ \mathbf{0} & \mathbf{0} & \hat{\mathbf{K}}_{31}^{(2)} & \hat{\mathbf{K}}_{32}^{(2)} & \hat{\mathbf{K}}_{33}^{(2)} & \hat{\mathbf{K}}_{34}^{(2)} \\ \mathbf{0} & \mathbf{0} & \hat{\mathbf{K}}_{41}^{(2)} & \hat{\mathbf{K}}_{42}^{(2)} & \hat{\mathbf{K}}_{43}^{(2)} & \hat{\mathbf{K}}_{44}^{(2)} \end{bmatrix} \quad (36)$$

$$\bar{\mathbf{Q}}=\begin{bmatrix} \mathbf{Q}_{12}^{(1)} \\ \mathbf{Q}_{34}^{(1)} \\ \mathbf{Q}_{56}^{(1)}+\hat{\mathbf{Q}}_{12}^{(2)} \\ \mathbf{Q}_{78}^{(1)}+\hat{\mathbf{Q}}_{34}^{(2)} \\ \hat{\mathbf{Q}}_{56}^{(2)} \\ \hat{\mathbf{Q}}_{78}^{(2)} \end{bmatrix} \quad (37)$$

$$\hat{\mathbf{M}}^{(2)}=\mathbf{A}^T\mathbf{M}^{(2)}\mathbf{A} \quad (38)$$

$$\hat{\mathbf{K}}^{(2)}=\mathbf{A}^T\mathbf{K}^{(2)}\mathbf{A} \quad (39)$$

$$\hat{\mathbf{Q}}^{(2)}=\mathbf{A}^T\mathbf{Q}^{(2)} \quad (40)$$

The equation of motion of the connected beam elements can be described as an ordinary differential equation by using the coordinate transformation matrix for the absolute nodal coordinate formulation. Since the coordinate transformation matrix is constant, the mass matrix and the bending stiffness matrix of Eq. (33) are constant even in the large displacement problem.

4. Numerical Simulation and Experimental Result

4.1 Vibration problem

In this section, we examine the validity of the frame structure modeling for the beam element formulated by the absolute nodal coordinate. Fig. 4 shows the 1/8 scale model of the container crane on a shaking table for the seismic experiment. The seismic characteristics of the container crane were investigated from the experiment in 1997 (Kanayama, and Kashiwazaki, 1998 ; Kuribara and Kobayashi, 2000). Fig. 5 shows the analytical

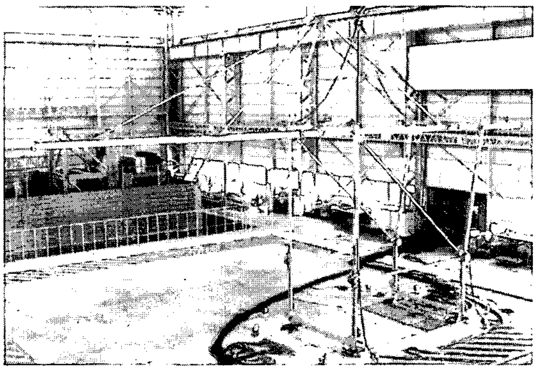

Fig. 4 Experiment of crane model (1/8 scale)

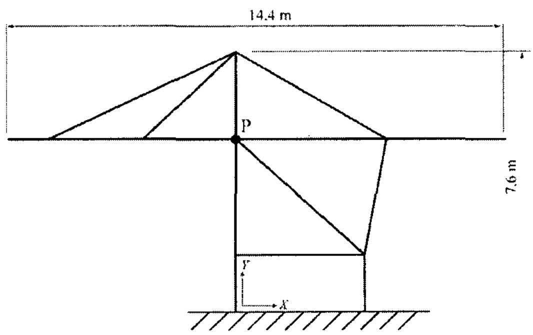

Fig. 5 Simulation model of crane

model of this structure. The table and both legs of the crane are connected by the revolute joints in this model.

Fig. 6(a) shows the horizontal acceleration of the shaking table. We used this data from the observed earthquake at Hachinohe harbor in Japan in 1968. The maximum value of this accerelation was set to 1.69 m/s^2, and the time axis was 1/8 times of the original data. Fig. 6(b) shows the experimental seismic response at the point P in Fig. 5. From this experimental result, we can observe that the 1st mode of this model is 1.69 Hz, and the damping ratio of this mode is 0.18%. In order to make the simulation model, the coefficient $\alpha(=2\zeta_1\omega_1)$ of the damping matrix $\mathbf{C}=\alpha\mathbf{M}$ is set to 0.0373. We use the software "MATLAB" to make simulation programs. Fig. 6(c) and (d) show the simulation results at the point P by the linear finite element method (FEM) and the proposed method.

The frequency of 6 Hz is included from 3 sec to 5 sec in the experimental result, but this is not included in both simulation results. Because the analysis model is different in detail to the real model. The small difference can be confirmed in the experimental result and the simulation results, but these results are good agreement with each other. It is proven that the proposed method is effective for the modeling of the frame structure.

4.2 Large displacement problem

Next, we examine the validity of the proposed method for the large displacement problem. Fig. 7 shows the example model of free falling of a

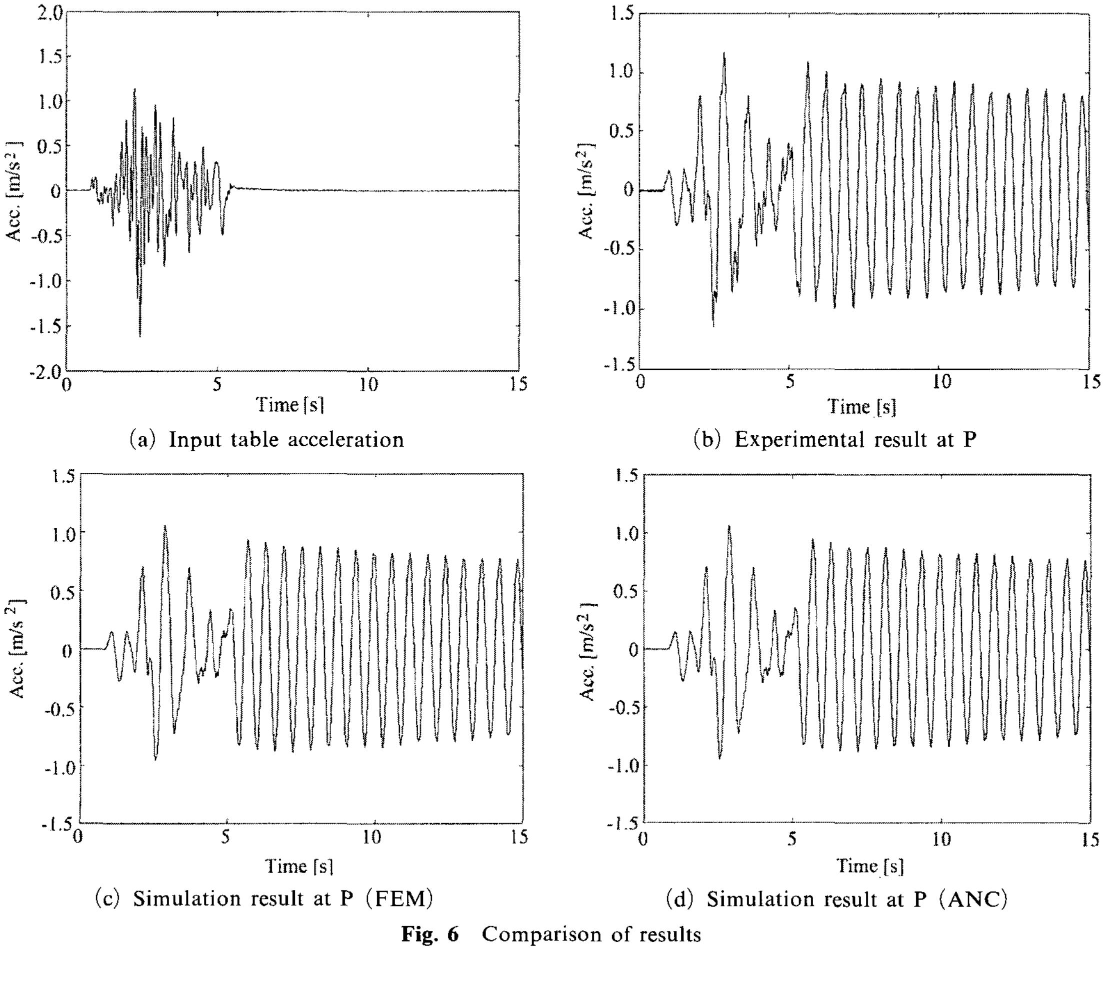

Fig. 6 Comparison of results

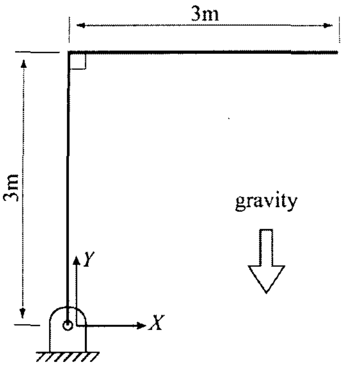

Fig. 7 Analysis model

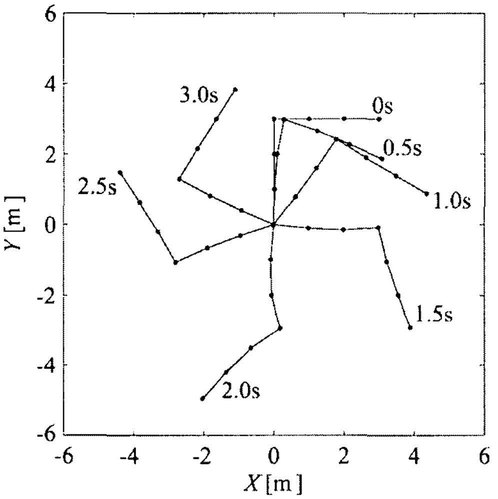

Fig. 8 Simulation result (RecurDyn)

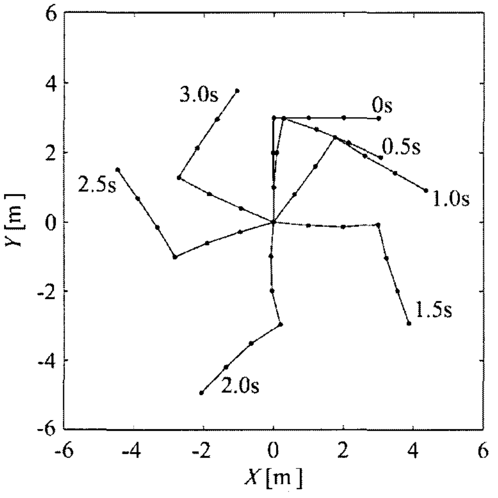

Fig. 9 Simulation result (proposed method)

flexible pendulum. This pendulum has the rectangular corner part, and this model is connected to ground by the revolute joint. The beam has a mass density of 10 kg/m^3, a modulus of elasticity of 5×10^8 N/m^2 and no damping effect. The cross section of the beam element is square, and the size is 1 cm$\times$1 cm. The pendulum is divided into 6 elements.

In order to verify the proposed method, we calculated this problem by using the MBD software "RecurDyn" Figs. 8 and 9 show the deformed shapes of the pendulum at different time. It can be seen that the results of the proposed method and RecurDyn software are fairly in good agreement. From these results, it can be confirmed that the proposed method is effective for the large displacement problem.

5. Conclusions

This paper proposed a formulation of frame structures by making use of the beam element described by the absolute nodal coordinate procedure. This beam element uses not the angles but the slopes as the nodal coordinates. Beam element connection can not be performed by assembling the matrix superposition of the beam element by the traditional FE transformation matrix. Though the connection can be made by using constraint equations, equations of motion of the whole system become differnetial algebraic equations. In this paper, the coordinate transformation matrix was first derived and the matrix was introduced when the element matrices are assembled to connect the beam elements in structural analysis. The obtained equations of motion become ordinary differnetial equation not DAE. The constant mass and bending stiffness matrices are formulated for large displacement problem.

To verify the proposed method, a vibration problem and a large displacement problem were solved and the result was compared with the results of the seismic experiment and the linear FE method. These results show in good agreement, thus the proposed method was confirmed to be effective for dynamical problems. Since the method can apply for the analysis of the frame structure model with rigid body motion, analysis of falling behavior of the cranes, simulation of large displacement excitation of a large scale shaking table, and analysis of structural behavior due to the movement of soil foundation caused by liquefaction, etc can be performed.

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