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Study on the Dynamic Model and Simulation of a Flexible Mechanical Arm Considering its Random Parameters

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Randomness exists in engineering. Tolerance, assemble-error, environment temperature and wear make the parameters of a mechanical system uncertain. So the behavior or response of the mechanical system is uncertain. In this paper, the uncertain parameters are treated as random variables. So if the probability distribution of a random parameter is known, the simulation of mechanical multibody dynamics can be made by Monte-Carlo method. Thus multibody dynamics simulation results can be obtained in statistics. A new concept called functional reliability is put forward in this paper, which can be defined as the probability of the dynamic parameters (such as position, orientation, velocity, acceleration etc.) of the key parts of a mechanical multibody system belong to their tolerance values. A flexible mechanical arm with random parameters is studied in this paper. The length, width, thickness and density of the flexible arm are treated as random variables and Gaussian distribution is used with given mean and variance. Computer code is developed based on the dynamic model and Monte-Carlo method to simulate the dynamic behavior of the flexible arm. At the same time the end effector's locating reliability is calculated with circular tolerance area. The theory and method presented in this paper are applicable on the dynamics modeling of general multibody systems.

Key Words : Multibody System, Flexible Body, Dynamics, Rondom Parameters, Monte-Carlo Method, Reliability, Flexible Mechanical Arm, Kane's Equation, Simulation, Matlab

1. Introduction

Multibody dynamics has wide application in engineering, such as space flexible mechanical arm, flexible linkage, antenna of satellite etc. In these systems, coupling between elastic deformation and rigid body movement usually exists, thus the dynamic behavior becomes complicated (Huston, 1991 ; Liu, 2000). Conventional dynamic methods usually treat such systems as deter mined, which means the dynamic parameters are constant without variation. Thus the results of dynamic calculation are also determined. But this isn't the truth. In practice, the geometric parameters of a mechanical system such as length are uncertain because of design tolerance, assembly error, wear and the material parameters such as density are also uncertain because of working temperature and material ununiformity. Thus, dynamic response of mechanical systems is also uncertain. In order to describe the exact dynamic behavior, it is necessary to take these uncertain parameters into account. These uncertain parameters can be classified to probabilistic parameters and fuzzy parameters and the probabilistic parameter can be further divided into random variables, random process and random field. In

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this paper, random variable will be discussed mainly. As an illustrative example, a flexible mechanical arm with probabilistic parameters is studied in this paper. Base on Kane's equation, dynamic model with "dynamic stiffness" is established and Monte-Carlo method is used for simulation. The simulation results are given in terms of statistics and locating reliability of the end effector is also calculated.

2. The Probabilistic Parameters and Functional Reliability of General Multibody Systems

In this section, the classification of probabilistic parameters, Monte-Carlo method and the concept of functional reliability will be discussed.

2.1 Classification of the probabilistic parameters

The probabilistic parameters of general multibody systems can be classified as follows:

(1) Load: The load of mechanical multibody systems is usually uncertain, For example, the end effector of the flexible arm can be used to grip objects and usually the weight of the objects are probabilistic.

(2) Material: The density, elastic modulus, poisson's ratio, damping, coefficient of friction of mechanical multibody systems may be probabilistic because of manufacture, working conditions etc.

(3) Geometric parameters: The length, width, thickness, cross section area inertia moment and other geometric parameters of mechanical multibody systems may be random due to tolerance, assembly, wear etc.

(4) Initial conditions and boundary conditions: The randomness of material and geometric parameters will be discussed mainly in this paper

2.2 Simulation by Monte-Carlo method

By means of numerical simulation, Monte-Carlo method can solve mathematics and engineering probabilistic problems effectively. Generally speaking, there are 3 steps to make Monte-Carlo simulation.

(1) Sampling of probabilistic parameters: Usually, the distribution function of the probabilistic parameters is assumed as known condition. By computer code such as MATLAB, it is easy to get random numbers satisfying the distribution function such as Gaussian distribution, uniform distribution.

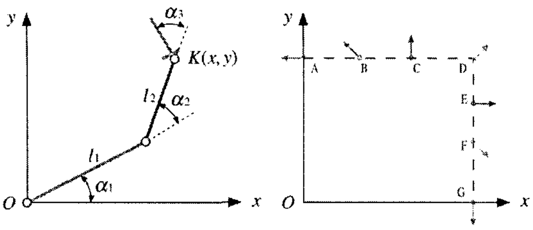
(2) Calculating the response of every sample: In this step, the dynamic equations are solved according to the value of sample. This process is the same as the determined method and each sample simulation can be called an experiment. This step can take much calculation time because there may be thousands of or even millions of samples.

(3) Calculating the simulation results in statistics. Through the first 2 steps, many sample values of the result or response are calculated and these numbers are used to calculate the mean, variance and even the distribution function of the simulation result.

2.3 Functional reliability of mechanical multibody systems

Reliability is usually related to fatigue, fracture and life of mechanical systems conventionally. But with the development of high precise device in robot, machinery and space engineering, the concept of functional reliability of mechanical mulibody systems is put forward and becomes more and more important (Schneider, 1994; Rao, 2001).

Functional reliability is the reliability of mechanical system fulfilling its scheduled motion mission. There may be some restrictions on the location, orientation, velocity and trace of some key pats of the mechanical system. If a tolerance value or area or space is defined at some time or position, then the function reliability can be calculated. Figure 1(a) is the schematic sketch of a two-arms manipulator and (b) is the location and orientation requirement of some special working points. The ideal location and orientation of these points may be A(0, 1, 180°), B(0.35,



(a) Schematic sketch (b) Requirement of points

Fig. 1 Sample of functional reliability

1, 135°), C(0.7, 1, 90°), D(1, 1, 45°), E(1, 0.7, 0°), F(1, 0.35, −45°), G(1, 0, −90°). If tolerance values or areas are defined for these points, then the locating or orient reliability can be calculated. In Figure 3, a circular tolerance area is defined for the end effector's locating reliability at the time of 2 second. If the end effector is in this area it is reliable else it is a failure. So the locating reliability can be expressed as

$$R_L=\frac{N_S}{N_T} \tag{1}$$

Where, N_S is the number of successful experiments and N_T is the total number of experiments.

3. Dynamic Modeling of a Flexible Mechanical Arm

In this section, the deformation description of a cantilever beam and dynamic model of a flexible mechanical arm will be introduced.

3.1 Deformation kinematics of a cantilever beam

In order to simplify the dynamic analysis, some assumptions are made: (1) deformation of the beam is elastic, small and in the paper plane (2) it is a Euler-Bernoulli beam (3) the axial line can not be elongated or compressed.

In Figure 2, O-$x_1x_2x_3$ is the local coordinate fixed on the beam and $\boldsymbol{b}_1$, $\boldsymbol{b}_2$, $\boldsymbol{b}_3$ are unit vectors. The axial line of the beam is in coincidence with Ox_1 when there is no deformation. Point P moves to Point P^* after deformation.

According to the assumptions (1)-(3) and geometrical deformation constraint method and

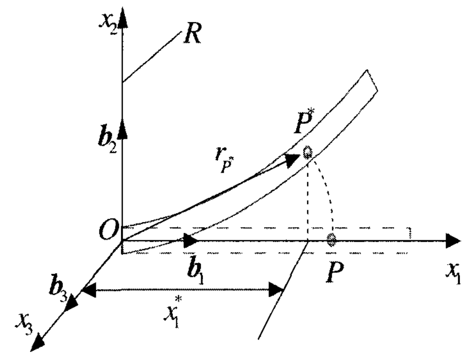


Fig. 2 Deformation of a cantilever beam

by modal condensation (He, 2003), the physical coordinates can be transferred into modal coordinates. Thus the deformation can be expressed as:

$$\begin{cases} u_1=-\frac{1}{2}\int_0^x\left(\frac{\partial u_2(\delta,\ t)}{\partial\delta}\right)^2 d\delta=-\frac{1}{2}H_{ij}q_iq_j \\ u_2=\phi_iq_i \qquad (i,\ j=1,\ 2,\ \cdots,\ N) \end{cases} \tag{2}$$

$$H_{ij}=\int_0^x\frac{d\phi_i(\delta)}{d\delta}\frac{d\phi_j(\delta)}{d\delta}d\delta \qquad (i,\ j=1,\ 2,\ \cdots,\ N) \tag{3}$$

Where u_1 and u_2 are the deformation of point P in Ox_1 and Ox_2 direction respectively. ϕ_i $(i=1,\ 2,\ \cdots,\ N)$ and q_i are the No. i modal shape function and modal coordinate. N is the total number of modals.

3.2 Dynamic analysis

A planar flexible mechanical arm is shown in Figure 3. L is the length, h_1 is the width, h_2 is the thickness, $A=h_1h_2$ is the cross-sectional area, ρ is the density, EI is the bending stiffness. One end of the arm is fixed on an electric motor and the other end is equipped with an end effector. τ is the driving torque and J_h is the moment of inertia of the electric motor. θ is the angular displacement of the arm. OX_1X_2 is the inertial coordinate system and Ox_1x_2 is the local coordinate system fixed on the arm.

Assume that $\boldsymbol{r}_{P^*}$ is the position vector of P^*, $\boldsymbol{\omega}$ is the angular velocity vector of the beam. V_P is the velocity vector of P^*, $\boldsymbol{a}_{P^*}$ is the acceleration vector of P^*, $\boldsymbol{\varepsilon}$ is the angular acceleration

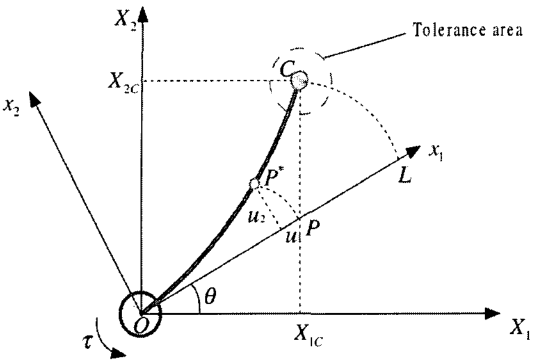


Fig. 3 Planar flexible mechanical arm system

vector of the beam. These vectors are expressed as:

$$\boldsymbol{r}_{P^*}=\left(x-\frac{1}{2}H_{ij}q_iq_j\right)\boldsymbol{b}_1+\phi_iq_i\boldsymbol{b}_2 \quad (4)$$

$$\boldsymbol{\omega}=\dot{\theta}\,\boldsymbol{b}_3 \quad (5)$$

$$\begin{aligned}\boldsymbol{V}_{P^*}=&-(\phi_iq_i\dot{\theta}+H_{ij}\dot{q}_iq_j)\,b_1\\&+\left(x\dot{\theta}+\phi_i\dot{q}_i-\frac{1}{2}\dot{\theta}H_{ij}q_iq_j\right)b_2\end{aligned} \quad (6)$$

$$\begin{aligned}\boldsymbol{a}_{P^*}=&-\left(x\dot{\theta}^2+\phi_iq_i\ddot{\theta}+2\phi_i\dot{q}_i\dot{\theta}-\frac{1}{2}\dot{\theta}H_{ij}q_iq_j\right)b_1\\&+(+H_{ij}\ddot{q}_iq_j+H_{ij}\dot{q}_i\dot{q}_j)\,\boldsymbol{b}_1+(x\ddot{\theta}+\phi_i\ddot{q}_i)\,\boldsymbol{b}_2\\&+\left(-\phi_iq_i\dot{\theta}^2-\frac{1}{2}\ddot{\theta}H_{ij}q_iq_{ij}-2\dot{\theta}H_{ij}\dot{q}q_j\right)\boldsymbol{b}_2\end{aligned} \quad (7)$$

$$\boldsymbol{\varepsilon}=\ddot{\theta}\,\boldsymbol{b}_3 \quad (8)$$

Take $\dot{\theta}$, $\dot{q}_1$, $\cdots$, $\dot{q}_N$ as generalize speed and the partial velocity vectors are listed in Table 1.

The generalize inertia force of the infinitesimal section where point P^* located is:

$$\mathrm{d}F_{\dot{\theta}}^*=-\rho A\mathrm{d}x\,\boldsymbol{a}_{P^*}\cdot\frac{\partial \boldsymbol{V}_{P^*}}{\partial\dot{\theta}} \quad (9)$$

$$\mathrm{d}F_{\dot{q}_i}^*=-\rho A\mathrm{d}x\,\boldsymbol{a}_{P^*}\cdot\frac{\partial \boldsymbol{V}_{P^*}}{\partial\dot{q}_i}\quad (i=1,2,\cdots,N) \quad (10)$$

The generalize inertia force of the electric motor is

$$F_{H\dot{\theta}}^*=-J_h\boldsymbol{\varepsilon}\cdot\frac{\partial\boldsymbol{\omega}}{\partial\dot{\theta}} \quad (11)$$

$$F_{H\dot{q}_i}^*=-J_h\boldsymbol{\varepsilon}\cdot\frac{\partial\boldsymbol{\omega}}{\partial\dot{q}_i}\quad (i=1,2,\cdots,N) \quad (12)$$

The generalize drive force is

$$F_{\dot{\theta}}=\tau(t) \quad (13)$$

$$F_{\dot{q}_i}=-K_{ij}q_j\quad (j=1,2,\cdots,N) \quad (14)$$

Table 1 Partial velocity vectors

	V_{P^*}	ω
$\dot{\theta}$	$-\phi_iq_i\boldsymbol{b}_1+\left(x-\frac{1}{2}H_{ij}q_iq_j\right)\boldsymbol{b}_2$ $(i,j=1,2,\cdots,N)$	$\boldsymbol{b}_3$
$\dot{q}_i$ $(i=1,2,\cdots,N)$	$-H_{ij}q_i\boldsymbol{b}_1+\phi_i\boldsymbol{b}_2$ $(i,j=1,2,\cdots,N)$	0

Where K_{ij} is the i-row and j-column element of the modal stiffness matrix-K.

The orthogonality of the matrix K gives:

$$K_{ij}=\int_0^L EI\frac{\mathrm{d}^2\phi_i}{\mathrm{d}x^2}\frac{\mathrm{d}^2\phi_j}{\mathrm{d}x^2}\mathrm{d}x=\begin{cases}\omega_i^2 & i=J\\ 0 & i\neq j\end{cases} \quad (15)$$

where ω_i is the i-th natural frequency of the beam.

3.3 Dynamic equation

Based on Kane's equation (Kane, 1985), the governing equation of the flexible mechanical arm can be written as:

$$\begin{cases}F_{\dot{\theta}}+\int_L\mathrm{d}F_{\dot{\theta}}^*+F_{H\dot{\theta}}^*=0\\ F_{\dot{q}_i}+\int_L\mathrm{d}F_{\dot{q}_2}^*+F_{H\dot{q}_i}^*=0\end{cases} \quad (16)$$

Substitute (9)-(14) in (16), make some simplification, and take the random variables into account, the equation (16) becomes:

$$\begin{cases}\left(J_h+\frac{1}{3}\tilde{\rho}\tilde{A}\tilde{L}^3\right)\ddot{\tilde{\theta}}+\sum_{i=1}^{N}\left(\tilde{\rho}\tilde{A}\int_0^{\tilde{L}}x\phi_idx\right)\ddot{\tilde{q}}_i=\tau\\ \left(\tilde{\rho}\tilde{A}\int_0^{\tilde{L}}x\phi_pdx\right)\ddot{\tilde{\theta}}+\ddot{\tilde{q}}_p+(\omega_p^2-\dot{\theta}^2)\,\tilde{q}_p\\ +\sum_{i=1}^{N}\left(\int_0^{\tilde{L}}\tilde{\rho}\tilde{A}xH_{pi}dx\right)\tilde{q}_i\dot{\tilde{\theta}}^2=0\end{cases} \quad (17)$$

where the symbol "~" represents the stochastic parameters.

4. An Illustrative Example

The schematic sketch of a flexible mechanical arm system is shown in Figure 3. The density, length, width and thickness of the flexible mechanical arm are chosen as random variables. Based on central limit law (Li, 1996), they are assumed to satisfy Gaussian distribution. Table 2 is the mean and variance of these parameters.

Table 2 Mean and variance

Stochastic parameter	Density (kg/m³)	Length (m)	Width (m)	Thickness (m)
mean	7.8×10^3	0.53	0.032	8.5×10^{-4}
variance	100	1×10^{-6}	1×10^{-8}	1×10^{-10}

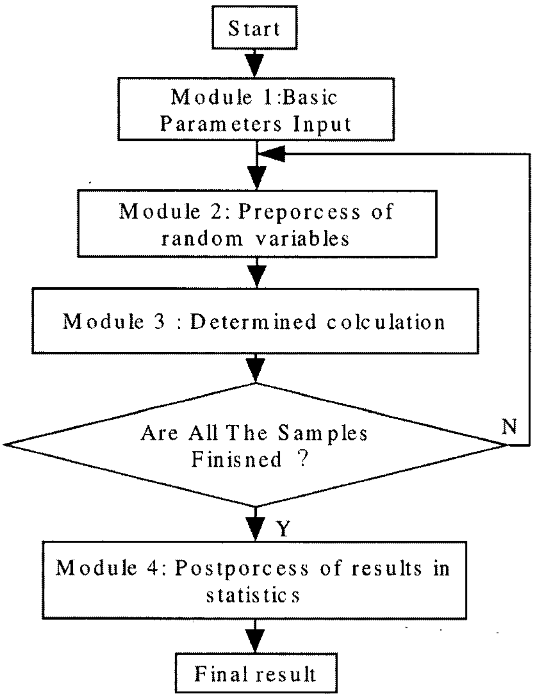


Fig. 4 Flow chart of the computer code

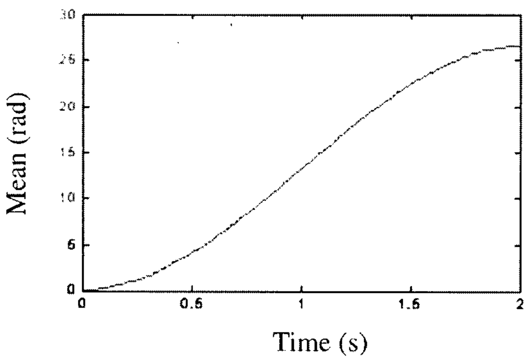


Fig. 5 Mean of the angular displacement

$J_h=1.46\times10^{-2}$ kgm², Elastic modulus is 2.22×10^{11} N/m² and $\tau=1-t$, where t is the simulation time. In order to calculate the locating reliability at $t=2s$, circular tolerance area is defined. The center of the circle is $A(X_{1A}, X_{2A})$ and the radius is R_A. $X_{1A}=-0.025$ m, $X_{2A}=0.5260$ m, $R_A=0.015$ m.

Computer code is developed by MATLAB (Figure 4) and in order to calculate the influence of each single stochastic parameter the other stochastic parameters can be treated as determined. In this paper, the density, length, width and thickness are treated as random variables at the same time (1000 samples). The result of angular displacement, angular velocity, end point deformation and end point deformation velocity are given in Figure 5~12.

For the circular tolerance area, the formula to calculate the locating reliability of the end

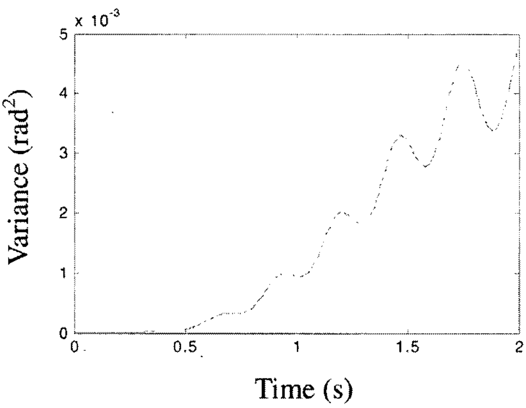


Fig. 6 Variance of angular displacement

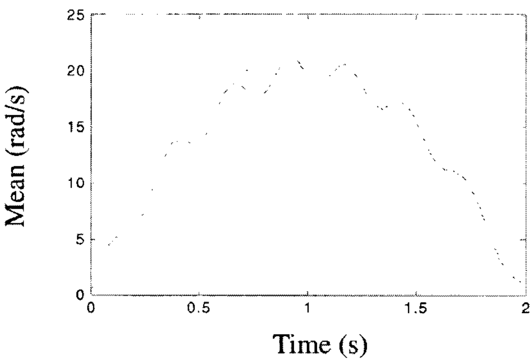


Fig. 7 Mean of angular velocity

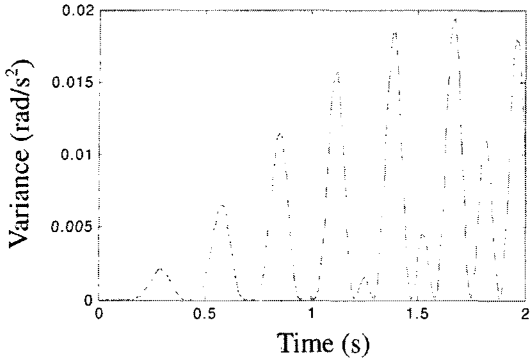


Fig. 8 Variance of angular velocity

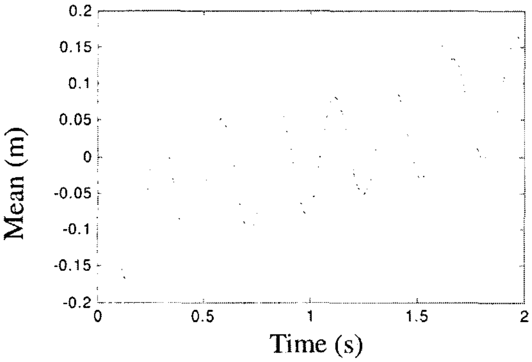


Fig. 9 Mean of end point deformation

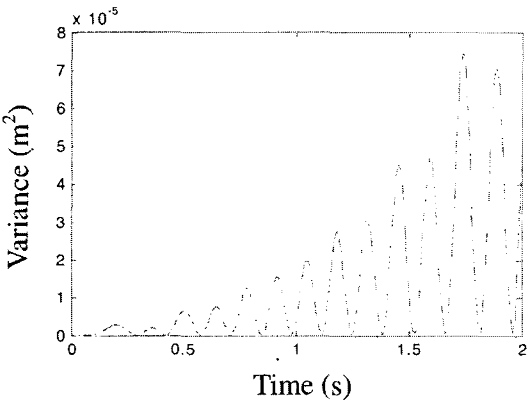


Fig. 10 Variance of end point deformation

effector is:

$$R_L=P(\sqrt{(X_{1C}-X_{1A})^2+(X_{2C}-X_{2A})^2}\le R_A) \quad (18)$$

Based on the 1000 samples, the locating reliability at $t=2s$ is $R_L=99.7\%$.

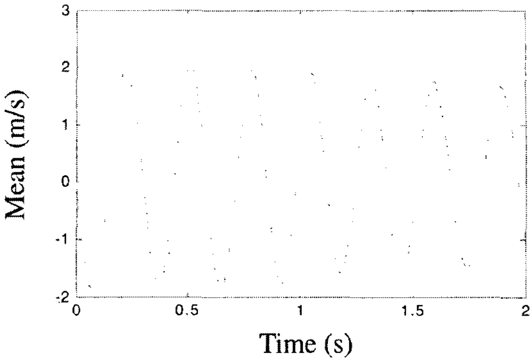


Fig. 11 Mean of end point deformation velocity

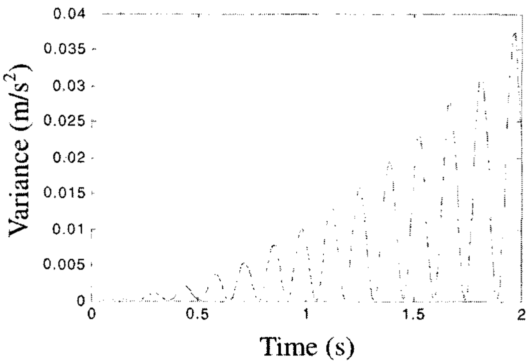


Fig. 12 Variance of end point deformation velocity

5. Conclusion

(1) Stochastic dynamic model can solve engineering problems more effectively and comprehensively compared to conventional determined methods. The dynamic response is usually in the form of statistics.

(2) Based on the stochastic dynamic model, function reliability can be calculated and this kind of reliability may be used to verify the motion function or accuracy of mechanical systems.

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