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# 고속 데이터 전송 채널을 위한 신호공간 검출

## (Signal Space Detection for High Data Rate Channels)

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### 요 약

본 논문에서는 신호공간 검출의 개념을 일반화하여 하나의 심볼 구간에서 하나 이상의 심볼들의 블록에 대한 검출을 수행하는 고정지연 트리 검색 신호검출기의 구성을 제안한다. 제안된 기법은 고속의 구현에 적합하다. 두 가지의 접근방법이 논의되며 이들은 모두 효율적인 신호공간 분할에 기반을 두고 있다. 첫 번째 방법에서는 심볼의 검출이 다중 클래스 분할에 기반을 둔다. 이 방법은 2개의 클래스에 기반을 둔 이진 심볼 검출방법을 일반화한 접근방법을 사용한다. 두 번째 방법에서는 이진 신호 검출이 look-ahead 기법과 결합된 고도의 병렬처리 신호검출 구조를 활용한다.

### Abstract

This paper generalizes the concept of the signal space detection to construct a fixed delay tree search (FDTS) detector which estimates a block of  $n$  channel symbols at a time. This technique is applicable to high speed implementation. Two approaches are discussed both of which are based on efficient signal space partitioning. In the first approach, symbol detection is performed based on a multi-class partitioning of the signal space. This approach is a generalization of binary symbol detection based on a two-class pattern classification. In the second approach, binary signal detection is combined with a look-ahead technique, resulting in a highly parallel detector architecture.

**KeyWords** : Fixed relay tree search, signal detection, multi-class partitioning, intersymbol interference.

## I. Introduction

Formulation of fixed delay tree search (FDTS) using the signal space partitioning method has been shown to yield an efficient sub-optimal sequence detector in severe intersymbol interference channels [1]-[3]. This formulation utilizes the fact that the decision process in FDTS with depth parameter  $\tau$  is equivalent to the two-class pattern classification in the  $(\tau + 1)$ -dimensional signal space. The concept of binary classification reduces the detection problem to finding required decision boundaries and the associated decision rule, which maps the observation

sample vector into the estimate of the input symbol. In this paper, we generalize the signal space approach to block-by-block detection which is applicable to high data rate applications. In this scheme, the detector estimates a block of  $n$  input symbols at a time based on  $(\tau + n)$  consecutive observation samples. Two approaches are discussed both of which are based on signal space partitioning. The first one is a generalization of the binary classification approach into multi-class detection. In this approach, the entire  $(\tau + n)$ -dimensional signal space is partitioned into  $2^n$  regions where each region is associated with  $n$  binary input symbols. Based on the partitioned space, the symbol detection is performed on a block-by-block basis. The other approach combines binary signal detection with a

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look-ahead technique. In this scheme, a block of  $n$  input symbols is estimated based on  $n$  parallel binary detectors. The difficulty associated with the dependence on past decisions is avoided by simply looking ahead all possibilities for the set of past decision symbols. Detailed construction examples taken from high-density channels are also discussed

## II. Block Detection by Multi-Class Classification

This approach is a generalization of binary classification using the efficient signal space partitioning. For this approach, we assume the observation sample  $r_k$  contains  $n-1$  more intersymbol interference (ISI) terms than in the binary detection case:

$$r_k = \sum_{i=0}^{\tau+n-1} f_i x_{k-1} + n_k = y_k + n_k \quad (1)$$

where  $f_k$ ,  $x_k$ ,  $n_k$  and  $y_k$  are the equalized channel impulse response, the binary input symbol, the noise and the generalized signal sample at the FDTS input, respectively. Also, we use the generalized version of observation vector  $r$  which has  $(\tau+n)$  consecutive modified observation samples as its elements:

$$r = [r'_k, r'_{k-1}, \dots, r'_{k-(\tau+n-1)}] \quad (2)$$

where the  $r'_k$ 's are also the generalized form of modified observation samples utilizing the previously detected symbols,  $\hat{x}_{k-l}$ ,  $l > \tau+n-1$ . The relationship between  $r_k$  and  $r'_k$  is:

$$r'_{k-j} = \begin{cases} r_{k-j}, & \text{for } j = 0 \\ r_{k-j} - \sum_{l=\tau+n-j}^{\tau+n-1} f_l \hat{x}_{k-j-l}, & \text{for } 1 \leq \tau+n-1 \end{cases} \quad (3)$$

The modified reference signal samples  $y_k^{(i)}$ 's and signal vectors  $y_i$  can also be generalized by simply substituting  $\tau$  in the binary case with  $\tau+n-1$  as follows:

$$y'_{k-j} = \begin{cases} y_{k-j}^{(i)}, & \text{for } j = 0 \\ y_{k-j}^{(i)} - \sum_{l=\tau+n-j}^{\tau+n-1} f_l x_{k-j-l}, & \text{for } 1 \leq \tau+n-1 \end{cases} \quad (4)$$

and

$$y_i = [y_k^{(i)}, y_{k-1}^{(i)}, \dots, y_{k-(\tau+n-1)}^{(i)}], \text{ for } 1 \leq i \leq 2^{\tau+n} \quad (5)$$

Given these generalized versions of the observation vector  $r$  and  $2^{\tau+n}$  reference signal vectors  $y_i$ , the goal of this detection scheme is to find the reference signal that has the minimum distance to the observation and release the associated set of  $n$  symbols as the decision. If we use the concept of the look-ahead tree used in the original tree search algorithm, the multi-class detection scheme is analogous to finding out which "group of paths" among  $2^n$  groups includes the path associated with the minimum accumulated metric. The next step is to identify the set of  $n$  symbols associated with that group and release them as a decision. In Fig. 1, the look-ahead tree for  $\tau=1$  and  $n=2$  are shown along with the 4 possible path groups that share the same values of 2 symbols,  $x_{k-1}$  and  $x_{k-2}$ . For example, the first group which contains path 1 and 2 is associated with  $(x_{k-1}, x_{k-2})=(1,1)$ , the second group with  $(x_{k-1}, x_{k-2})=(-1,1)$ , and so on.

For multi-class detection, the first step is to construct the Voronoi Diagram (VOD) on the  $2^{\tau+n}$  possible signal vectors<sup>[4][5]</sup>. As in the binary detection case, the next step is to find in-class Delaunay neighbors (DNs) which do not contribute to the overall decision boundary. The decision rule is essentially the same as the one in the binary case except the decision regions are divided into  $2^n$  instead of two: If the generalized observation vector falls into one of  $2^n$  decision regions, then the set of symbol values associated with that region is released as the decision.

For the detailed construction procedure, we consider the channel with impulse response  $f_k = \{1.0, 0.7, 0.2\}$  and the random binary input data

is assumed. Let us assume that we are interested in the detection of two symbols ( $n = 2$ ) at every cycle with  $\tau = 1$  performance. The first step is to find  $2^{\tau+n} = 2^3 = 8$  possible reference signal vectors and to construct the VOD on them. The DN pairs are summarized in Table 1. The index pairs marked with 'o' or 'x' are the DNs to each other. The pairs with 'x' satisfy the redundancy condition and are not needed for our 4-class detection. Based on the DN pair table, we can partition the  $(\tau + n = 3)$ -dimensional signal space into 4 regions. In terms of Voronoi regions, they can be expressed as  $V_1 \cup V_2, V_3 \cup V_4, V_5 \cup V_6,$  and  $V_7 \cup V_8,$  respectively. Each region is associated with a set of symbol values  $(x_{k-1}, x_{k-2})$ . The decision can be summarized as (6) in terms of the required half spaces  $H_{ij}$  which are formed by reference signals  $y_i$  and  $y_j$ :

$$(\hat{x}_{k-1}, \hat{x}_{k-2}) = \begin{cases} (1,-1), & \text{if } \mathbf{r} \in (H_{13} \cap H_{15}) \\ & \quad Y(H_{23} \cap H_{24} \cap H_{25} \cap H_{26}) \\ (-1,1), & \text{if } \mathbf{r} \in (H_{31} \cap H_{32} \cap H_{35} \\ & \quad \cap H_{36} \cap H_{37}) \\ & \quad Y(H_{42} \cap H_{46} \cap H_{47} \cap H_{48}) \\ (1,-1), & \text{if } \mathbf{r} \in (H_{51} \cap H_{52} \cap H_{53} \cap H_{57}) \\ & \quad Y(H_{62} \cap H_{63} \cap H_{64} \\ & \quad \cap H_{67} \cap H_{68}) \\ (-1,-1), & \text{otherwise} \end{cases} \quad (6)$$

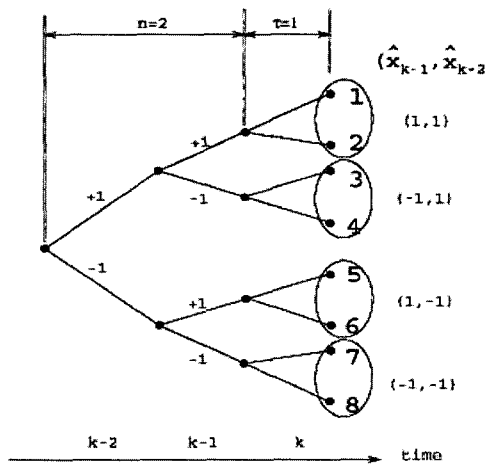


그림 1. 4 개의 클래스 ( $\tau = 1, n = 2$ ) 신호 검출에 해당하는 미리보기 트리  
 Fig. 1. The look-ahead tree corresponding to the four-class detection ( $\tau = 1$  and  $n = 2$ ).

표 1. 추정블록의 크기  $n = 2$ 이고 지연시간  $\tau = 1$ 일 때 채널응답  $f_k = \{1.0, 0.7, 0.2\}$ 에 대한 DN 결례

Table 1. The DN pairs for the estimation block size  $n = 2$  with  $\tau = 1$  for the channel  $f_k = \{1.0, 0.7, 0.2\}$ .

	1	2	3	4	5	6
1		X	O		O	
2	X		O	O	O	O
3	O	O		X	O	O
4		O	X			O
5	O	O	O			X
6		O	O	O	X	
7			O	O	O	O
8				O		O

where  $H_{ij}$  is defined as the region in which all the points are closer to  $y_i$  than to  $y_j$ . The discriminant function  $h_{ij}(\mathbf{r})$  can be used for determining whether the observation  $\mathbf{r}$  is in the region  $H_{ij}$  or not which is defined as follows:

$$h_{ij}(\mathbf{r}) = \frac{1}{2}(\mathbf{y}_i - \mathbf{y}_j) \cdot \mathbf{r} - \frac{1}{4}(\mathbf{y}_i + \mathbf{y}_j) \cdot (\mathbf{y}_i - \mathbf{y}_j) \quad (7)$$

where  $h_{ij}(\mathbf{r})$  has positive value when  $\mathbf{r}$  is in  $H_{ij}$  and negative value when  $\mathbf{r}$  is in  $H_j$ .

### III. Parallel Processing of the Detection

In this section, the construction procedure for a high speed FDTS with depth  $\tau$  using signal space partitioning and a look-ahead technique is described in detail. The main idea in this approach is to estimate a block of  $n$  channel input symbols in parallel. In this way, we can effectively decrease the symbol period by about a factor of  $n$ . In this block-based detection approach, most of the detection procedures for each symbol in the block are carried out in parallel without waiting for the previously detected input symbols within a prescribed period.

Let us assume that a block of observation

samples of size  $\tau + n$  is given by  $[r_k, r_{k-1}, \dots, r_{k-(\tau+n-1)}]$  for the detection of a block of  $n$  input symbols,  $[x_{k-\tau}, x_{k-(\tau+1)}, \dots, x_{k-(\tau+n-1)}]$ . For the estimation of one input symbol  $x_{k-(\tau+n-1)}$  with depth  $\tau$  performance, an observation vector, whose elements are the  $\tau+1$  most recent observation samples, is examined in the  $(\tau+1)$ -dimensional space using the nearest neighbor rule. This observation vector is defined as

$$\mathbf{r}_{k-(n-1)} = [r'_{k-(n-1)}, r'_{k-n}, \dots, r'_{k-(\tau+n-1)}] \quad (8)$$

where  $r'_k$  is defined in (3) assuming binary detection case. At the time the detection process for  $x_{k-(\tau+n-1)}$  begins, the processes for detecting the other symbols can begin without waiting for the detection on  $x_{k-(\tau+n-1)}$  by utilizing a look-ahead technique. Similar modification should be made to define the vector  $\mathbf{r}_{k-(n-2)}$  for detecting the symbol,  $x_{k-(\tau+n-2)}$ . The problem occurs, however, when we try to define the modified observation vector  $\mathbf{r}_{k-(n-2)}$  in a similar way shown in (3) and (8). An estimated symbol  $\hat{x}_{k-(\tau+n-1)}$ , is needed in this modification which is not yet available. Although this undetected channel input symbol is present in the observation samples, we can still estimate the conditional symbol value  $x_{k-(\tau+n-2)}$  given  $x_{k-(\tau+n-1)} = 1$  or  $-1$ , respectively, without waiting for the final decision on  $x_{k-(\tau+n-1)}$ . In general, we can define the observation samples  $r'_k(\tilde{\mathbf{x}}_{k-m})$ , for the estimation of  $x_{k-(\tau+n-m-1)}$   $1 \leq m \leq n-1$ , as follows:

$$r'_{k-l}(\tilde{\mathbf{x}}_{k-m}) = r_{k-l} - \sum_{i=\tau+n-l-m}^{\tau+n-l-1} f_i \tilde{x}_{k-i-l} - \sum_{i=\tau+n-l}^{\tau+n-1} f_i \hat{x}_{k-i-l}, \quad (9)$$

for  $n-m-1 \leq l \leq \tau+n-m-1$

where  $n \geq 2$  and  $\tilde{\mathbf{x}}_{k-m}$  is the vector of size  $m$ , whose elements consist of the look-ahead values:

$$\tilde{\mathbf{x}}_{k-m} = [\tilde{x}_{k-(\tau+n-m)}, \tilde{x}_{k-(\tau+n-m+1)}, \Lambda, \tilde{x}_{k-(\tau+n-1)}]. \quad (10)$$

Based on modified observation samples utilizing look-ahead values as well as previously detected symbols, a new conditional observation vectors,

$\mathbf{r}_{k-(n-m-1)}(\tilde{\mathbf{x}}_{k-m})$ ,  $1 \leq m \leq n-1$ , are defined as follows:

$$\mathbf{r}_{k-(n-m-1)}(\tilde{\mathbf{x}}_{k-m}) = \begin{bmatrix} r'_{k-(n-m-1)}(\tilde{\mathbf{x}}_{k-m}) \\ r'_{k-(n-m)}(\tilde{\mathbf{x}}_{k-m}) \\ \mathbf{M} \\ r'_{k-(\tau+n-m-1)}(\tilde{\mathbf{x}}_{k-m}) \end{bmatrix}^T. \quad (11)$$

These vectors are used for the conditional estimations of  $x_{k-(\tau+n-m-1)}$  given the look-ahead symbol values  $\tilde{\mathbf{x}}_{k-m}$ .

The final step is to make decisions on the  $n$  channel input symbols,  $x_{k-\tau}, x_{k-(\tau+1)}, \dots, x_{k-(\tau+n-2)}$ , based on the conditional estimations  $\hat{x}_{k-\tau}(\tilde{\mathbf{x}}_{k-(n-1)})$ ,  $\hat{x}_{k-(\tau+1)}(\tilde{\mathbf{x}}_{k-(n-2)})$ ,  $\dots$ ,  $\hat{x}_{k-(\tau+n-2)}(\tilde{\mathbf{x}}_{k-1})$ , and the detected symbol value  $\hat{x}_{k-(\tau+n-1)}$ . The procedures for detecting the conditional symbols and the symbol  $x_{k-(\tau+n-1)}$  are the same, since they all utilize the same geometric information in the  $(\tau+1)$ -dimensional signal space.

#### IV. Construction Example

As a construction example, we consider the implementation of FDTTS with depth parameter  $\tau=1$  and estimation symbol block size  $n=2$  in a high density channel. The equalized observation samples  $r_k$  are given by

$$r_k = x_k + f_1 x_{k-1} + f_2 x_{k-2} + n_k = y_k + n_k \quad (12)$$

where, with no loss of generality,  $f_0$  is normalized to one and it is assumed that  $f_1 > 0$  and close to one. The first step is to find the VOD and DNs of the 4 possible reference signals  $y_i$ ,  $1 \leq i \leq 4$ , in the 2-dimensional space, where  $y_i$  is defined as

$$y_i = [y_k^{(i)} \ y_{k-1}^{(i)}] = [x_k^{(i)} + f_1 x_{k-1}^{(i)}, x_{k-1}^{(i)}]. \quad (13)$$

Fig. 2. shows the placements of the reference

signals and their VOD. Solid line segments partition the space into 4 Voronoi regions,  $V_i, 1 \leq i \leq 4$ , while dotted lines connect two signals which are DNs to each other. As shown in the figure, the boundary  $B_{12}$  is not needed to construct the region, ( $V_1 \cap V_2$ ) which is associated with the input symbol "1", since the boundaries  $B_{13}$  and  $B_{24}$  are parallel to each other. For detecting  $x_{k-2}$ , a modified observation vector  $r_{k-1}$  is defined as follows:

$$\begin{aligned} r_{k-1} &= [r'_{k-1}, r'_{k-2}] \\ &= [r_{k-1} - f_2 \hat{x}_{k-3}, r_{k-2} - f_1 \hat{x}_{k-3} - f_2 \hat{x}_{k-4}] \end{aligned} \quad (14)$$

where we utilize the past decisions,  $\hat{x}_{k-3}$  and  $\hat{x}_{k-4}$ . Based on the required boundaries obtained from the VOD and the redundancy test, the detection rule can be set up as follows in terms of the discriminant function defined in (7):

$$\hat{x}_{k-2} = \begin{cases} 1 & \text{if } (h_{13}(r_{k-1}) > 0) \\ & Y(h_{23}(r_{k-1}) > 0 \text{ I. } h_{24}(r_{k-1}) > 0) \\ -1 & \text{otherwise.} \end{cases} \quad (15)$$

For the conditional estimation of  $x_{k-1}$ , another modified observation vector  $r_k(\tilde{x}_{k-2})$  is defined as follows:

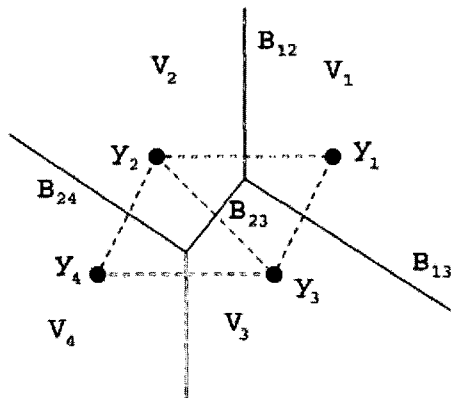


그림 2. 2 차원공간에서 FDTS ( $\tau = 1$ ) 에 관련된 4 개의 기준 신호에 대한 VOD (실선) 과 DN (점선으로 연결)

Fig. 2. The VOD (solid lines) and DNs (connected by dotted lines) of the 4 reference signals in the 2 dimensional space for the FDTS with  $\tau = 1$ .

$$\begin{aligned} r_k(\tilde{x}_{k-2}) &= [r'_k(\tilde{x}_{k-2}), r'_{k-1}(\tilde{x}_{k-2})] \\ &= [r_k - f_2 \tilde{x}_{k-2}, r_{k-1} - f_1 \tilde{x}_{k-2} - f_2 \hat{x}_{k-3}] \end{aligned} \quad (16)$$

where  $\tilde{x}_{k-2}$  is the look-ahead value which is either 1 or -1. The block diagram for the detector is shown in Fig. 3. Three identical structures (inside the dotted rectangles) are placed in parallel for detecting  $x_{k-2}$  and  $x_{k-1}(\tilde{x}_{k-2})$ 's for high speed operation. At the final stage, a 2-by-1 multiplexer is used to make the final estimation on  $x_{k-1}$ .

### V. Conclusion

Two methods are discussed which implement a high speed FDTS. These approaches are based on multi-class detection and parallel estimation of a block of  $n$  input symbols using efficient space partitioning techniques. The approaches can be generalized to the case for arbitrary values of  $\tau$  and block size  $n \geq 2$ . The illustrative examples from high density channels show the significant

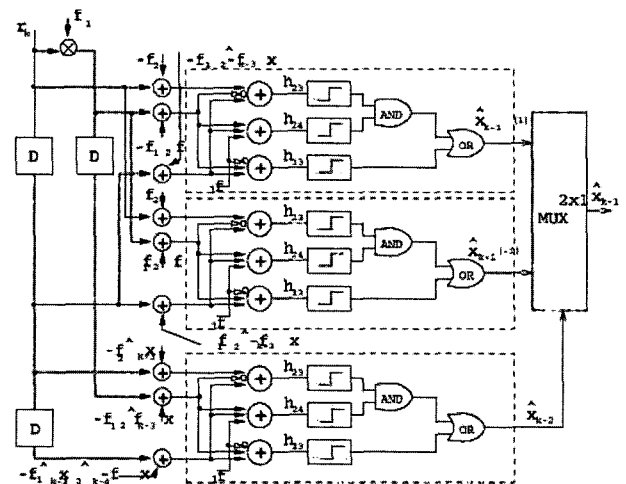


그림 3. 추정블럭의 크기  $n = 2$  이고 지연시간  $\tau = 1$  를 가지는 FDTS 의 고속구현 (덧셈기의 입력에 서작은 원과 백색 화살표는 가산연산을 나타내고 반면 흑색은 감산연산을 나타낸다)

Fig. 3. The high speed implementation of the FDTS with  $\tau = 1$  and the input symbol block size  $n = 2$  (the white arrow heads with small circles at the input of the adder represent the subtraction, while dark ones represent the addition).

improvement in the detection speed which is suitable for high data rate communication channels.

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