Journal of the Korea Society of Mathematical Education Series D: Research in Mathematical Education
Vol. 9, No.3, September 2005, 275–284

韓國數學教育學會誌 시리즈 D:
<數學教育硏究>
제 9 권 제 3 호 2005 년 9 월, 275–284

Girl-Favored Tessellations Using Technology[1]

Choi-Kho, Sangsook
Department of Mathematics Education, College of Education, Dankook University,
147, Hannam-ro, Yongsan-gu, Seoul 140-714, Korea;
E-mail: sangch@dankook.ac.kr

(Received August 29, 2005)

Tessellations are the pattern of iterations of geometric symmetry and translation. We can find them in the works of Escher who is the famous Dutch artist, and the American Indean life. Also, we can find the beauty of tessellations in the Korean traditional house door, Buddist temple architecture, palace's fence, etc. In the article, the figures of patterns we present are bird, fish, cat, pig, elephant, penguin, child and horse riding man, including Escher's, which are constructed using the computer geometric program, GSP (Geometer's Sketchpad). We want to talk about girl's disposition toward mathematics related to the figures. If they are supported by this kind of interesting figures constructed by their own hands, students will have more interest in learning geometric figures.

Keywords: gender difference, Mathematical disposition, Geometric figures, technology, tessellation
ZDM Classification: C40, U60, U70
MSC2000 Classification: 97U60, 97U70

INTRODUCTION

Equity for women in education has been a focus among many issues for a long time but, still today the position of women in mathematics especially has not been improved as much as we pursued in the mathematics literature. Recently, researchers all over the world have investigated the issue of gender difference in various domains of literature (*e.g.* Burton 1990; Chipman, Brush & Wilson 1985; Fennema 1985; Fennema & Leder 1990; Fox, Brodin & Tobin 1980). The most common domains in which research has

[1] This paper will be presented at the Tenth International Seminar of Mathematics Education on Creativity Development at Korea Advanced Institute of Science and Technology, Daejeon, Korea, October 9, 2005.

been done are learning styles (Honigsfeld & Dunn 2003), attitudes toward mathematics and mathematical achievement (Royster, Harris, & Scheps 1999; Utsumi &Mendes 2000), general knowledge (Lynn, Irwing & Cammock 2001), and intelligence (Bennet 1996; Furnham, Clark, & Bailey 1999; Furnham & Fong 2000).

Schools have responsibility to acknowledge that today's curriculum not only does not meet the needs of females, but of other under-represented groups (Sullivan 1994). The issue no longer is whether gender inequities exist in mathematics. Gender equity has to be pursued with in a more concrete way.

A large number of variables (cognitive, affective and socio-cultural) are thought to contribute to gender differences in the learning and teaching mathematics. In their study, Honigsfeld & Dunn (2003) found that their subjects, who were from five different countries (Bermuda, Brunei, Hungary, New Zealand and Sweden), differed in their preferences for learning styles according to their gender types. It was found that females were more self-motivated, persistent and responsible than males, whereas males were more kinesthetic and peer oriented. Some researchers also concluded that students' attitudes might change from positive to less positive as they promote to a higher level in their schooling. In other words, as the content of the subject becomes more difficult in the upper years of schooling, students' attitudes towards the subject matter change and differ accordingly.

This study is to investigate how differently some females, compared to male students, learn transformation of mathematics, with respect to mathematical disposition. We want to compare the degree of their mathematical disposition including affective elements between two genders so that we could be more aware of the characteristics of females' learning style of mathematics and apply the implications of the study to teaching them mathematics. Also, the study reports the correlation between subcomponents of Mathematical disposition at a middle school in Korea.

THEORETICAL BACKGROUNDS

1. What is a tessellation?

Tessellations mean to completely cover the plane without gaps or overlaps. Mathematicians call such an *arrangement of shapes a tessellation or a tiling.* The word, tessellation comes from the small square ceramic tiles, called tesserae that Romans used to create mosaic tile designs. When a tessellation uses only one shape as in a honeycomb, it's called a *monohydric tiling.* Tessellations are found in many natural forms other than honeycombs. The molecular structures of some crystals show tessellation patterns. Also, in Korean life, we can find them in our palace walls, our wall paper, our traditional windows and temple decoration called "*Dan Cheng.*"

In general, the only regular polygons that create monohydric tessellations are equilateral triangles, squares, and regular hexagons. The tessellations in which tiles are congruent regular polygons whose edges exactly match are called *regular tessellations.* The tessellations, called semi regular tessellations can involve more than one type of shape, such as an octagon-square combination, and a dodecagon-equilateral triangle combination.

2. Escher's works and abstract patterns

Dutch, Escher (1898–1972) traveled to Spain and became fascinated with the tile patterns of the Alhambra. Escher spent days in Alhambra, sketching the tessellations on the walls and ceilings. Escher spent many years learning how to use translations, rotations, and glide reflections on the grids of equilateral triangles and parallelograms to create tessellations of birds, fish, reptiles, and human. Especially, Escher cared for regularities which existed in art and put the value of a color and a figure by comparing them in relationships. He thought any image, a figure, even light and shade, or color did not exist by itself, but existed in relation to others. The works of Escher who was not familiar with mathematics demonstrated some important properties of mathematics that mathematicians can read into. Escher who found beauty and peace in the categories of regularity, periodic repetition, and regeneration, presented abstraction and infinite perpetuity using tessellations.

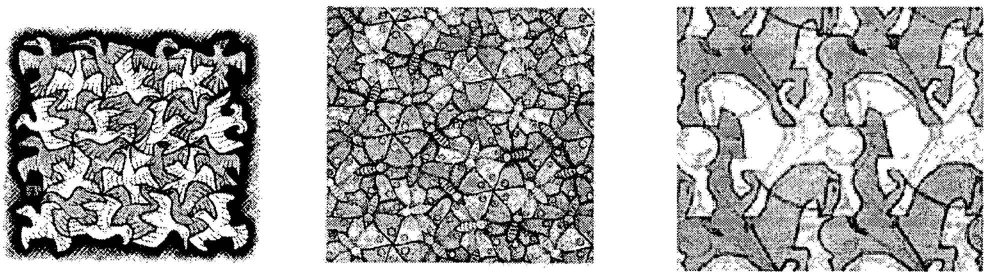

Figure 1. Escher's works & other patterns

We want to pay attention to the designs of Indian-Americans of North America. Medeleine (1993) investigated the patterns these people used. These geometric and abstract dèsigns looked much modernized. They used to engrave their own people's identity in baskets, ceramics, carpets, leather, wooden crafts and the bark of a tree. Why did they design their goods with abstract patterns? They are nomadic people who traveled the plains of North America all the times, hunting buffalo, and fishing salmon. They needed to have goods which matched well with others and presented their identity regardless of specific times and places. Geometric and abstract patterns are what they thought is useful for that purpose. Those patterns Indian-Americans used in past are still attractive to us even today. Since the pattern with regularity, periodic repetition, or regeneration has abstract figures in a natural way, modern people of the 21st century still feel beautiful.

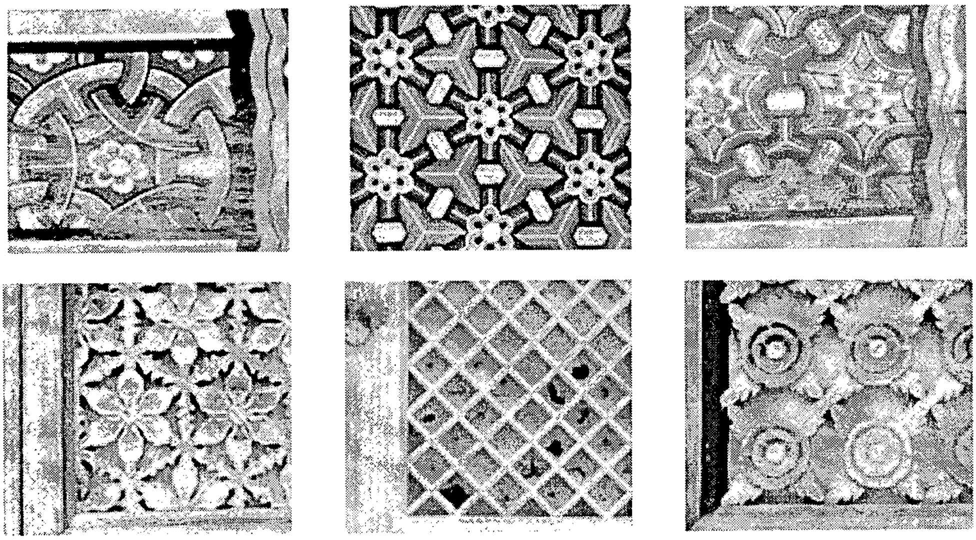

Figure 2. Korean tessellations from the old architecture

TECHNOLOGY IN MATHEMATICS EDUCATION METHOD

The sample was composed of 70 students, attending at a middle school in Kyunggi province. Since some students were absent on the day the test was executed, 27 female students and 32 male students were arbitrary selected in the school which a teacher who has an interest in gender differences and use of technology works for. Students these days so called "internet generation" did not need much time to get used to this geometrical software.

The teacher provided the instructional materials including 5 lessons about tessellations and gave the pre and post tests to collect the data. Before the instructional materials were given to the students, some training hours for use of technology were designed for students to be familiar with using computer software, GSP (Geometer's Sketchpad).

Instructional Materials

5 lessons consisted of three stages of learning, exploring (1), performing (3), and

reflecting (1). Some instructions that students followed at their performing stage are present below as an example. Many figures were referred to Serra (1997)'s book, *Discovering Geometry*.

Translations with rectangles

1) Pigs
   (1) Start with a grid of squares and create a curve using three points. Take all points to color inside so that they look like a pig.
   (2) Hide lines of squares and color hats and legs.
   (3) Complete tessellations using translations up and down.

2) Korean masks (Reds')
   Start with a grid of parallelograms and create curves using a lot of three points to get our line of a mask. After you get one mask, click all points in order to color inside with the color you want in figure 4. Then repeat translations as many as you wish to get complete tessellations with multi-masks.

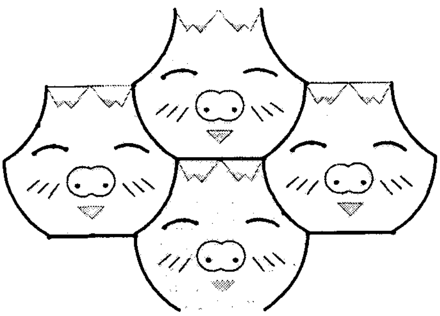

Figure 3. Pigs from students' works

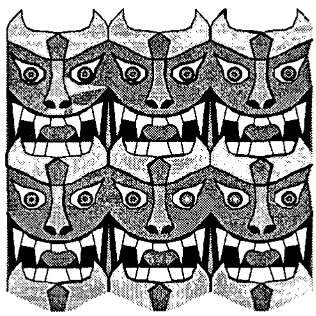

Figure 4. Korean masks (Reds')

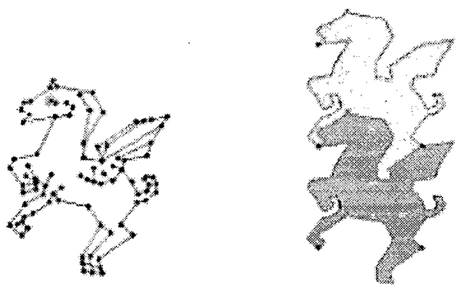

Figure 5. Pegasus

3) Pegasus

Start with a grid of rectangles and create curves using three points as many times as it needs on the bottom side of a rectangle. Then translate this side to the top of a rectangle. In a similar way, construct the front of a Pegasus and translate the front side to the right hand side. As Figure 9, fix a point away from the original figure with the length of rectangles and translate a whole figure of Pegasus. Constructing a tessellation of Pegasus takes more effort due to its complication.

Instrument

Mathematical Disposition Test (MDT) for students' affective domain includes 24 questions from 6 areas (4 questions for each) which relate to Mathematical Confidence, Mathematical Flexibility, Mathematical Willpower, Mathematical Curiosity, Mathematical reflection, and Mathematical Value. Each question contains five response alternatives per item: strongly positive, less positive, in-between, less negative, and strongly negative. Typical test reliability of the MDT is around 0.8397.

Table 1. Analysis of Quality of Test Instruments (MDT)

Item		1	2	3	4	5	6	7
Reliability		0.7782	0.8524	0.7740	0.7759	0.8480	0.8283	0.8397
Internal Validity	Infit	1.51	0.48	1.02	1.38	0.70	0.80	0.98
	Outfit	1.24	0.55	1.11	0.96	0.71	0.73	0.88
Difficulties		– 0.17	0.32	– 0.72	0.85	– 0.29	0.02	0.00
Discrimination		0.87	0.60	0.88	0.87	0.53	0.68	1.00

*Note. 1 = Mathematical Confidence; 2 = Mathematical Flexibility; 3 = Mathematical Intention; 4 = Mathematical Curiosity; 5 = Mathematical Reflection; 6 = Mathematical Value; 7 = Mathematical Disposition Index.

Procedure

Mathematical concepts including in learning materials are translation, reflection, rotation, and composition of more than two of transformations. A total of 3 units consisting of 5 lessons (each for 60–70 minutes) were given to the students who were asked to work in a group of 2 or 3 students. At the last lesson, they should hand out their final project using what they learned about tessellations with the presentation of it. This project was evaluated using the revised form of the evaluation tool that Korean Educational Development Institute (KEDI) developed. Pre and Post Tests were performed and evaluated by the researcher at the end of first semester. A test booklet and pencils were provided to each participant. The MDT was administered according to the instructions in the manual.

Data Analysis

To analyze the data collected, the frequency and percentage per type to find out the Mathematical Styles in mathematical situations of seven grade, average, standard deviation, *t*-test. Data were processed through SPSS/PC 10.0K static program for Windows. In order to evaluate item-internal consistency reliability and discrimination, Cronbach α. was calculated using SPSS 10.0K. Internal validity and difficulty were calculated using BIGSTEPS (Livacre & Wright 2003) based on Rasch's 1-parameter item-response model.

Table 2. Differences of Mathematical Disposition of children with performance for Gender

Components	Gender	*N*	Mean	Std. Deviation	*t*-value	Probability
Mathematical Confidence	Male	27	11.7037	3.93030	– 2.135	0.037*
	Female	32	14.0938	4.56041		
Mathematical Flexibility	Male	27	9.8148	2.85599	– 3.553	0.001**
	Female	32	13.0938	4.01095		
Mathematical Willpower	Male	27	11.3704	3.30673	– 2.702	0.009**
	Female	32	14.0313	4.11515		
Mathematical Curiosity	Male	27	10.7037	2.49330	– 2.308	0.025*
	Female	32	13.0625	4.78531		
Mathematical Reflection	Male	27	13.2963	1.48880	0.066	0.947
	Female	32	13.2500	3.36011		
Mathematical Value	Male	27	9.5556	2.96561	– 2.071	0.043*
	Female	32	11.4063	3.75765		
Mathematical Disposition Index	Male	27	66.4444	10.69627	– 3.015	0.004**
	Female	32	78.9375	19.13776		

*Note. $*p < 0.05$; $**p < 0.01$

RESULTS

Comparative study

The aim of the analysis is an attempt to examine the differences of Mathematical Disposition Ability on the student gender among the Korean students. Table 2 provides the means and standard deviations of the items in the MDT and of the student gender, and the t-value on the difference. The result of t-test (see Table 2) showed a significant difference on Mathematical Disposition Tests in favoring female. Statistically significant difference was found on the component of Mathematical Confidence ($t = -2.135$, $p = 0.037$), Mathematical Flexibility ($t = -3.553$, $p = 0.001$), Mathematical Intention ($t = -2.702$, $p = 0.009$), Mathematical Curiosity ($t = -2.308$, $p = 0.025$), Mathematical Value ($t = -2.071$, $p = 0.043$), and Mathematical Disposition Index ($t = -3.015$, $p = 0.004$). No statistically significant difference was found on only the component of Mathematical Reflection ($t = 0.066$, $p = 0.947$).

Table 3. Correlation of the Mathematical Disposition

	1	2	3	4	5	6	7
Mathematical Confidence	1	0.463(**)	0.777(**)	0.733(**)	0.302(*)	0.451(**)	0.865(**)
Mathematical Flexibility		1	0.479(**)	0.326(*)	0.026	0.253	0.596(**)
Mathematical Intention			1	0.713(**)	0.433(**)	0.439(**)	0.876(**)
Mathematical Curiosity				1	0.504(**)	0.573(**)	0.870(**)
Mathematical Reflective					1	0.339(**)	0.534(**)
Mathematical Value						1	0.676(**)
Mathematical Disposition Index							1

*Note. **Correlation is significant at the 0.01 level (2-tailed); *Correlation is significant at the 0.05 level (2-tailed).

Correlation study

The aim of the analysis is an attempt to examine the correlation between subcomponents of the MDT. The following table shows that there was significant relationship between each component except Mathematical Flexibility. Mathematical

Flexibility had no relationship with Mathematical Reflection Mathematical Value. This indicates that students might have been more exposed to Mathematical Fluency since the school mathematics focuses on fluency with speed and correctness, rather than on mathematical reflection and mathematical value.

Conclusion

The main purpose of this study was to find out whether or not there were any gender differences in mathematical disposition to improve of female students' learning in math classrooms. Through analyzing the data, the results showed that these male and female students differ significantly in most elements of their disposition except mathematical reflection. Mathematical reflection might not have been challenged by using a computer or learning of tessellation. This requires that the teacher needs to be more concern about giving a chance for student to reflect what they are doing through appropriate questions.

However, the study concluded that the female students enjoyed more their learning with computer, which was very different outcome from what most educators predicted. The study convinced that female students have more favor in constructing colorful figures as the researcher observed the study, because they wanted to put more beautiful color and to be more concern about such external properties as color and balanced shape. Usually, they could not finish their work within given time since they spent most of time in getting better shapes instead of finding out mathematical concepts.

In conclusion, we need to investigate the area of cognitive understanding along with Mathematical Disposition and expand the lesson hours for their learning with a computer to get more meaningful research outcome.

REFERENCES

Bennet, M. (1996): Men's and women's self-estimates of intelligence. *The Journal of social Psychology* **136**, 411–412.

Burton, L. (1990): *Gender and mathematics: An international perspective.* London, England: Cassell.

Chipman, S. F.; Brush, L. R. & Wilson, D. M. (1985): *Women and Mathematics: Balancing the Equation*, Hillsdale, NJ: Lawrence Erlbaum Associates. MATHDI **1985g.**02353

Fennema, E. (1985): Explaining Sex-related Differences in Mathematics: Theoretical Models. *Educational Studies in Mathematics* **16(3)**, 303–320. MATHDI **1986d.**07477

Fennema E. & Leder, G. (1990): *Mathematics and Gender.* New York: Teachers College Press. MATHDI **1991d.**02905

Furnham, A.; Clark, K. & Bailey, K. (1999): Sex differences in estimates of multiple intelligences.

European Journal of Personality **13**, 247–259.

Furnham, A. & Fong, G. (2000): Self-estimated and psychometrically measured intelligence; A cross-cultural and sex differences study of British and Singaporean students. *North American Journal of Psychology* **2**, 191–200.

Honigsfeld, A. & Dunn, R. (2003): High school male and female learning-style similarities and differences in diverse nations. *Journal of Educational Research* **96(4)**, 195–206. MATHDI **2003d.**03671

Livacre, J. M. & Wright, B. D. (2003): *A User's Guide to BIGSTEPS Rasch-Model Computer Programs*. Winsteps.com.

Lynn, R.; Irwing, P. & Cammock, T. (2001): Sex differences in general knowledge Intelligence. **30(1)**, 27–39.

Serra, M. (1997): *Discovening Geometry, An inductive approach*. Berkeley, CA: Key Curriculum Press. MATHDI **1997d.**02483

Royster, D. C.; Herris, M. k. & Schoeps, N. (1999): Dispositions of college mathematics students. *International Journal of Mathematical Education in science & Technology* **30(3)**, 317–333. MATHDI **1999e.**03613

Sullivan, E. (1994): Achieving equity in mathematics education. *Thrust for Educational Leadership* **23(7)**, 35–39.

Utsumi, M.C. & Mendes, C. R. (2000): Researching the attitudes towards mathematics in basic education. *Educational Psychology* **20(2)**, 237–243.