The Korean Communications
in Statistics Vol. 12 No. 3, 2005
pp. 807-818

Regression Quantiles Under Censoring and Truncation[1]

Jinho Park[2] and Jinmi Kim[3]

Abstract

In this paper we propose an estimation method for regression quantiles with left-truncated and right-censored data. The estimation procedure is based on the weight determined by the Kaplan-Meier estimate of the distribution of the response. We show how the proposed regression quantile estimators perform through analyses of Stanford heart transplant data and AIDS incubation data. We also investigate the effect of censoring on regression quantiles through simulation study.

Keywords : censoring, regression quantile, Kaplan-Meier estimate.

1. Introduction

When we investigate the effects of covariates on the censored response, one often uses the proportional hazard model (Cox, 1972) which describes the hazard rate as a function of covariates. As pointed out by Portnoy (2003), the proportional hazard model is not easy to interpret since it models the hazard rate rather than the survival time directly. An alternative is to model the mean of the response instead of the hazard rate as in Miller (1976), Buckley and James (1979). The mean regression model allows the direct interpretation of the effects of covariates on the survival times. However it is not appropriate to analyze the data with non-homogeneous variability and it is not robust in the sense that a few observations can have a serious influence on the analysis of the model. Another approach is to model the quantile (median) of the survival time as a function of covariates. The regression quantile model offers easy interpretation and it can also deal with heterogeneous variability.

The regression quantile model with the censored response has been an object of attention in econometric literature. Starting with Powell (1984, 1986), an estimation of censored regression quantiles was studied by Newey and Powell (1990), Buchinsky (1995), Koenker and Park

1) This research was supported by Inha University Research Grant (INHA-31630).
2) Associate Professor, Department of Statistics, Inha University, Incheon 402-751, Korea.
E-mail : jhpark@stat.inha.ac.kr
3) Graduate Student, Department of Statistics, Inha University, Incheon 402-751, Korea.

(1996), Buchinsky and Hahn (1998), Chen and Khan (2001), Honore, Khan and Powell (2002) among others. The censored regression quantile was also considered in statistics literature. Ying, Jung and Wei (1995) investigated the median regression with the censored response. Lindgren (1997) and Portnoy (2003) suggested alternative estimation methods using Kaplan-Meier estimates.

For a random variable Y with distribution function $F_Y(\cdot)$, the α-th quantile q_α is defined as

$$q_\alpha = \inf \ \{y : F_Y(y) \geqq \alpha\}.$$

Koenker and Basset (1978) showed that the α-th quantile is the value which minimizes

$$E[\rho_\alpha(Y-t)]$$

over t, where $\rho_\alpha(y) = (\alpha - I(y<0))y$ and $I(A)$ is the indicator function. For a random sample $Y_1, Y_2, \cdots, Y_n$, we can estimate q_α as the value which minimizes

$$\frac{1}{n}\sum_{i=1}^{n} \rho_\alpha(Y_i - t)$$

over t.

The above estimation method can be extended to the regression model. For a given covariate vector $X=x$, the conditional α-th quantile $q_\alpha(x)$ of the response Y is defined as

$$q_\alpha(x) = \inf \ \{y : P(Y \leqq y | X=x) \geqq \alpha\}.$$

To study the relationship between covariates and the response, we usually assume that $q_\alpha(x)$ is a function of covariates with unknown parameter θ_α. For example, we can assume that $q_\alpha(x)$ is a linear function of x so that $q_\alpha(x) = x^t\theta_\alpha$. Then we can estimate θ_α as the value which minimizes

$$\frac{1}{n}\sum_{i=1}^{n} \rho_\alpha(Y_i - X_i^t\theta)$$

over θ.

In biostatistical applications, the survival times are often right censored. Some of patients are lost to follow up and some are alive at the end of study. When the survival times are right censored, we observe

$$(X_i, \widetilde{Y}_i, \delta_i), \ i = 1, 2, \cdots, n,$$

where $\widetilde{Y}_i = \min(Y_i, C_i)$, $\delta_i = I(Y_i \leqq C_i)$, and C_i is a censoring time.

The Tobit model (Tobin, 1958) assumes that the observed response has following form

$$\widetilde{Y}_i = \min(C, X_i^t\theta_0 + \epsilon_i),$$

where ϵ_i is the error term. So the response cannot be observed above the fixed censoring time C. Powell (1986) estimated the unknown parameter θ_0 by minimizing

$$\frac{1}{n}\sum_{i=1}^{n} \rho_\alpha(\widetilde{Y}_i - \min(X_i^t\theta, C)).$$

The assumption that the fixed censoring times are all the same for every observations was removed by Newey and Powell (1990). But they assumed that the censoring times are fixed and available. Honore, Khan and Powell (2002) generalized the Tobit model to allow that the censoring times are random.

In statistics literature, Lindgren (1997) suggested a weighted estimation method using local Kaplan-Meier estimate. Portnoy (2003) proposed a recursively reweighted estimation procedure motivated by Kaplan-Meier estimator. But these methods require iterations and they are computationally intensive. Ying, Jung and Wei (1995) also considered the median regression under random censoring.

In this paper we propose an alternative approach to the regression quantiles under random censoring and truncation. The estimation procedure is based on the weight determined by the Kaplan-Meier estimates of the distribution of the response. We apply the estimation method to Stanford heart transplant data and AIDS incubation data. We also investigate the effect of censoring on regression quantiles through simulation study.

2. Estimation of Regression Quantiles Under Censoring and Truncation

In this paper we will assume that the response is left truncated in addition to right censoring. If the response is only right censored, you may consider that the response is left truncated at $-\infty$. Let C_i and T_i denote a right censoring variable and a left truncation variable, respectively. Suppose that the (C_i, T_i) are independent of the (X_i, Y_i). If the response is subject to right censoring and left truncation, we make observation only when $\widetilde{Y}_i \geqq T_i$. Let

$$(X_i, \widetilde{Y}_i, \delta_i, T_i),\ i = 1, 2, \cdots, n \ \text{with}\ \widetilde{Y}_i \geqq T_i$$

denote the observed data.

Let $S(t)$ denote the survival function defined by $S(t) = P(Y \geqq t)$, and let $G(t) = P(T \leqq t \leqq C)$. Define

$$\underline{\tau} = \inf\ \{t : G(t) > 0\}$$

$$\bar{\tau} = \inf\ \{t > \underline{\tau} : S(t) = 0 \ \text{or}\ \ G(t) = 0\}.$$

Then $\underline{\tau}$ and $\bar{\tau}$ are the left and right boundaries of the interval within which we can observe the data under left truncation and right censoring. Lai and Ying (1991) showed that the conditional distribution

$$F_{\underline{\tau}}(y) = P(Y \leqq y \mid Y \geqq \underline{\tau})$$

can be nonparametrically estimated for $y < \bar{\tau}$ from left-truncated and right-censored data.

Suppose a and b are some constants such that $a > \underline{\tau}$ and $b < \bar{\tau}$. Let $\hat{F}_a(y)$ be the product-limit estimator of $F_a(y) = P(Y \leqq y | Y \geqq a)$ given by

$$\hat{F}_a(y) = 1 - \prod_{i : a \leqq y_{(i)} \leqq y} \left[1 - \frac{d_{(i)}}{n_{(i)}} \right]$$

where $y_{(1)} < y_{(2)} < \cdots$ are the distinct uncensored observations; $d_{(i)}$ is the number of deaths at $y_{(i)}$; $n_{(i)}$ is the size of the risk set at $y_{(i)}$, i.e., $n_{(i)} = \sum_{j=1}^{n} I(T_j \leqq y_{(i)} \leqq \tilde{Y}_j)$. And let $\hat{S}_a(y)$ be an estimator of the conditional survival function $S_a(y) = P(Y \geqq y | Y \geqq a)$ given by

$$\hat{S}_a(y) = 1 - \hat{F}_a(y-).$$

While $E[h(X, Y)]$ may not be estimable because of incomplete information about the distribution of Y, Gross and Lai (1996) showed that $E[h(X, Y) | a \leqq Y \leqq b]$, for a function h, can be consistently estimated by

$$\frac{1}{\hat{F}_a(b)} \sum_{i=1}^{n} \delta_i I(a \leqq \tilde{Y}_i \leqq b) h(X_i, \tilde{Y}_i) \frac{\hat{S}_a(\tilde{Y}_i)}{\#(\tilde{Y}_i)}$$

where $\#(\tilde{Y}_i) = \sum_{j=1}^{n} I(T_j \leqq \tilde{Y}_i \leqq \tilde{Y}_j)$. This implies that $E[h(X, Y) | a \leqq Y \leqq b]$ can be estimated using the weight

$$W_i = \frac{\delta_i I(a \leqq \tilde{Y}_i \leqq b)}{\hat{F}_a(b)} \frac{\hat{S}_a(\tilde{Y}_i)}{\#(\tilde{Y}_i)}$$

instead of the equal weight to each observation.

In the parametric regression quantile model, the conditional α-th quantile $q_\alpha(x) = \inf \ \{y : P(Y \leqq y | X = x) \geqq \alpha\}$ of the response Y is usually written as the following form

$$q_\alpha(x) = x^t \theta_\alpha,$$

and the true value θ_α minimizes $E[\rho_\alpha(Y - X^t\theta)]$ over θ. However, as discussed before, $E[\rho_\alpha(Y - X^t\theta)]$ is not estimable because of incomplete information about the distribution of the response due to censoring and truncation. Instead we can estimate $E[\rho_\alpha(Y - X^t\theta) | a \leqq Y \leqq b]$ on the interval $[a, b]$ where we can make observations of the response. Because of this restriction, we need to modify the regression quantile model to the following form

$$q_\alpha^c(x) = x^t \theta_\alpha,$$

where $q_\alpha^c(x) = \inf \ \{y : P(Y \leqq y | X = x, a \leqq Y \leqq b) \geqq \alpha\}$. The quantile $q_\alpha^c(x)$ is different from the original quantile $q_\alpha(x)$. We may interpret $q_\alpha^c(x)$ as an approximation of the original quantile $q_\alpha(x)$ in the presence of censoring and truncation. While $q_\alpha(x)$ is not estimable, $q_\alpha^c(x)$ is estimable using available data.

Since we can estimate $E[\rho_\alpha(Y-X^t\theta)|a \leqq Y \leqq b]$ using the weight W_i to each observation, an estimate of unknown parameter θ_α in the model $q_\alpha^c(x) = x^t\theta_\alpha$ can be defined as the value which minimizes

$$\sum_{i=1}^{n} \rho_\alpha(\widetilde{Y}_i - X_i^t\theta)W_i$$

$$= \frac{1}{\widehat{F}_a(b)}\sum_{i=1}^{n} \delta_i I(a \leqq \widetilde{Y}_i \leqq b)\rho_\alpha(\widetilde{Y}_i - X_i^t\theta)\frac{\widehat{S}_a(\widetilde{Y}_i)}{\#(\widetilde{Y}_i)}$$

over θ.

If $F(\underline{\tau}) = 0$ and $F(\bar{\tau}) = 1$ so that the survival function is estimable without the condition $a \leqq Y \leqq b$, then $E[h(X,Y)]$ can be estimated by

$$\sum_{i=1}^{n} \delta_i h(X_i, \widetilde{Y}_i)\frac{\widehat{S}(\widetilde{Y}_i)}{\#(\widetilde{Y}_i)},$$

where $\widehat{S}(t) = \prod_{i:y_{(i)}<t}\left[1 - \frac{d_{(i)}}{n_{(i)}}\right]$ and we can use the original regression quantile model $q_\alpha(x) = x^t\theta_\alpha$. So an estimate of θ_α in the model $q_\alpha(x) = x^t\theta_\alpha$ can be defined as the value which minimizes

$$\sum_{i=1}^{n} \delta_i\rho_\alpha(\widetilde{Y}_i - X_i^t\theta)\frac{\widehat{S}(\widetilde{Y}_i)}{\#(\widetilde{Y}_i)}$$

over θ. Note that the weight $\delta_i\frac{\widehat{S}(\widetilde{Y}_i)}{\#(\widetilde{Y}_i)}$ is different from Lindgren (1997) who used the weight determined by the local Kaplan-Meier estimates of the distribution function of censoring time.

3. Examples

In this section we will investigate the proposed estimation procedure using two real data sets and some simulated data sets. We have used *quantreg* package in R to estimate the parameter θ_α. The only difference between the suggested estimation method and the regular regression quantile estimation methods for the complete data is that we need to use the weight determined by the Kaplan-Meier estimates of the distribution of the response rather than the equal weight to each observation. So we can use any algorithm to estimate the regression quantiles. We just need to modify it to accommodate the weight. When we use *quantreg* package in R, we can give the weight to each observation.

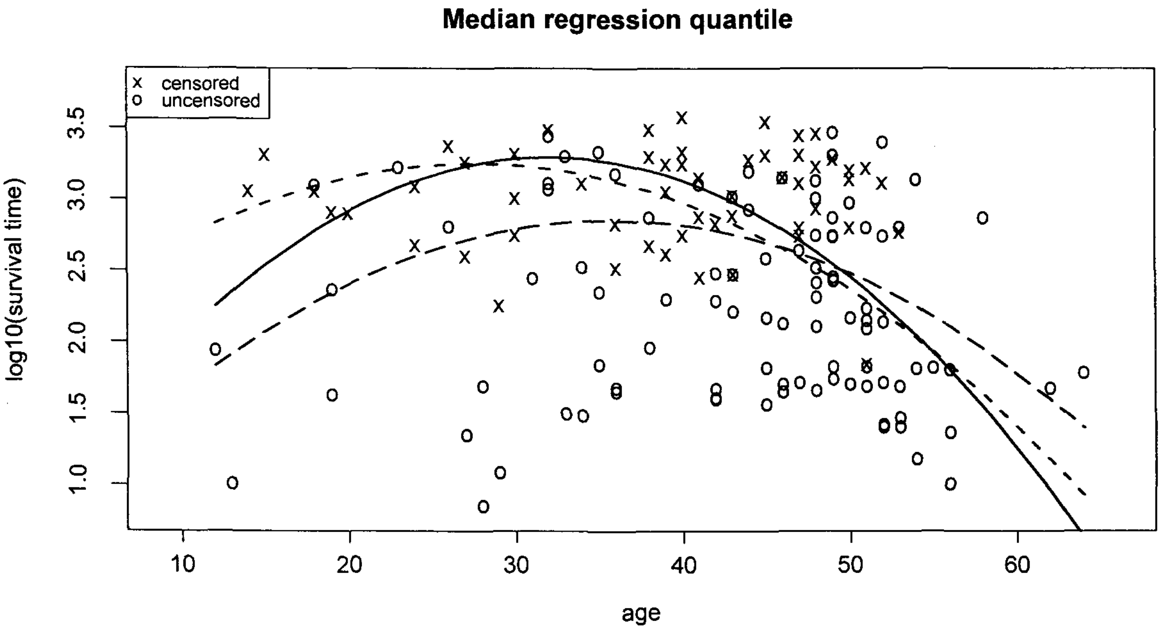
Example 1 Stanford heart transplant data.

The data consist of survival times of 157 patients who had received heart transplants from October 1967 to February 1980. The survival times of 55 patients are censored among 157 cases. The Stanford heart transplant data have been studied by several authors and the results of analyses are well summarized in Leurgans (1987) and Zhou (1992). The main issue is the effect of age on the survival times. Age is believed to have a quadratic effect on the logarithm of survival times.

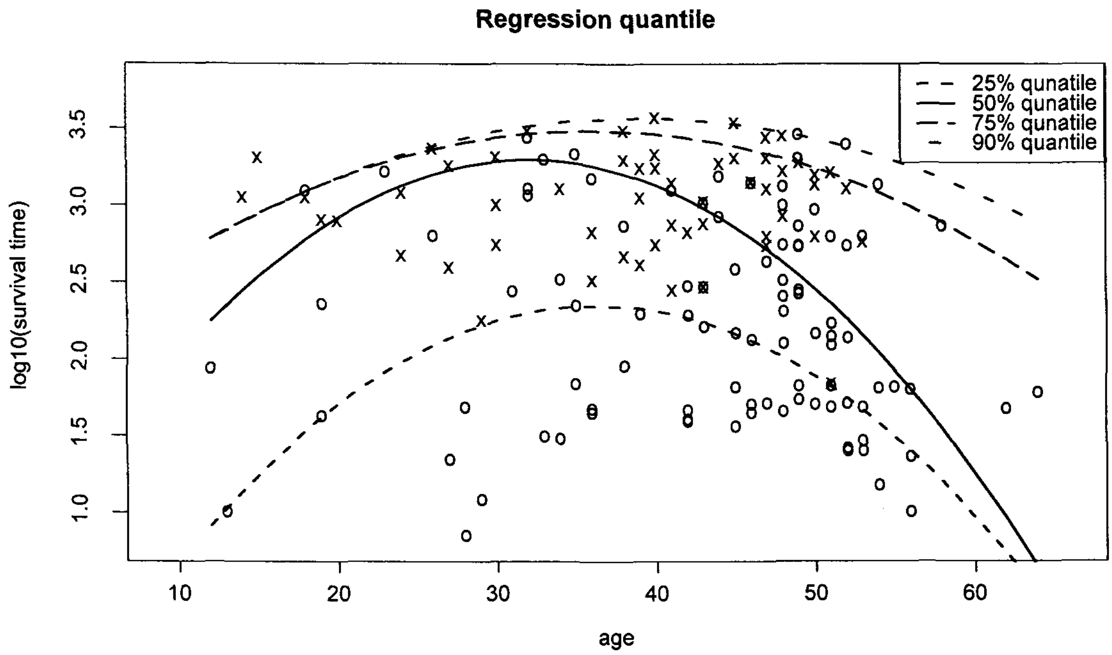
As pointed out in Leurgans (1987), the simple random censorship model in which the censoring times are assumed i.i.d. may not be appropriate for this data. One way to avoid the difficulty is to use stratification. Following Leurgans (1987), the data are stratified into 4 groups using age as a criterion of stratification; less than 30, 30-39, 40-49, and 50 or older. The weight is obtained at each stratification, and they are combined to fit the whole data.

<Figure 1> shows regression quantile lines of log10(survival time) in the model with age and the square of age as covariates. It also shows the fitted mean regression line and the deepest regression line (see Park and Hwang, 2003). The fitted regression line is the least squares line with the same weight. As discussed in Park and Hwang (2003), the least squares line is heavily influenced by a few observations with age less than 20. Because the survival times of uncensored observations at early age are very low compared to the censored observations, those observations have the effect of pulling down the least squares line at early age. While the deepest regression line is not so efficient as the least squares line, it is more robust in the sense that it is not much influenced by a few uncensored observations at early age. We can see that there is a noticeable difference between the deepest regression line and the least squares line in the area that the patient's age is less than 30. And the median regression quantile lies between two lines in the area. So we can argue that the median regression quantile is not affected as much as the least squares line by the uncensored observations at early age though it is not so robust as the deepest regression line.

<Figure 2> shows 25%, 50%, 75% and 90% regression quantiles. We can see that there is a heterogeneous variability of the survival times across age. And the survival times have a skewed distribution. The distribution of the survival times is skewed to the left before age 40, while the distribution is skewed to the right after age 50. The third quartile is closer to the median than the first quartile up to age 40.



<Figure 1> The solid line is the median regression quantile. The long dashed line is the least square regression line and the short dashed line is the deepest regression line



<Figure 2> 25%, 50%, 75% and 90% regression quantiles for the stanford heart transplant data.

Example 2 AIDS incubation data.

The data include 295 cases of HIV infection by blood or blood-product transfusion reported to the Center for Disease Control prior to January 1, 1987, and diagnosed prior to July 1, 1986. The data consist of three variables; INF is the month of infection with 1=January of 1978; DIAG is the duration of the induction period in months; and AGE is the age+1 (in years) at the time of infection. Following Kalbfleisch and Lawless (1989), the response variable Y is the incubation period defined as DIAG-0.5. Since only the patients diagnosed prior to July 1, 1986 are recruited into the study, the data are right truncated. The right truncation variable T is 101.5-INF. We observe (Y, T) when $Y \leq T$. The relationship between the age of patients (AGE=age+1) and the incubation time (DIAG) was investigated by Gross and Lai (1996) using a linear model $-$ (DIAG) $= \beta_0 + \beta_1$(AGE)$+ \epsilon$. Then response $Y_0 = -\log$(DIAG) is left truncated by $T_0 = -\log$(101.5-INF). Because of the different characteristics among children (age ≤ 4), adults ($5 \leq$ age ≤ 59) and elderly (age ≥ 60) patients, the data set is divided into three groups. <Table 1> shows the estimates of regression quantiles and their bootstrap standard errors. The standard errors are estimated by 1000 bootstrap simulations. We use the same values for a and b as in Gross and Lai (1996).

<Table 1> Estimates of median regression quantiles and their bootstrap standard errors

AGE	$\hat{\beta}_0$	$se(\hat{\beta}_0)$	$\hat{\beta}_1$	$se(\hat{\beta}_1)$
$a = -4.38$, $b = -1.84$				
age ≤ 4	-1.866	0.119	-0.436	0.043
$5 \leq$ age ≤ 59	-4.248	0.197	0.005	0.004
age ≥ 60	-2.812	1.015	-0.019	0.015
$a = -3.5$, $b = -1.84$				
age ≤ 4	-1.897	0.186	-0.405	0.119
$5 \leq$ age ≤ 59	-3.630	0.323	0.009	0.007
age ≥ 60	-2.909	0.835	-0.004	0.012

We can compare the estimates of β_0 and β_1 with the least squares estimates and the deepest regression estimates. The least squares estimates and the deepest regression estimates using the same weight are given in Park and Hwang (2003). The differences between estimates are not significant considering their standard errors. However the standard errors of the estimates of regression quantiles are greater than the standard errors of the least squares estimates but they are less than the standard errors of the deepest regression estimates. This reflects that regression quantile estimates are more efficient than the deepest regression estimates but they are not efficient as much as the least squares estimates.

Example 3 Simulation study

The simulated data are generated from the model

$$Y|X \sim N\left(X, \left(\frac{1}{5}+\frac{2}{5}X\right)^2\right),$$

where the covariate X is from Uniform(0,5). The censoring variables are generated from gamma distributions whose parameters are determined so that the responses are 10%, 30% and 50% censored. From the given model, the standard deviation of the response increases linearly in X, and the true α-th quantile $q_\alpha(x)$ of the response is given by

$$q_\alpha(x) = \frac{1}{5}z_\alpha + \left(1+\frac{2}{5}z_a\right)x,$$

where z_α is the α-th quantile of the standard normal distribution.

<Table 2> shows the estimates of regression quantiles and their standard errors. The estimates and the standard errors are based on 1000 replications. As expected, the biases and the standard errors of the estimates of regression quantiles increase as the censoring percentage increases and they decrease as the sample size increases. Note that the standard errors of the estimates of 75% regression quantiles are greater than those of the estimates of 25% regression quantiles. For complete data without censoring, the standard errors of the estimates of 75% regression quantiles should be approximately same as the standard errors of the estimates of 25% regression quantiles because of symmetry. The greater standard errors of the estimates of 75% regression quantiles are due to right censoring. Because of the smaller size of risk set at right tail compared to left tail, the variability increases at right tail. This causes the greater standard errors of the estimates of 75% regression quantiles.

The table also shows the estimates and the standard errors of the least squares estimates which can be compared with the estimates and the standard errors of 50% regression quantiles. However it may not be fair to compare these two estimates since the response does not have equal variances across the covariate and the usual least squares estimation is not appropriate. As you can see at <Table 2>, the least squares estimates have larger biases. Note that the standard errors are also greater than 50% regression quantiles for most cases, which might be unexpected.

<Table 2> Estimates of regression quantiles and their standard errors

		true quantiles intercept	true quantiles slope	10% censoring intercept	10% censoring slope
n=50	25% quantile	-0.135	0.730	-0.156(0.227*)	0.743(0.157)
	50% quantile	0	1	0.002(0.213)	1.004(0.147)
	75% quantile	0.135	1.270	0.156(0.244)	1.259(0.169)
	90% quantile	0.256	1.513	0.330(0.352)	1.468(0.232)
	least squares			0.023(0.274)	0.988(0.155)
n=100	25% quantile	-0.135	0.730	-0.138(0.149)	0.735(0.107)
	50% quantile	0	1	0.004(0.135)	0.999(0.102)
	75% quantile	0.135	1.270	0.138(0.147)	1.269(0.119)
	90% quantile	0.256	1.513	0.283(0.218)	1.499(0.165)
	least squares			0.005(0.186)	0.998(0.107)
n=500	25% quantile	-0.135	0.730	-0.135(0.063)	0.731(0.048)
	50% quantile	0	1	0.000(0.059)	1.001(0.046)
	75% quantile	0.135	1.270	0.138(0.066)	1.270(0.051)
	90% quantile	0.256	1.513	0.263(0.088)	1.509(0.067)
	least squares			0.004(0.082)	0.999(0.047)
		30% censoring intercept	30% censoring slope	50% censoring intercept	50% censoring slope
n=50	25% quantile	-0.171(0.266)	0.754(0.186)	-0.220(0.423)	0.785(0.271)
	50% quantile	-0.018(0.254)	1.022(0.189)	-0.018(0.358)	1.026(0.261)
	75% quantile	0.172(0.342)	1.263(0.249)	0.292(0.508)	1.170(0.316)
	90% quantile	0.477(0.604)	1.383(0.342)	0.749(0.915)	1.177(0.427)
	least squares	0.025(0.332)	0.981(0.185)	0.083(0.415)	0.934(0.226)
n=100	25% quantile	-0.148(0.164)	0.742(0.120)	-0.153(0.215)	0.746(0.164)
	50% quantile	-0.001(0.162)	1.007(0.127)	-0.013(0.218)	1.016(0.187)
	75% quantile	0.140(0.217)	1.282(0.182)	0.198(0.294)	1.243(0.247)
	90% quantile	0.357(0.339)	1.470(0.250)	0.574(0.617)	1.311(0.337)
	least squares	0.011(0.241)	0.995(0.134)	0.064(0.297)	0.957(0.169)
n=500	25% quantile	-0.139(0.070)	0.734(0.055)	-0.140(0.082)	0.732(0.067)
	50% quantile	0.000(0.070)	0.999(0.056)	-0.002(0.088)	1.002(0.077)
	75% quantile	0.137(0.084)	1.270(0.071)	0.141(0.127)	1.269(0.114)
	90% quantile	0.269(0.125)	1.508(0.114)	0.329(0.216)	1.472(0.186)
	least squares	0.004(0.110)	0.997(0.062)	0.025(0.152)	0.983(0.085)

* standard errors

4. Conclusion

The quantile regression model offers direct interpretation compared to the proportional hazard model for censored data and it can also deal with heterogeneous variability. In this paper we have developed an estimation method for regression quantiles under right censoring and left truncation. The estimation method is based on the Kaplan-Meier estimates of the distribution of the response. From the analysis of two real data sets and simulation study, we can see the suggested estimates lies between the efficient least squares estimates and more robust estimates such as estimates using depth. It is more robust than the least squares estimates and it is more efficient than depth estimates.

References

[1] Buchinsky, M. (1995). Estimating the asymptotic covariance matrix for quantile regression models, *J. Ecometrics*, Vol. 68, 303-338.

[2] Buchinsky, M. and Hahn, J. (1998). An alternative estimator for the censored quantile regression model, *Econometrica*, Vol. 66, 653-671.

[3] Buckley, J. and James, I. (1979). Linear regression with censored data, *Biometrika,* Vol. 66, 429-464.

[4] Chen, S. and Khan, S. (2001). Estimation of a partially linear censored regression model, *Econometrics Theory*, Vol. 17, 567-590.

[5] Cox, D.R. (1972). Regression models and life-tables (with discussion), *J. Roy. Statist. Soc. Ser. B*, Vol. 34, 187-220.

[6] Gross, S.T. and Lai, T.L. (1996). Nonparametric estimation and regression analysis with left-truncated and right-censored data, *J. Amer. Statist. Assoc.,* Vol. 91, 1166-1180.

[7] Honore, B., Khan, S. and Powell, J.L. (2002). Quantile regression under random censoring, *J. Ecometrics*, Vol. 109, 67-105.

[8] Kalbfleisch, J.D. and Lawless, J.F. (1989). Inference based on retrospective ascertainment: An analysis of the data on transfusion-related AIDS, *J. Amer. Statist. Assoc.,* Vol. 84, 360-372.

[9] Koenker, R. and Basset, G. (1978). Regression quantiles, *Econometrica*, Vol. 46, 33-50.

[10] Koenker, R. and Park, B.J. (1996). An interior point algorithm for nonlinear quantile regression. *J. Ecometrics*, Vol. 71, 265-283.

[11] Lai, T.L. and Ying, Z. (1991). Estimating a distribution function with truncated and censored data, *Ann. Statist.*, Vol. 19, 417-442.

[12] Leurgans, S. (1987). Linear models, random censoring and synthetic data, *Biometrika,* Vol. 74, 301-309.

[13] Lindgren, A. (1997). Quantile regression with censored data using generalized L_1 minimization, *Comput. Statist. Data Anal.*, Vol. 23, 509-524.

[14] Miller, R.G. (1976). Least squares regression with censored data, *Biometrika,* Vol. 63, 449-464.

[15] Newey, W.K. and Powell, J.L. (1990). Efficient estimation of linear and type I censored regression models under conditional quantile restrictions, *Econometrics Theory*, Vol. 6, 295-317.

[16] Park, J. and Hwang, J. (2003). Regression depth with censored and truncated data, *Comm. Statist.*, Vol. 32, 997-1008.

[17] Powell, J. (1984). Least absolute deviations estimation for the censored regression model, *J. Ecometrics*, Vol. 25, 303-325.

[18] Powell, J. (1986). Censored Regression Quantiles, *J. Ecometrics*, Vol. 32, 143-155.

[19] Portnoy, S. (2003). Censored regression quantiles, *J. Amer. Statist. Assoc.,* Vol. 98, 1001-2003.

[20] Tobin, J. (1958). Estimation of relationships for limited dependent variables, *Econometrica*, Vol. 26, 24-36.

[21] Ying, Z., Jung, S.H. and Wei, L.J. (1995). Survival analysis with median regression models, *J. Amer. Statist. Assoc.,* Vol. 90, 178-184.

[22] Zhou, M. (1992). M-estimation in censored linear models, *Biometrika,* Vol. 79, 837-841.

[Received March 2005, Accepted November 2005]