Hall Effect on Unsteady Couette Flow with Heat Transfer Under Exponential Decaying Pressure Gradient

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The unsteady Couette flow of an electrically conducting, viscous, incompressible fluid bounded by two parallel non-conducting porous plates is studied with heat transfer taking the Hall effect into consideration. An external uniform magnetic field and a uniform suction and injection are applied perpendicular to the plates while the fluid motion is subjected to an exponential decaying pressure gradient. The two plates are kept at different but constant temperatures while the Joule and viscous dissipations are included in the energy equation. The effect of the ion slip and the uniform suction and injection on both the velocity and temperature distributions is examined.

Key Words : Fluid Mechanics, Hydromagnetic Flow, Couette Flow, Heat Transfer, Finite Difference

Nomenclature

a : Viscosity parameter
b : Thermal conductivity parameter
c_p : Specific heat at constant pressure
Ec : Eckert number
Ha : Hartmann number
$\vec{i}$: Unit vector in the axial direction
$\vec{j}$: Unit vector in the vertical direction
$\vec{k}$: Unit vector in the z-direction
$\vec{J}$: Current density
k : Thermal conductivity
h : Separation between the two plates
P : Pressure gradient
Pr : Prandtl number
Re : Reynolds number
S : Suction parameter
t : Time
T : Temperature of the fluid
T_1 : Temperature of the lower plate
T_2 : Temperature of the upper plate
m : Hall parameter
u : Velocity component in the x-direction
w : Velocity component in the z-direction
U_o : Velocity of the upper plate
v_o : Suction velocity
x : Axial direction
y : Distance in the vertical direction
z : Distance in the z-direction
μ : Viscosity of the fluid
ρ : Density of the fluid
σ : Electrical conductivity of the fluid
β : Hall factor

1. Introduction

The magnetohydrodynamic flow between two parallel plates, known as Hartmann flow, is a classical problem that has many applications in magnetohydrodynamic (MHD) power generators, MHD pumps, accelerators, aerodynamic heating, electrostatic precipitation, polymer technology, petroleum industry, purification of crude oil and fluid droplets and sprays. Hartmann and Lazarus (1937) studied the influence of a transverse uniform magnetic field on the flow of a conducting fluid between two infinite parallel, stationary, and

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insulated plates. Then, a lot of research work concerning the Hartmann flow has been obtained under different physical effects (Tao, 1960 ; Alpher, 1961 ; Sutton et al., 1965 ; Cramer et al., 1973 ; Nigam et al., 1960 ; Tani, 1962 ; Soundalgekar et al., 1979 ; 1986 ; Abo-El-Dahab,1993 ; Attia, 2005). In the above mentioned work the Hall term was ignored in applying Ohm's law. In fact, the Hall effect is important when the Hall parameter, which is the ratio between the electron-cyclotron frequency and the electron-atom-collision frequency, is high. This happens when the magnetic field is high or when the collision frequency is low (Sutton et al., 1965 ; Cramer et al., 1973). The Hall term was ignored in applying Ohm's law, as it has no marked effect for small and moderate values of the magnetic field. However, the current trend for the application of magnetohydrodynamics is towards a strong magnetic field, so that the influence of the electromagnetic force is noticeable under these conditions, and the Hall current is important and has a marked effect on the magnitude and direction of the current density and consequently on the magnetic-force term (Sutton et al., 1965 ; Cramer et al., 1973). Tani (1962) studied the Hall effect on the steady motion of electrically conducting and viscous fluids in channels. Soudalgekar et al. (1979 ; 1986) studied the effect of the Hall currents on the steady MHD Couette flow with heat transfer. The temperatures of the two plates were assumed either to be constant (Soundalgekar et al., 1979) or to vary linearly along the plates in the direction of the flow (Soundalgekar et al., 1986). Abo-El-Dahab (1993) studied the effect of Hall current on the steady Hartmann flow subjected to a uniform suction and injection at the bounding plates. Later, Attia (1998) extended the problem to the unsteady state with heat transfer, with constant pressure gradient applied.

In the present study, the unsteady Couette flow and heat transfer of an incompressible, viscous, electrically conducting fluid between two infinite non-conducting horizontal porous plates are studied with the consideration of the Hall current. The upper plate is moving with a uniform velocity while the lower plate is kept stationary. The fluid is acted upon by an exponential decaying pressure gradient, a uniform suction and injection and a uniform magnetic field perpendicular to the plates. The induced magnetic field is neglected by assuming a very small magnetic Reynolds number (Sutton et al., 1965 ; Cramer et al., 1973). The two plates are maintained at two different but constant temperatures. This configuration is a good approximation of some practical situations such as heat exchangers, flow meters, and pipes that connect system components. The cooling of these devices can be achieved by utilizing a porous surface through which a coolant, either a liquid or gas, is forced through a suction or injection process. Therefore, the results obtained here are important for the design of the wall and the cooling arrangements of these devices. The governing equations are solved numerically taking the Joule and the viscous dissipations into consideration. The effect of the magnetic field, the Hall current and the suction and injection on both the velocity and temperature distributions is studied.

2. Description of the Problem

The two non-conducting plates are located at the $y=\pm h$ planes and extend from $x=-\infty$ to ∞ and $z=-\infty$ to ∞ as shown in Fig. 1. The upper plate is moving with a uniform velocity U_o while the lower plate is kept stationary. The lower and upper plates are kept at the two constant temperatures T_1 and T_2, respectively, where $T_2 > T_1$. The fluid flows between the two plates under the influence of an exponential decaying pressure gradient dP/dx in the x-direction, and a uniform suction from above and injection from below which are applied at $t=0$. The whole system is subjected to a uniform magnetic field B_o in the positive y-direction. This is the total magnetic field acting on the fluid since the induced magnetic field is neglected. From the geometry of the problem, it is evident that $\partial/\partial x=\partial/\partial z=0$ for all quantities apart from the pressure gradient dP/dx. The existence of the Hall term gives rise to a z-component of the velocity. Thus, the velocity vector of the fluid is

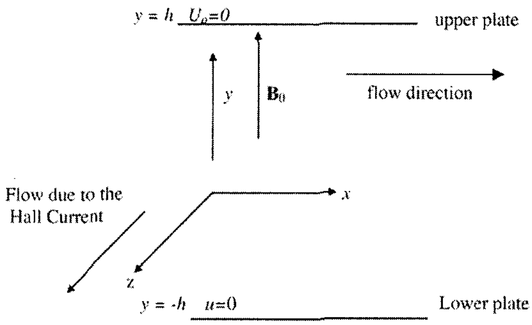


Fig. 1 The geometry of the problem

$$\vec{v}(y,\ t)=u(y,\ t)\vec{i}+v_o\vec{j}+w(y,\ t)\vec{k}$$

with the initial and boundary conditions $u=w=0$ at $t\leq 0$, $u=w=0$ at $y=-h$ for $t>0$ and $u=U_o$ and $w=0$ at $y=h$ for $t>0$. The temperature $T(y,\ t)$ at any point in the fluid satisfies both the initial and boundary conditions $T=T_1$ at $t\leq 0$, $T=T_2$ at $y=+h$, and $T=T_1$ at $y=-h$ for $t>0$. The fluid flow is governed by the momentum equation

$$\rho\frac{D\vec{v}}{Dt}=\mu\nabla^2\vec{v}-\vec{\nabla}P+\vec{J}\wedge\vec{B}_o \qquad (1)$$

where ρ and μ are, respectively, the density and the coefficient of viscosity of the fluid. If the Hall term is retained, the current density $\vec{J}$ is given by

$$\vec{J}=\sigma\{\vec{v}\wedge\vec{B}_o-\beta(\vec{J}\wedge\vec{B}_o)\}$$

where σ is the electric conductivity of the fluid, and β is the Hall factor (Sutton et al., 1965; Cramer et al., 1973). This equation may be solved in $\vec{J}$ to yield

$$\vec{J}\wedge\vec{B}_o=-\frac{\sigma B_o^2}{1+m^2}\{(u+mw)\vec{i}+(w-mu)\vec{k}\} \qquad (2)$$

where $m=\sigma\beta B_o$, is the Hall parameter (Sutton et al., 1965; Cramer et al., 1973). Thus, in terms of Eq. (2), the two components of Eq. (1) read

$$\rho\frac{\partial u}{\partial t}+\rho v_o\frac{\partial u}{\partial y}=-\frac{dP}{dx}+\mu\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_o^2}{1+m^2}(u+mw) \qquad (3)$$

$$\rho\frac{\partial w}{\partial t}+\rho v_o\frac{\partial w}{\partial y}=\mu\frac{\partial^2 w}{\partial y^2}-\frac{\sigma B_o^2}{1+m^2}(w-mu) \qquad (4)$$

To find the temperature distribution inside the fluid we use the energy equation (Schlichting, 1968)

$$\rho c_p\frac{\partial T}{\partial t}+\rho c_p v_o\frac{\partial T}{\partial y}=k\frac{\partial^2 T}{\partial y^2}+\mu\left[\left(\frac{\partial u}{\partial y}\right)^2+\left(\frac{\partial w}{\partial y}\right)^2\right]+\frac{\sigma B_o^2}{1+m^2}(u^2+w^2) \qquad (5)$$

where c_p and k are, respectively, the specific heat capacity and the thermal conductivity of the fluid. The second and third terms on the right-hand side represent the viscous and Joule dissipations, respectively.

The problem is simplified by writing the equations in the non-dimensional form. We define the following non-dimensional quantities

$$\hat{x}=\frac{x}{h},\ \hat{y}=\frac{y}{h},\ \hat{z}=\frac{z}{h},\ \hat{u}=\frac{u}{U_o},\ \hat{P}=\frac{P}{\rho U_o^2},\ t=\frac{tU_o}{h}$$

$Re=\rho hU_o/\mu$ is the Reynolds number,
$S=v_o/U_o$ is the suction parameter,
$Pr=\mu c/k$ is the Prandtl number,
$Ec=U_o^2/c_p(T_2-T_1)$ is the Eckert number,
$Ha^2=\sigma B_o^2h^2/\mu$ where Ha is the Hartmann number,
$\tau_{\hat{x}_L}=\left(\frac{\partial\hat{u}}{\partial\hat{y}}\right)_{\hat{y}=-1}$ is the axial skin friction coefficient at the lower plate,
$\tau_{\hat{x}_U}=\left(\frac{\partial\hat{u}}{\partial\hat{y}}\right)_{\hat{y}=-1}$ is the axial skin friction coefficient at the upper plate,
$\tau_{\hat{z}_L}=\left(\frac{\partial\hat{w}}{\partial\hat{y}}\right)_{\hat{y}=-1}$ is the transverse skin friction coefficient at the lower plate,
$\tau_{\hat{z}_U}=\left(\frac{\partial\hat{w}}{\partial\hat{y}}\right)_{\hat{y}=-1}$ is the transverse skin friction coefficient at the upper plate,
$Nu_L=\left(\frac{\partial\theta}{\partial\hat{y}}\right)_{\hat{y}=-1}$ is the Nusselt number at the lower plate,
$Nu_U=\left(\frac{\partial\theta}{\partial\hat{y}}\right)_{\hat{y}=-1}$ is the Nusselt number at the upper plate.

In terms of the above non-dimensional variables and parameters, the basic Eqs. (3)-(5) are written as (the "hats" will be dropped for convenience)

$$\frac{\partial u}{\partial t}+S\frac{\partial u}{\partial y}=-\frac{dP}{dx}+\frac{1}{\mathrm{Re}}\frac{\partial^2 u}{\partial y^2}-\frac{Ha^2}{\mathrm{Re}(1+m^2)}(u+mw) \quad (6)$$

$$\frac{\partial w}{\partial t}+S\frac{\partial w}{\partial y}=\frac{1}{\mathrm{Re}}\frac{\partial^2 w}{\partial y^2}-\frac{Ha^2}{\mathrm{Re}(1+m^2)}(w-mu) \quad (7)$$

$$\frac{\partial T}{\partial t}+S\frac{\partial T}{\partial y}=\frac{1}{\mathrm{RePr}}\frac{\partial^2 T}{\partial y^2}+\frac{Ec}{\mathrm{Re}}\left[\left(\frac{\partial u}{\partial y}\right)^2+\left(\frac{\partial w}{\partial y}\right)^2\right]+\frac{EcHa^2}{\mathrm{Re}(1+m^2)}(u^2+w^2) \quad (8)$$

The initial and boundary conditions for the velocity become

$$t\le 0:\ u=w=0,\ t>0:\ u=w=0 \quad y=-1,\ u=1,\ w=0,\ y=1 \quad (9)$$

and the initial and boundary conditions for the temperature are given by

$$t\le 0:\ T=0,\ t>0:\ T=1,\ y=+1,\ T=0,\ y=-1 \quad (10)$$

where the pressure gradient is assumed in the form $dP/dx=Ce^{-at}$, where C and a are constants.

3. Numerical Solution of the Governing Equations

Equations (6)-(8) are solved numerically using finite differences (Ames, 1977) under the initial and boundary conditions (9) and (10) to determine the velocity and temperature distributions for different values of the parameters Ha and S. The Crank-Nicolson implicit method is applied. The finite difference equations are written at the mid-point of the computational cell and the different terms are replaced by their second-order central difference approximations in the y-direction. The diffusion term is replaced by the average of the central differences at two successive time levels. The viscous and Joule dissipation terms are evaluated using the velocity components and their derivatives in the y-direction which are obtained from the numerical solution of the momentum equations. Finally, the block tri-diagonal system is solved using Thomas' algorithm. All calculations have been carried out for $C=-5$, $a=1$, Re=1, Pr=1 and $Ec=0.2$. It should be mentioned that the results obtained herein reduce to those reported by Attia (1998) for the case of constant pressure gradient ($a=0$). These comparisons lend confidence in the accuracy and correctness of the solutions.

4. Results and Discussion

Figure 2 shows the profiles of the velocity components u and w and temperature T for various values of time t. The figure is plotted for $Ha=1$, $m=3$ and $S=1$. It is observed that the velocity components and temperature increase with time

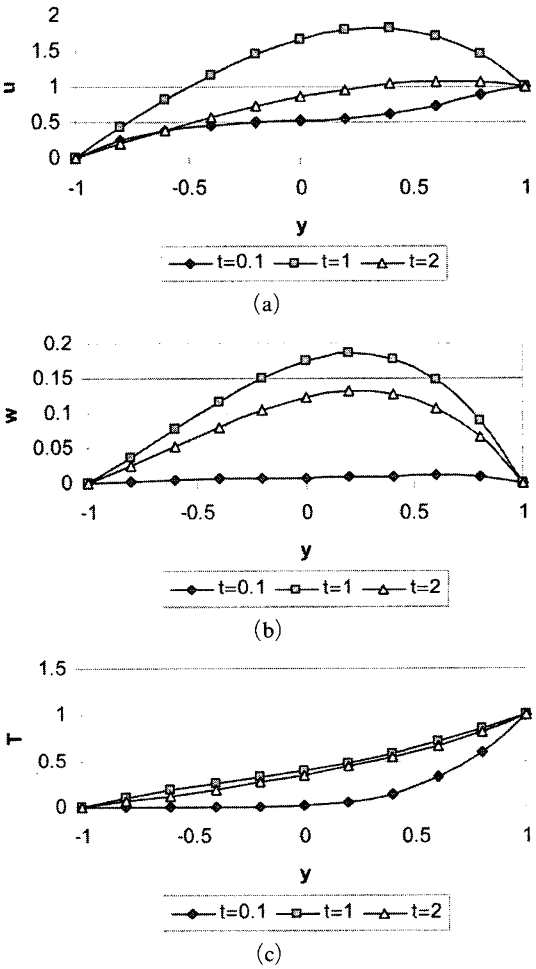
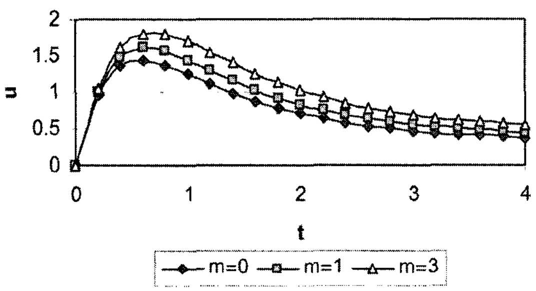


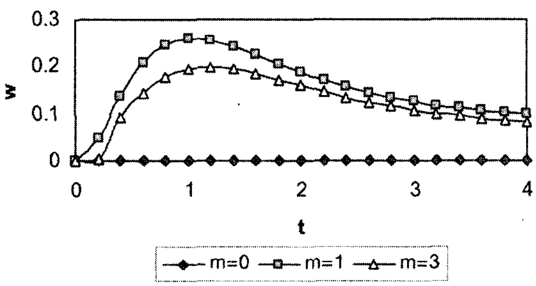
Fig. 2 Time development of the profile of: (a) u; (b) w; and (c) T ($Ha=1$, $m=3$ and $S=1$)

for small values of time and then decrease as time develops. For some times they exceed their steady state values and then go down towards steady state. This can be explained as, the velocity u increases from its zero rest value which increases the driving force of w and then, increases w with time. The increase in w increases the resistive force of u and, in turn, decreases u and consequently w. The increase and decrease in u and w are expected to cause, respectively, corresponding increase and decrease in viscous and Joule dissipations and, in turn, in temperature T.

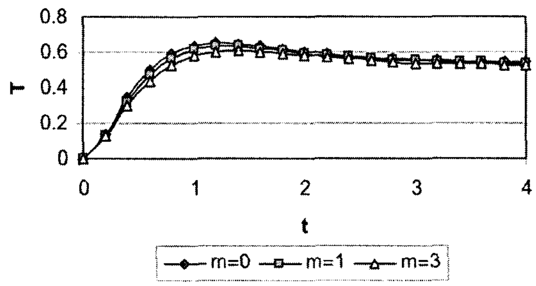
Figure 3 shows the time evolution of u, w and T at the centre of the channel $y=0$ for various values of the Hall parameter m. In this figure, $Ha=1$ and $S=0$. It is clear from Fig. 3(a) that increasing the parameter m increases u. This is because the effective conductivity $(\sigma/(1+m^2))$ decreases with increasing m which reduces the magnetic damping force on u. In Fig. 3(b), the velocity component w increases with increasing the parameter m slightly ($m=0$ to 1), since increasing m increases the driving force term $(mHa^2u/(1+m^2))$ in Eq. (7) which pumps the flow in the z-direction. However, increasing m more decreases the effective conductivity that results in a reduced driving force and then, decreases w. Figure 2(c) indicates that increasing m slightly decreases T for all values of t as a result of decreasing the Joule dissipation. In general, the effect of m on the temperature T can be neglected especially for higher values of t where the velocity components u and w become small and consequently, the viscous and Joule dissipations are neglected.



(a)



(b)



(c)

Fig. 3 Effect of m on the time variation of: (a) u at $y=0$; (b) w at $y=0$ and (c) T at $y=0$ ($Ha=1$ and $S=0$)

Figure 4 shows the time evolution of u, w and T at the centre of the channel $y=0$ for various values of the Hartmann number Ha. In this figure, $m=3$ and $S=0$. Figure 4(a) indicates that increasing Ha decreases u as a result of increasing the damping force on u. Figure 4(b) shows that increasing Ha increases w since it increases the driving force on w. However, increasing Ha more increases w at small t but decreases it at large t. This can be attributed to the fact that large Ha decreases the main velocity u, which increases with time, and reduces the driving force on w which results in decreasing w at large t. Figure 4(c) shows that increasing Ha slightly increases T due to increasing the Joule dissipation. However, for higher Ha, the effect of Ha on T depends on time. For small t, increasing Ha increases T due to increasing the Joule dissipation. But, for large t, increasing Ha decreases T as a result of decreasing the velocities u and w and consequently decreases the viscous and Joule dissipations. It should be mentioned that, the effect of Ha on the temperature T can be neglected especially for higher values of t where the velocity components u and w become small and consequently, the viscous and Joule dissipations are neglected.

Figure 5 presents the time evolution of u, w and T at the centre of the channel $y=0$ for various values of the suction parameter S. In this figure $Ha=1$ and $m=3$. Figures 5(a) and 5(b) show that increasing the suction decreases u, for all t, due to the convection of the fluid from regions in the lower half to the centre which has higher fluid speed. The effect of S on u can be neglected for very small time, but becomes more pronounced as time develops. As shown in Fig. 5 (b), for small t, increasing S increases w, but increasing S more decreases it. As time develops, increasing S always decreases w as a result of decreasing u which decreases the source term of w. The increase in w occurs at very small time can be attributed to the fact that the flow started impulsively from rest and u increases from its zero value which increases the driving force of w and consequently increases w. The suction has the effect of decreasing u which, in turn, decreases the damping force of w and its effect appears later as time develops. Higher suction velocity results in a shorter time duration at which the decrease in w occurs. Figure 4(c) shows that increasing S decreases the temperature at the centre of the channel. This is due to the influence of convection in pumping the fluid from the cold lower half

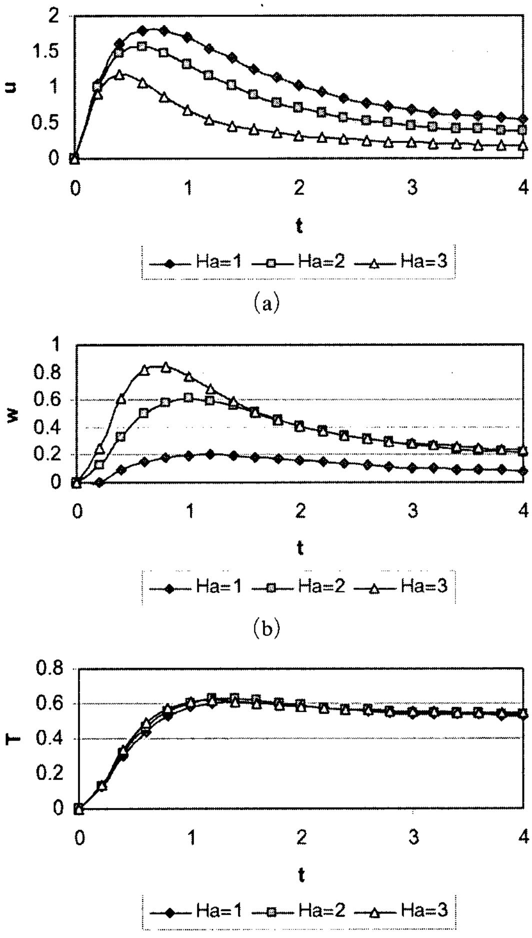


Fig. 4 Effect of Ha on the time variation of: (a) u at $y=0$; (b) w at $y=0$ and (c) T at $y=0$ ($m=3$ and $S=0$)

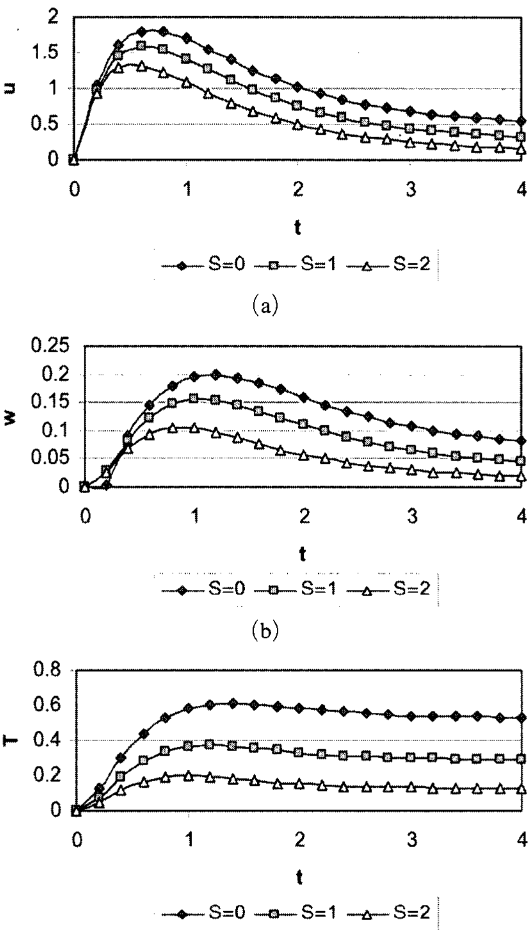


Fig. 5 Effect of m on the time variation of: (a) u at $y=0$; (b) w at $y=0$; and (c) T at $y=0$ ($Ha=1$ and $m=3$)

Table 1 Variation of the steady state values of τx_L, τx_U, τz_L, τz_U, Nu_L and Nu_U with the Hall parameter m ($Ha=1$, $S=1$)

	$m=0$	$m=1$	$m=3$	$m=5$
τx_L	2.8639	3.1217	3.4701	3.5439
τx_U	−3.2919	−4.0565	−5.0559	−5.2647
τz_L	0	0.3485	0.2738	0.1844
τz_U	0	−1.0121	−0.7834	−0.5262
Nu_L	0.8593	0.9501	1.0748	1.1015
Nu_U	−0.2410	−0.3400	−0.4843	−0.5157

towards the centre of the channel.

Table 1 presents the variation of the steady state values of the skin friction coefficients at both the lower and upper plates (τx_L, τx_U, τz_L, and τz_U) and the Nusselt number at the lower and upper plates (Nu_L and Nu_U) with the Hall parameter m for $Ha=1$ and $S=1$. It is clear that increasing m increases τx_L and the magnitude of τx_U but decreases τz_L and the magnitude of τz_U. Also, increasing m increases Nu_L and the magnitude of Nu_U.

5. Conclusions

The unsteady Couette flow of a conducting fluid under the influence of an applied uniform magnetic field has been studied considering the Hall effect in the presence of uniform suction and injection and an exponential decaying pressure gradient. Introducing the Hall term gives rise to a velocity component w in the z-direction and it affected the main velocity u in the x-direction. The effect of the magnetic field, the Hall parameter and the suction and injection velocity on the velocity and temperature distributions has been investigated. As time develops, increasing the Hall parameter m increases the velocity component u and increases the velocity component w for small m and decreases it for large m. It is found that the effect of large Ha on w depends on time. Also, it is of interest to detect that the effect of the parameter S on the velocity component w depends on time. In general, the effect of Ha or m on the temperature T can be neglected especially for higher values of time.

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