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Mass and Heat Transfer Characteristics of Vertical Flat Plate with Free Convection

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Abstract : This paper has dealt with the characteristics of mass and heat transfer of vertical flat plate with free convection. The theory of similarity transformations applied to the momentum and energy equations for free convection. To derive the similarity equation of mass transfer, the equation for conservation of species was added to the continuity, momentum and energy equations. The momentum, energy and species equations set numerically to obtain the velocity, temperature and mass fraction of species as dimensionless. For cases where momentum transport dominates, the thermal boundary layers are shorter than the momentum boundary layer. The relationships between momentum, energy and species were clarified from this study.

Key words : Mass and heat transfer, Similarity transformations, Mass fraction, Free convection

Nomenclature

C : Concentration, kmol/m^3
f : Similarity function for momentum
g : Gravitational acceleration, m/s^2
Gr_x : Local Grashof number
M : Molecular weight, kg/kmol
Pr : Prandtl number
Sc : Schmidt number
T : Temperature, K
u : Velocity in x direction, m/s
v : Velocity in y direction, m/s
a : Thermal diffusivity, m^2/s
β : Thermal expansion coefficient, K^{-1}
δ : Boundary layer thickness, m
η : Similarity variable
θ : Non-dimensional temperature
ν : Kinematic viscosity, m^2/s
ρ : Density, kg/m^3
ϕ : Non-dimensional concentration
ψ : Stream function

Subscript

A : Species A
B : Species B
w : Wall surface
∞ : Free stream

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1. Introduction

Recently, many concerns to adsorption technique have been rising in industrial fields. Especially, the adsorption technique has been exposed to the attention in the field of air conditioning system[1]-[3].

The adsorption operations exploit the ability of certain solids preferentially to concentrate specific substances from solution onto their surfaces. In this manner, the components of either gaseous or liquid solutions can be separated from each other. A few examples will indicate the general nature of the separations possible and at the same time demonstrate the great variety of practical applications[4]. In the field of gaseous separations, adsorption is used to dehumidify air and other gases, to remove objectionable odors and impurities from industrial gases such as carbon dioxide, to recover valuable solvent vapors from dilute mixtures with air and other gases, and to fractionate mixtures of hydrocarbon gases containing such substances as methane, ethylene, ethane, propylene, and propane[5].

The types of adsorption must be distinguished two types of adsorption phenomena, physical and chemical. Physical adsorption, or van der Waals adsorption, a readily reversible phenomenon, is the result of intermolecular forces of attraction between molecules of the solid and the substance adsorbed. The adsorbed substance does not penetrate within the crystal lattice of the solid and does not dissolve in it but remains entirely upon the surface[6]. If, however, the solid is highly porous, containing many fine capillaries, the adsorbed substance will penetrate these interstices if it wets the solid. On the other hand, Chemisorption, or activated adsorption, is the result of chemical interaction between the solid and the adsorbed substance[7]. The strength of the chemical bond may vary considerably, and identifiable chemical compounds in the usual sense may not actually form, but the adhesive force is generally much greater than that found in physical adsorption.

In this study, to find the mechanism of transport phenomena, the vertical flat plate which occurs free convection at the surface was considered. The objectives of this study are to confirm the mass and heat transfer mechanism on free convective boundary layer over a vertical flat plate which leads to a set of non-linear, ordinary differential equations. The momentum, energy and species conservation equations of this model are transformed using the theory of similarity solutions and these equations are set numerically to obtain the velocity, temperature and mole fraction profiles as dimensionless.

2. Governing boundary layer equations

Mass transfer may be encountered as evaporation, sublimation, diffusion through a semi-permeable membrane, flow through a porous wall, etc. The vapor pressure of species B in the convective fluid A must be less than its saturation pressure at the surface temperature. For steady, two-dimensional diffusion in a binary mixture of A and B, the conservation of

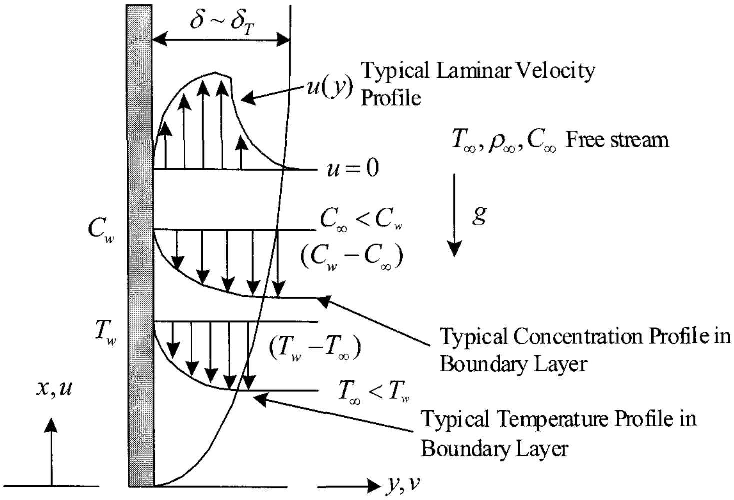

Fig. 1 Boundary layer in free convection of a fluid along a vertical surface

mass applied to species B gives equation (2), where D_{BA} is the mass diffusivity of component B into a binary mixture of A and B as defined by Fick's law[8], has the form as equation (1).

$$N''_B = -D_{BA}\frac{\partial C_A}{\partial y} \tag{1}$$

$$u\cdot\frac{\partial C_B}{\partial x} + v\cdot\frac{\partial C_B}{\partial y} = D_{BA}\cdot\frac{\partial^2 C_B}{\partial y^2} \tag{2}$$

Where, the components of velocity in the x and y directions are indicated by u and v respectively.

Equation (2) can also be written in terms of a non-dimensional mass fraction, when C_B is replaced by $\omega_B = C_B/C$. The corresponding continuity, momentum and energy equations are given by Equations (3), (4) and (5).

$$\frac{\partial u}{\partial x}u + \frac{\partial v}{\partial y}v = 0 \tag{3}$$

$$u\cdot\frac{\partial u}{\partial x} + v\cdot\frac{\partial u}{\partial y} = g\cdot\frac{(\rho_\infty - \rho)}{\rho} + v\cdot\frac{\partial^2 u}{\partial y^2} \tag{4}$$

Momentum equation where the body force is caused by density differences due to both temperature gradient and density gradient of species B.

$$u\cdot\frac{\partial T}{\partial x} + v\cdot\frac{\partial T}{\partial y} = \alpha\cdot\frac{\partial^2 T}{\partial y^2} \tag{5}$$

In equation (5), the thermal diffusivity, specific heat and dynamic viscosity are treated as constants.

The final relationship required is an equation of state, $\rho = \rho(C_B, T)$. The body force in equation (4) can be written as equation (6) by assuming like follows.

$$g\cdot\frac{\rho_\infty - \rho}{\rho} = g\cdot\left[\frac{1}{\frac{T_\infty}{T} + \left(1 - \frac{M_A}{M_B}\right)\cdot\omega_B}\right] \tag{6}$$

- Perfect gas behavior
- Local thermodynamic equilibrium
- No heat generation with mass transfer
- Density of injected component B is zero in the free steam

· Free convection laminar boundary layer

3. Similarity transformation

Numerous solutions to the laminar free convection boundary layer equations have been obtained, and a special case that has received much attention involves free convection from an isothermal vertical surface in an extensive quiescent medium (Fig. 1). For this geometry equations (2)~(5) must be solved subject to boundary conditions of the following conditions

$$y=0:\ u=v=0\quad T=T_w\quad C_B=C_w$$
$$y\to\infty:\ u\to 0\quad T\to T_\infty\quad C_B\to C_\infty \tag{7}$$

A similarity solution has been obtained by Ostrach[9]. The solution involves transforming variables by introducing a similarity parameter of the form as equation (8) and representing the velocity components in terms of stream function defined as equation (9).

$$\eta \equiv \frac{y}{x}\left(\frac{Gr_x}{4}\right)^{1/4} \tag{8}$$

$$\psi(x,y)\equiv f(\eta)\left[4\nu\left(\frac{Gr_x}{4}\right)^{1/4}\right],\qquad Gr_x=\frac{g\cdot\beta\cdot(T_w-T_\infty)\cdot x^3}{\nu^2} \tag{9}$$

With the definition of the stream function, the x-velocity component may be expressed as equation (10).

$$u=\frac{\partial\psi}{\partial y}=\frac{\partial\psi}{\partial\eta}\frac{\partial\eta}{\partial y}=4\nu\left(\frac{Gr_x}{4}\right)^{1/4}f'(\eta)\frac{1}{x}\left(\frac{Gr_x}{4}\right)^{1/4}$$
$$=\frac{2\nu}{x}Gr_x^{1/2}f'(\eta) \tag{10}$$

Where primed quantities indicate differentiation with respect to η. Hence $f'(\eta)\equiv df/d\eta$. Evaluating the y-velocity component $v=-\partial\psi/\partial x$ in a similar fashion and introducing the dimensionless temperature and density as equation (11).

$$\theta\equiv\frac{T-T_\infty}{T_w-T_\infty},\quad \phi\equiv\frac{C-C_\infty}{C_w-C_\infty} \tag{11}$$

The four original partial differential equations (2)~(5) may then be reduced to three ordinary differential equations.

$$f'''+3\cdot f\cdot f''-2\cdot(f')^2+\theta=0 \tag{12}$$

$$\theta''+3\cdot \mathrm{Pr}\cdot f\cdot\theta'=0 \tag{13}$$

$$\phi''+3\cdot Sc\cdot f\cdot\phi'=0 \tag{14}$$

Where the Pr and Sc are defined as $Pr = \nu / \alpha$ and $Sc = \nu / D_{AB}$, respectively.

The transformed boundary conditions required to solve the momentum and energy equations (12)~(14) are of the form as equation(15).

$$\eta=0:\ f=f'=0\quad \theta=1\quad \phi=1$$
$$\eta\to\infty:\ f'\to 0\quad \theta\to 0\quad \phi\to 0 \tag{15}$$

Here, It needs to solve the unknown initial values of f'', $\theta''$and ϕ'at $\eta = 0$. To find these values the iterative method were used. If the end value of η is too large, there may exist no convergence so the maximum value of η is restricted as four. Also, the Pr and Sc are treated as constants.

4. Solution algorithms

The solution for the free convection on a vertical flat plate has different coupling between the momentum, energy and species equations. The forced convection problem is only loosely coupled while the free convection problem is fully coupled. As a result, the three problems require three different solutions.

The solution of the momentum profile is achieved by using the multiple shooting method with a simple, first-order Euler method used to determine the values of the function derivatives. In this method, each derivative value is calculated using the derivative value of the next higher order.

$$f^{(i)} = f^{(i+1)} \cdot \Delta\eta + f_0^{(i)} \tag{16}$$

Where i is the order of the derivative, and the subscript zero denotes the value of the function at the previous grid point.

Each step along the integration can be done using equations (12), (13), (14) and (16). First, an estimate is made of the value of $f''(0)$. Then, this value is used to calculate f'. The f' value is used to calculate f, and the third derivative of f may be obtained using equation (12). Having a value for the third derivative, the new value of f'' is obtained from equation (16). This cycle is continued until the profile is complete. And also the problem of free convection on vertical flat plate is based on a set of first, second and third-order Taylor series expansions to calculate the second, first and zeroth-order derivatives, respectively.

5. Results and discussion

Fig. 2 indicates the calculated results of all ordinary, non-linear differential equations and the tendencies of non-dimensional profiles show good agreed with reference data[10] reasonably.

The calculated behavior of the non-dimensional profiles is demonstrated by varying the Prandtl and Schmidt number for the calculation and observing the result. Fig. 3 shows the variation of non-dimensional profiles with respect to the variation of Pr and Sc.

In the case of Pr less than one($Pr < 1$), the thermal boundary layers extend out beyond the momentum boundary layer. For cases where momentum transport dominates, the thermal boundary layers are shorter than the momentum boundary layer. From these results, the relationships between momentum, energy, and species were clarified.

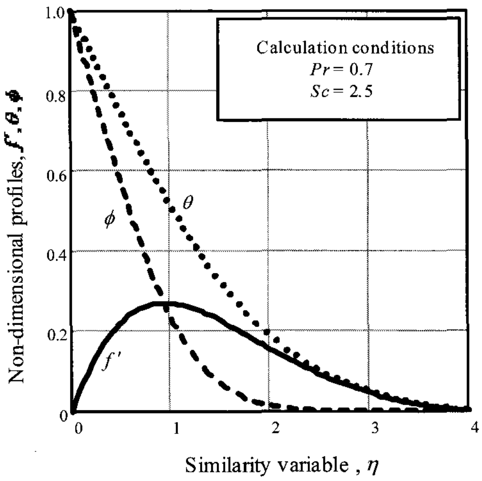

Fig. 2 Non-dimensional profiles of laminar, free convection boundary layer

6. Conclusions

The mass and heat transfer mechanism on free convection boundary layer over a vertical flat plate has mainly been

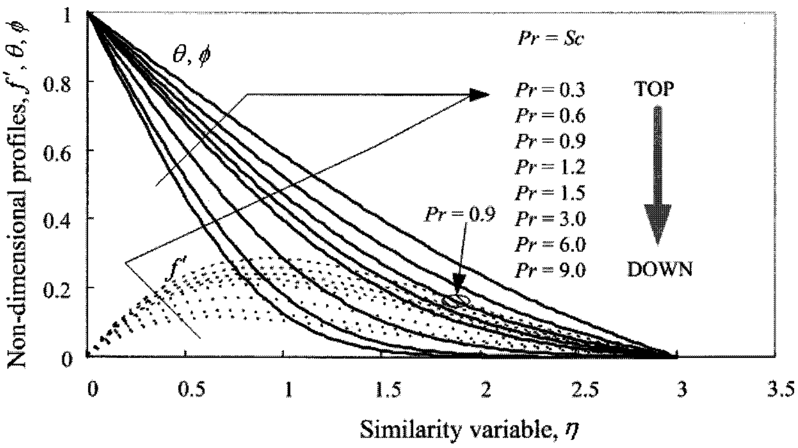

Fig. 3 Non-dimensional profiles for different values of the Pr and Sc

investigated numerically under the conditions of various Prandtl and Schmidt number. The main conclusions and the results of investigation are summarized as follows :

- Confirmed the mass and heat transfer mechanism on free convection boundary layer over a vertical flat plate which leads to a set of non-linear, ordinary differential equations.
- The momentum, energy and species conservation equations of this model were transformed using the theory of similarity solutions and these equations were set numerically to obtain the velocity, temperature and mole fraction profiles as dimensionless.
- In the case of $Pr < 1$, the thermal boundary layers extend out beyond the momentum boundary layer. For cases where momentum transport dominates, the thermal boundary layers were shorter than the momentum boundary layer. Also, the relationships between momentum, energy, and species were clarified.

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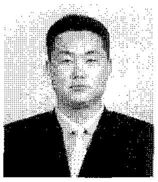
Author Profile



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