LAG TIME RELATIONS TO CATCHMENT SHAPE DESCRIPTORS AND HYDROLOGICAL RESPONSE

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Abstract: One of the most important factors for estimating a flood runoff from streams is the lag time. It is well known that the lag time is affected by the morphometric properties of basin which can be expressed by catchment shape descriptors. In this paper, the notion of the geometric characteristics of an equivalent ellipse proposed by Moussa(2003) was applied for calculating the lag time of geomorphologic instantaneous unit hydrograph(GIUH) at a basin outlet. The lag time was obtained from the observed data of rainfall and runoff by using the method of moments and the procedure based on geomorphology was used for GIUH. The relationships between the basin morphometric properties and the hydrological response were discussed based on application to 3 catchments in Korea. Additionally, the shapes of equivalent ellipse were examined how they are transformed from upstream area to downstream one. As a result, the relationship between the lag time and descriptors was shown to be close, and the shape of ellipse was presented to approach a circle along the river downwards. These results may be expanded to the estimation of hydrological response of ungauged catchment.

Keywords: Lag time; Catchment shape descriptors; Equivalent ellipse; Geomorphologic instantaneous unit hydrograph(GIUH); GIUH velocity

1. INTRODUCTION

Hydrologists would consider why and how the channel networks have been created as well as how those particular networks respond to rainfall events. Unfortunately, the exact solution of such problems has not been obtained until now. Nevertheless, there have been many hydrologist who have attempted to quantify the channel networks. For instance, Horton(1945), Strahler (1952), and Smart(1968) laid the foundation of quantitative geomorphology of drainage basins. Shumm(1956) has advocated the description of catchment shape by using the elongation ratio that is defined as the ratio of the diameter of circle equivalent to the catchment area to the catchment length taken as the mainstream length. From such a viewpoint, Moussa(2003) has tried to define catchment shape descriptors to characterize the hydrological response of the basin and then to analyze the relationship between these descriptors and the scale of observation. In order to study the relationship between morphometric properties of basins and basin hydrological response, he has proposed a new catchment-shape index, i.e., the equivalent ellipse which has the same center of gravity, the same principal inertia axes, the same area and the same ratio of minimal inertia moment to maximal inertia moment as the catchment. The use of digital elevation model(DEM) is eligible for calculating the better results on the factors mentioned above(Moussa, 2003).

The Moussa's research(2003) was based on hydrological properties of the basins' responses such as the lag time and GIUH. The first special workshop on scale problems for the basins' response whose contents are appeared in Journal of Hydrology(Klemes, 1983) was held in 1982. The following workshop in a series on this general topic was held in 1984, whose contents are introduced by Gupta et al(1986). Considering the historical backgrounds, to study the response of a basin to its morphologic or topographic features is one of very important roles of hydrologists. Chutha, and Dooge(1990) have chosen to reformulate the Rodriguez-Iturbe, and Valdes model on a deterministic basis and thus alleviated the difficulties for some hydrologists of a probabilistic treatment of catchment response to an individual rainstorm, and avoided the necessity of using the unfamiliar mathematics of Markov processes. Thet concluded that the shape of GIUH is closely approximated by Nash model of a cascade of equal linear reservoirs for their three assumptions about storage variations.

The main object of the present paper is to determine the representative velocity of GIUH, since, as seen generally, the velocities result in different values according to each storm event as a result of time-varying system. From the views of Rosso(1984) and Chutha, and Dooge(1990) that the shapes of GIUH and Nash model are very close, both of them are assumed to be equal in this paper so as to determine the representative GIUH velocity. The main reason to do so is to attempt to search for the feasibility of practical use of them both in determining the representative GIUH velocity. Also, the lag times for the fittest velocity of GIUH are related to catchment shape descriptors to see whether they will give the better results or not by using the regression method.

2. BACKGROUND

Nash model is a very common and relatively simple analytical form for the IUH, which is known as:

$$h(t) = \frac{1}{k\Gamma(n)}\left(\frac{t}{k}\right)^{n-1} e^{-\frac{t}{k}} \tag{1}$$

where n is the number of equal linear reservoirs, k is the storage constant, and $\Gamma(\cdot)$ is gamma function. This formulation was first derived by Nash(1957) under the conception of a cascade of equal linear reservoirs. By means of the method of moments, two parameters n and k can be directly estimated from the observed data. However, they have, per ce, different values according to each storm event even on the same basin area owing to nonlinearity and time-varying character of the watershed transformation system and thus the procedure not easy to determine the representative value of n or k such as averaging or optimization is needed for practical use. In order to avoid such a difficulty, in the present study Nash model and GIUH combine to obtain GIUH velocity v with the lag time t_L' in view of the results given by Rosso(1984) and Chutha, and Dooge(1990): Nash model is very close to GIUH.

For a basin of order Ω, according to the Strahler ordering scheme, the general formulation of GIUH can be put in the form:

$$h(t) = \sum_{s \in S} P(s) f_{T_s}(t) \quad (2)$$

where S is the population of the path s, $P(s)$ and $f_{T_s}(t)$ are the path probability and the probabilistic density function(pdf) of the holding time of the path s, respectively, $P(s)$ and $f_{T_s}(t)$ are defined as:

$$P(s) = \theta_i(0) \times p_{ij} \times \cdots \times p_{k\Omega} \quad (3)$$

$$f_{T_s}(t) = f_{T_i}(t) * f_{T_j}(t) * \cdots * f_{T_\Omega}(t) \quad (4)$$

where $\theta_i(0)$ is the initial state probability, p_{ij} is the state transition probability and $f_{T_i}(t)$ is the pdf of the holding time of a stream of order i In the original version of GIUH (Rodriguez-Iturbe, and Valdes, 1979), the holding time of an overland is neglected and the pdf of the holding time of a stream is assumed to be exponential:

$$f_{T_i}(t) = \lambda_i e^{-\lambda_i t} \quad (5)$$

where λ_i is the inverse of mean holding time of the stream of order i, and expressed by $v / \overline{L_i}$, where $\overline{L_i}$ is the mean length of the streams of order i. Due to the characteristics of exponential, the stream of the highest order is assumed to be composed of two equal linear reservoirs. Fig. 1 shows the schematic diagram of the basin of order 3. In the next year(1980), Gupta, Waymire, and Wang presented the generalized GIUH based on the path function probability which can be taken by arbitrary probability distribution for the holding times whose probability distribution is to be exponential type inevitably in view of the Markovian postulate made in the theory of Rodriguez-Iturbe, and Valdes(1979). Another model of path types(Jin, 1992) for GIUH is following the method of Gupta et al.(1980) who considered all possible paths of streams over the whole catchment as stated above. The model was attempted to apply a third parameter, i.e., stream flow velocity(termed as GIUH velocity in this paper) in addition to two parameters of n and k adopted by Nash(1957) to gamma-type GIUH model.

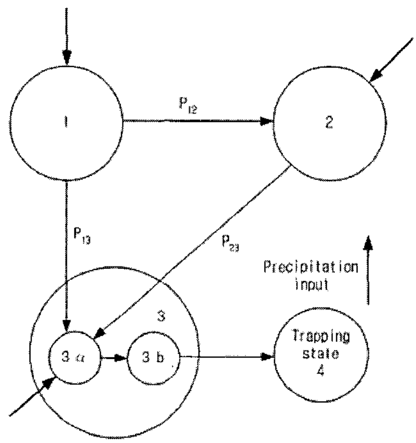


Fig. 1 Representation of a third order basin as a continuous Markov process (Rodriguez-Iturbe, and Valdes, 1979)

In this study, as assumed by Rodriguez-Iturbe, and Valdes(1979), the average stream velocity(v herein) was treated as the same approximately throughout the whole drainage network at any moment for a given rainfall-runoff. In other words, the average stream velocity could be approximated to the lag time of the IUH relevant to the whole basin. In accordance with such a condition, GIUH of Eq. (2) can be obtained from Eqs. (4) and (5).

In order to determine λ_i, the determination of dynamic parameter of GIUH, namely, v mentioned above, which is assumed to be constant thoroughout the basin, is required. Therefore, the representative value of v should be determined for the practical affairs. From such a standpoint, the representative lag time for the fittest GIUH velocity is attempted to attain the present aims as the lag time $t_L^{'}$ is related to new catchment shape descriptors which will be presented in the following section.

3. NEW CATCHMENT SHAPE DESCRIPTORS

An equivalent ellipse as the new catchment shape index is defined using four geometric properties(Moussa, 2003).

1) The center of the ellipse is the center of the basin.
2) The principal axes of the ellipse correspond to the principal axes of the basin.
3) The ellipse has the same area as the catchment area.
4) The ellipse has the same ratio of the minimal inertia moment to the maximal inertia moment as the catchment.

3.1 Inertia Moments of the Basin

Fig. 2 shows the example of an arbitrary catchment plan-form and the corresponding equivalent ellipse. Let G be the center of gravity of the catchment plan-form $\sum$ and $x'Gy'$ be the plane obtained by translating the xAy plane to G. According to the well-known technique of derivation of inertia moment, it can be demonstrated that two principal axes of inertia $\overline{GX}$ and $\overline{GY}$ are given as a function of the rotation angle ω:

$$\tan 2\omega = \frac{2I_{x'y'}}{I_{y'} - I_{x'}} \tag{6}$$

where $I_{x'}, I_{y'}$ and $I_{x'y'}$ are the inertia moments of $\sum$ in the $x'Gy'$ plane. In the XGY plane, $\sum$ also has the maximal inertia moment I_{max} and the minimal inertia moment I_{min}. The ratio of I_{max} to I_{min} is defined as the basin elongation R_i:

$$R_i = \frac{I_{min}}{I_{max}} \tag{7}$$

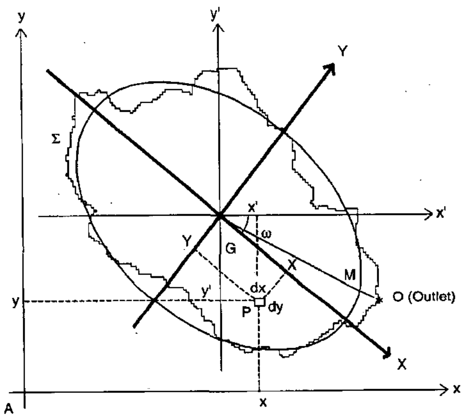


Fig. 2 The catchment plan-form $\sum$ and the equivalent ellipse(Moussa, 2003)

3.2 The Equivalent Ellipse

A point lies on the ellipse if and only if its coordinates satisfies Eq. (8).

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \tag{8}$$

where a and b are the semi-major and semi-minor axes of the ellipse. The area and the minimal and maximal inertia moments of the ellipse are given as:

$$A_e = \pi ab \quad I_{min,e} = \frac{\pi ab^3}{4} \quad I_{max;e} = \frac{\pi a^3 b}{4} \tag{9}$$

Let S_0 be the area of the catchment plan-form $\sum$. According to the definition of the equivalent ellipse, we have

$$S_0 = \pi ab \quad R_i = \left(\frac{b}{a}\right)^2 \tag{10}$$

Combining the two relations in Eq. (10) gives

$$a = \left[\frac{S_0}{\pi}\left(\frac{1}{R_i}\right)^{\frac{1}{2}}\right]^{\frac{1}{2}} \quad b = \left[\frac{S_0}{\pi}(R_i)^{\frac{1}{2}}\right]^{\frac{1}{2}} \tag{11}$$

3.3 Geometric Properties of the Equivalent Ellipse in Relation with the Basin Outlet Position

In order to estimate the mean length between the catchment plan-form and the outlet, two catchment shape descriptors are defined by geometric properties of the equivalent ellipse:

$$a+b \quad a+b+\varepsilon OM \tag{12}$$

where ε is the weight whose value is $+1$ or -1 depending on the outlet position, and OM is the distance between the outlet and the ellipse border. It is noted that the equivalent ellipse isn't identical with the catchment plan-form. Therefore, some parts of the basin are always located on inside or outside of the equivalent ellipse. The last term of the latter characterize them, especially the outlet position.

4. DETERMINATION OF GIUH VELOCITY

The lag time t_L', the first moment of the IUH about the origin, is given as Eq. (13) by Nash(1957):

$$t_L' = Q_1' - I_1' \tag{13}$$

where Q_1' and I_1' are the first moments of the direct runoff hydrograph and the effective hyetograph about the origin, respectively. The lag time of GIUH, i.e., t_L is obtained from Eq. (2) as:

$$t_L = \frac{1}{V}\sum_{s\in S} p(s)\sum_{k=i} \overline{L_k} \tag{14}$$

Following the assumption stated previously, t_L' averaged from the given real rainfall-runoff data of each subbasin is approximately equivalent to t_L:

$$t_L' ; t_L \tag{15}$$

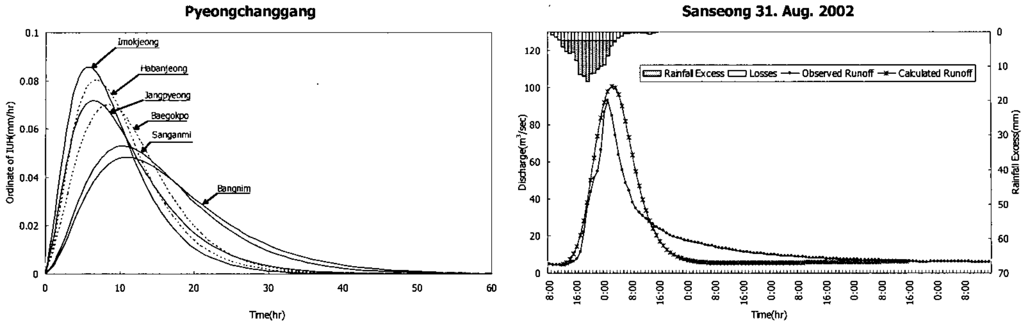
Therefore, once t_L' is determined from Eq. (13) by the observed data, the velocity V in Eq. (14) can be estimated, for the geomorphologic parameters which can be determined from the topographic map or digital elevation model (DEM) data of a catchment. In other words, the velocity V can be expressed as the function of only t_L' that is a single scaling factor such as the delay time for the reservoirs at the outlet from each first order:

$$V = f(t_L') = \frac{1}{t_L'}\sum_{s\in S} p(s)\sum_{k=i} \overline{L_k} \tag{16}$$

In this study, the descriptor $a+b$ or $a+b+\varepsilon OM$ is related to the lag times t_L', separately, using the regression method to determine the representative lag time.

The procedures of estimation above are compacted as follows:

1) The lag times corresponding to each storm event are estimated from Eq. (13).
2) The values of t_L in Eq. (14) are estimated



(a) (b)

Fig. 5 Representative IUHs(a) and Calculated and observed hydrograph(b)

the lag time than others mentioned above.

5.3 Estimation of Representative IUH

Fig. 5(a) shows shapes of the representative IUHs of 6 subbasins in Pyeongchanggang obtained from the results of this study. The solid and dotted lines stand for the cases of mainstream and tributary, respectably, in here. Lower peaks and longer times to peak of the IUHs appear along the mainstream downwards. The calculated hydrograph and the observed one of Sanseong subbasin in Bocheongcheon were compared for testing this study, whose data has not been used in determining the lag time. In Fig 5(b), baseflow and rainfall excess were estimated drawing a horizontal line and using Φ index, respectively. The proposed scheme results in a reasonable approach for obtaining the response function of a catchment as shown in Fig. 5(a), (b). Such features have also been given similarly in either case of Pyeongchanggang or Wicheon. From such result, it is expected that catchment shape descriptors based on the equivalent ellipse such as $a+b$ or $a+b+\varepsilon OM$ could be the useful tool for the hydrological application. It is desirable to conduct a lot of case studies on many catchments in other regions to obtain the better results.

6. CONCLUSIONS

1) The relationship between the lag time and catchment shape descriptors was shown to be close and tested to be superior to the existing geomorphologic factors.
2) The shape of ellipse tended to approach a circle along the river downwards.
3) By comparing the calculated outflows with the observed ones not used in analyzing the present model for obtaining the fittest lag time or GIUH velocity, the proposed GIUH was shown to be applicable, reasonable approach for obtaining the response function of a catchment.
4) The results obtained in this study may be expanded to the estimation of hydrological response of ungaged catchment.

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