Journal of the Korea Society of Mathematical Education Series D: Research in Mathematical Education
Vol. 9, No. 3, September 2005, 243–256

韓國數學敎育學會誌 시리즈 D: <數學敎育硏究>
제 9 권 제 3 호 2005 년 9 월, 243–256

Abacus Numerals for Rapid and Sufficient Mathematics Learning for Enhancing Creativity[1]

Hayakawa, Yoshinori
Department of Clinical Engineering, Toin University of Yokohama,
1614 Kurogane-cho, Aoba-ku, Yokohama, Kanagawa-ken 225-8502, Japan;
E-mail: hayakawa@cc.toin.ac.jp

(Received August 31, 2005)

Abacus numerals were developed using the concept of the binary system to form decimal numerals. This would allow addition, subtraction, multiplication, and division to be performed based solely on the knowledge of the 14 forms of the numerals and three simple rules. These numerals were taught to 260 elementary school pupils of 3rd and 4th grade. After 90 minutes of instruction, they, nearly all, were able to understand principles to add, and to subtract, and partly to multiply using Abacus Numerals. Protected Abacus Numerals are proposed against forgery. An International Numeration System is proposed based on the form of Abacus Numerals to facilitate international communication. A new type of abacus is proposed.

Keywords: abacus numerals, binary system, base-10 numerals, logical thinking, calculation, protected numerals, international numeration system
ZDM Classification: D50
MSC2000 Classification: 97C50, 97D50

INTRODUCTION

Mathematics is the science of logic. In the actual education of mathematics in early stages, memorizing is most important to add, subtract, multiply and divide. Pupils are forced to remember addition table and multiplication table. Without remembering these tables, they cannot advance to further learning of mathematics. An Abacus Numerals were invented by the author approximately 13 years ago to improve this situation

[1] This paper will be presented at the Tenth International Seminar of Mathematics Education on Creativity Development at Korea Advanced Institute of Science and Technology, Daejeon, Korea, October 8, 2005.

(Hayakawa 1995). By remembering three basic rules to create Abacus Numerals and 14 actual forms of Abacus Numerals, the ability to perform addition, subtraction, multiplication, and division appears to be attainable within a few months. The process of calculation is logical. In addition, pupils seem to learn the beauty of mathematical logic in their early ages through learning of Abacus Numerals.

The form of the decimal numerals is based on the concept of the binary system. However, knowledge of the binary system is not necessary in order to learn these numerals. A medieval Chinese numeral system (Table 1) by which addition and subtraction can be achieved by memorizing the form of the numerals has been reported previously (Ifrah 1981).

Table 1. Medieval Chinese Numerals Used for Calculation.

1	2	3	4	5	6	7	8	9	0
𝍠	𝍡	𝍢	𝍣	𝍤	𝍥	𝍦	𝍧	𝍨	O

However, because the Chinese numerals were developed on the quasi-quandary system, multiplication and division can only be performed by memorizing the multiplication table.

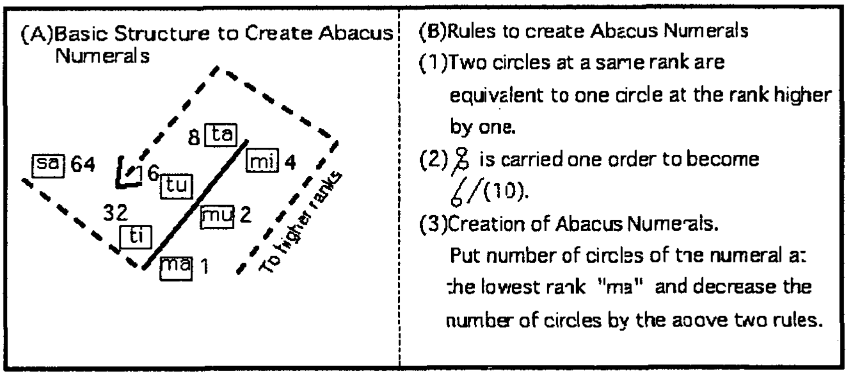


Figure 1. Rules Governing Formation of Abacus Numerals

A kind of decimal abacus (Papy's minicomputer) based on binary system has been reported to facilitate addition and subtraction (Papy 1970). Calculations of multiplication and division by Papy's minicomputer are possible without memorizing the multiplication table only for simple cases such as multiplying or dividing by 2. The Abacus N umeral system provides an improvement over the Chinese numeral system and Papy's minicomputer because multiplication and division can be performed easily without memorizing the multiplication table. The rules guiding formation of Abacus Numerals are shown in Figure 1.

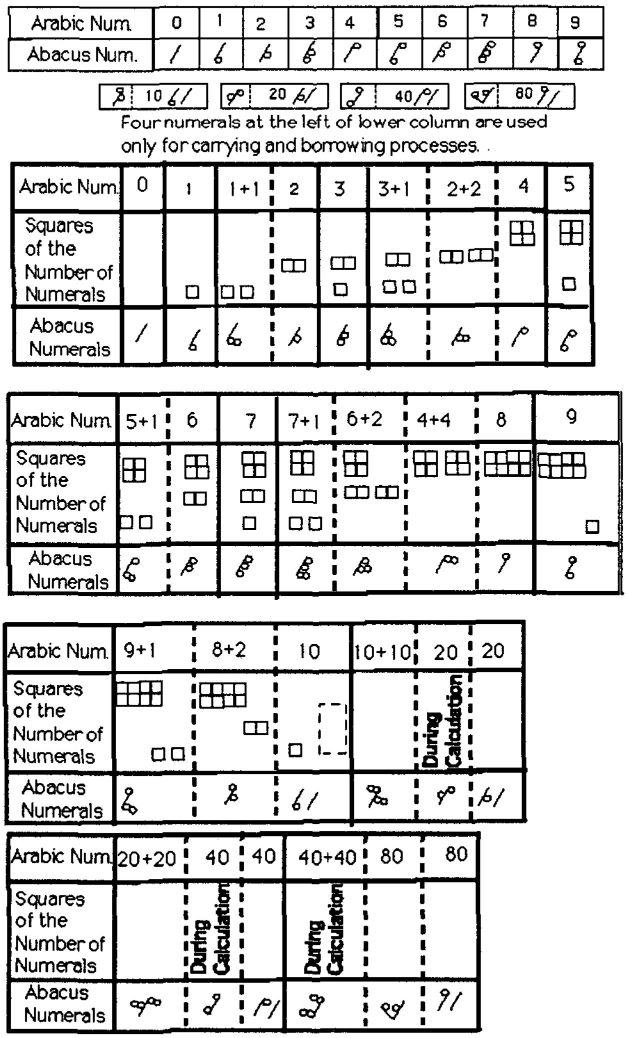
RULES TO CREATE ABACUS NUMERALS

(A) Order of height of ranks of numerals from the lowest are referred to as "ma", "mu", "mi", "ta", "tu", "ti", "sa". Two circles at a certain rank correspond with a circle one rank higher. Abacus numerals for a given number are formed by putting open circles of that number at the lowest rank "ma" and then shifting the location of the rank of the circles by the above rule to minimize the number of circles.

(B) Rules to create Abacus Numerals.

The actual forms of the numerals are shown in Table 2.

Table 2. Table of Abacus Numerals.



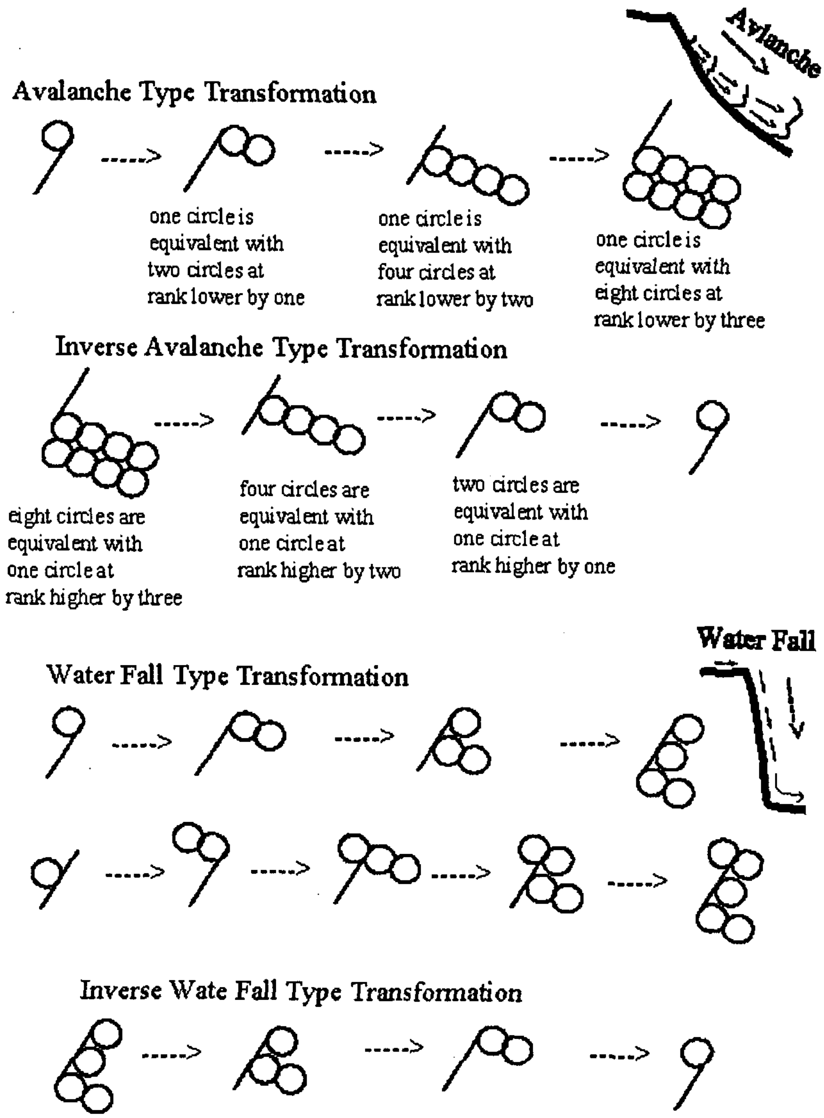


Figure 2. Transformation Frequently Used for Calculation.

EXAMPLES OF CALCULATIONS USING ABACUS NUMERALS

Examples of addition by Abacus Numerals are shown in Figure3. The numerals to be added are joined to yield the resultant summed numeral.

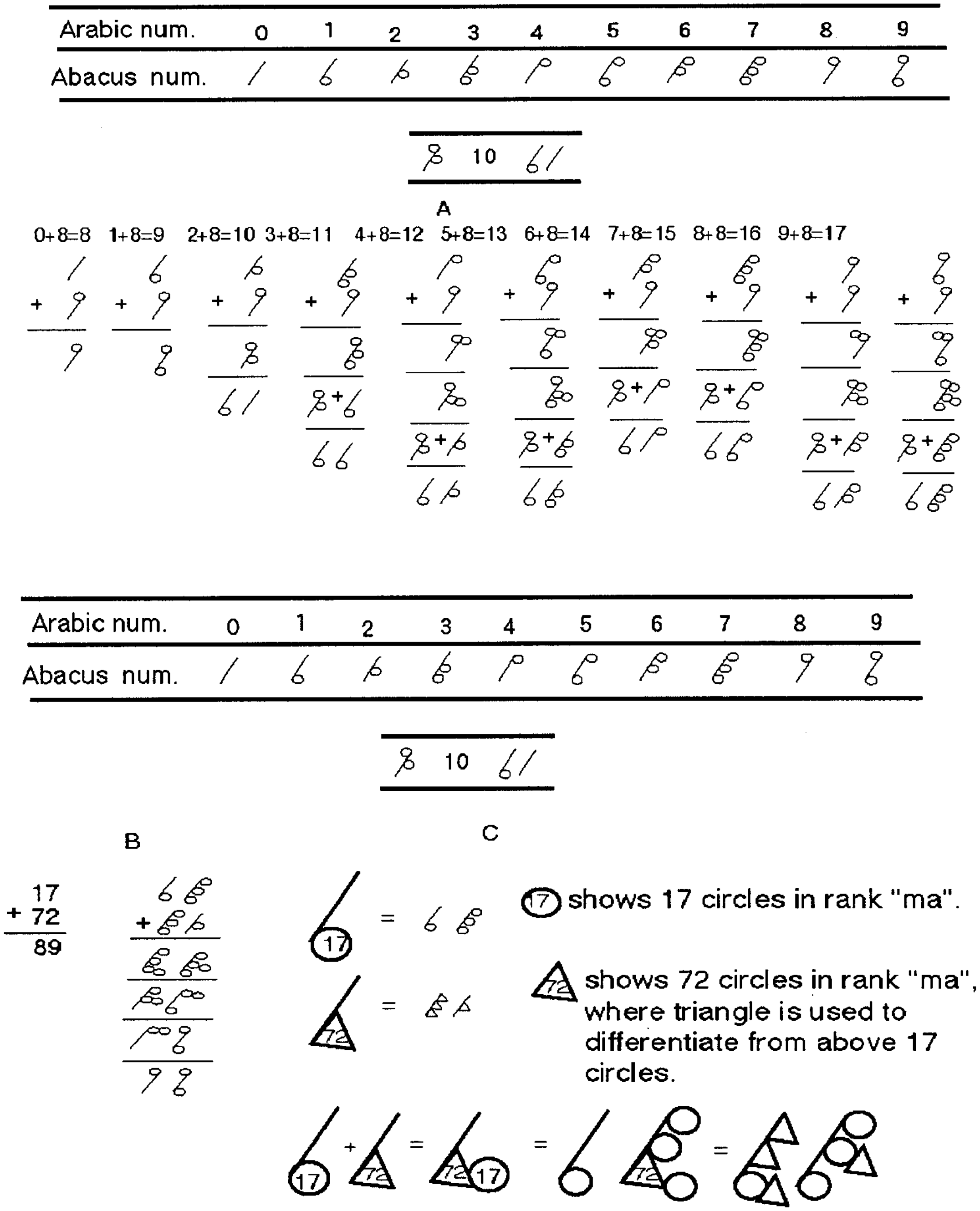


Figure 3. Examples of addition: Resultant Abacus Numerals are obtained by Joining Two Abacus Numerals. Actual Calculations can be simplified by Moving Coins or Stones Instead of Circles (Figure 7), as is the Case of a Traditional Abacus.

Examples of subtraction using Abacus Numerals are shown in Figure 4.

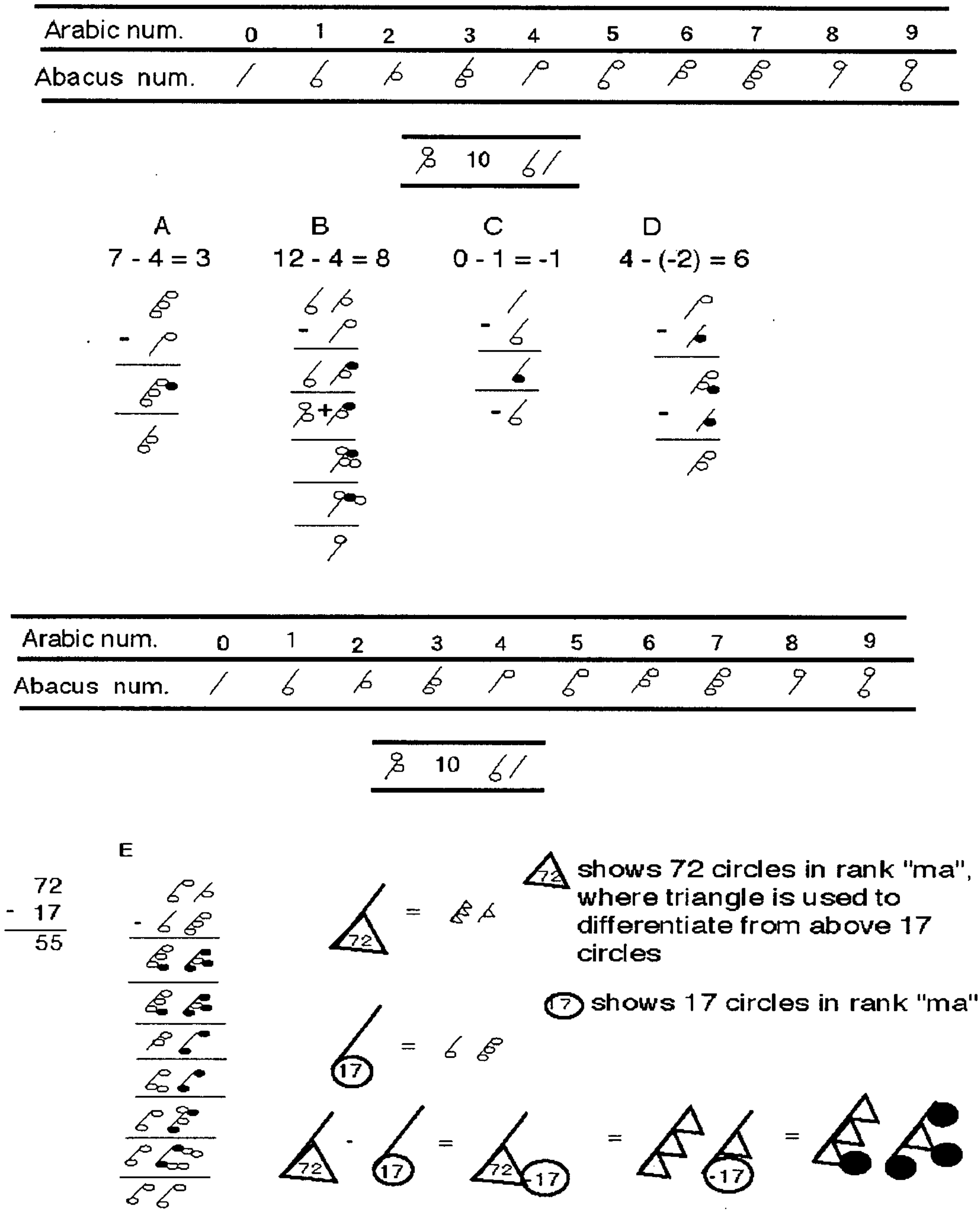
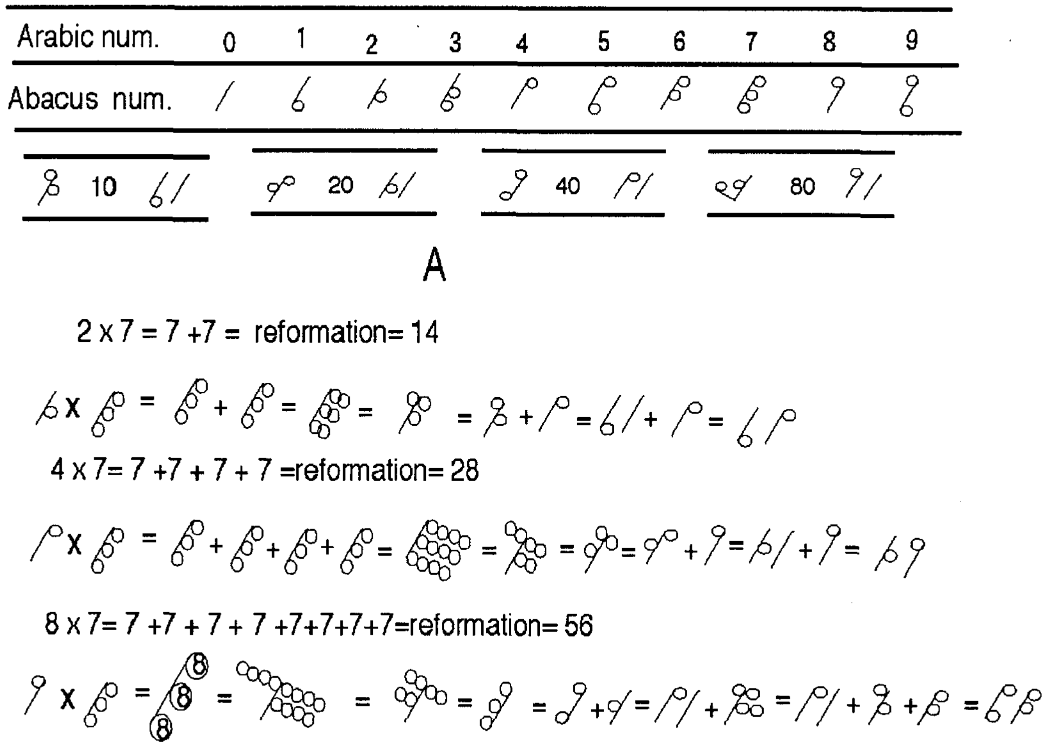


Figure 4. Examples of subtraction: Numerals to be subtracted are joined as filled circles or crosses. A filled circle and an open circle at the same rank cancel each other out. Actual calculations can be simplified by moving coins or stones instead of circles (Figure 7), as is the case of a traditional abacus.

In the case of subtraction, the numerals to be subtracted are joined as filled circles or crosses to the subtracted numerals with open circles to yield the resultant numerals. When a filled circle or a cross and an open circle appear at the same rank, they cancel each other out. Multiplication is performed by reforming the numerals as follows: Multiplication by 1 entails the addition of a given number to zero once, resulting in the formation of the original numeral. Multiplication by 2 entails the addition of a given number to zero twice, resulting in the formation of a numeral with two open circles in the original ranks, which results in one open circle shifted one rank higher from the original rank [The Abacus Numeral for 2 consists of single circle in rank "mu" (one rank higher than rank "ma") and is equivalent to 2 circles in rank "ma"] (Figure 5A). Multiplication by 8 is performed by joining the numeral 8 times, resulting in 8 open circles in the original rank. That result in a numeral with each circle shifted three ranks higher [the abacus numeral for 8 is equivalent to 8 circles in rank "ma" and consists of one open circle in rank "ta" (three ranks higher than rank "ma")]. Multiplication by number of higher order is shown in Figure 5B.



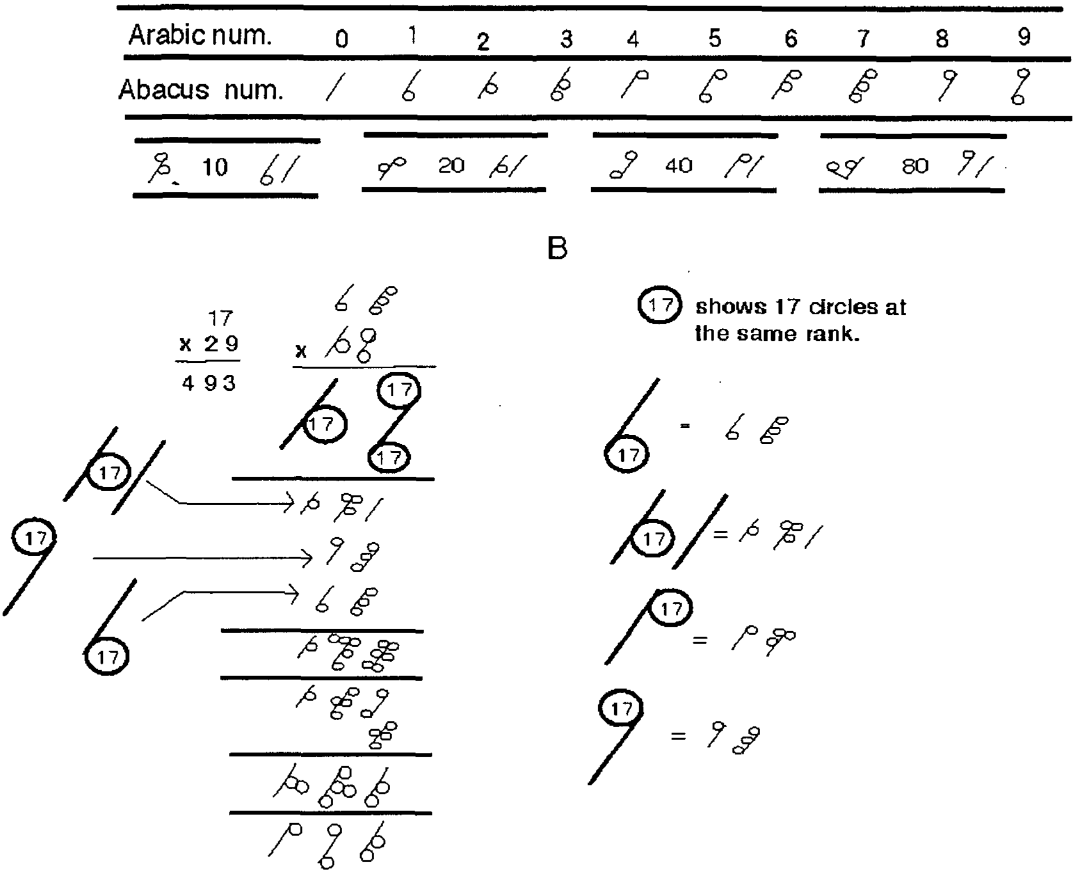
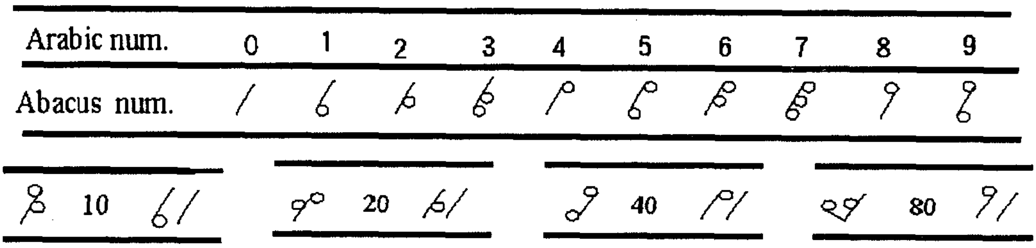
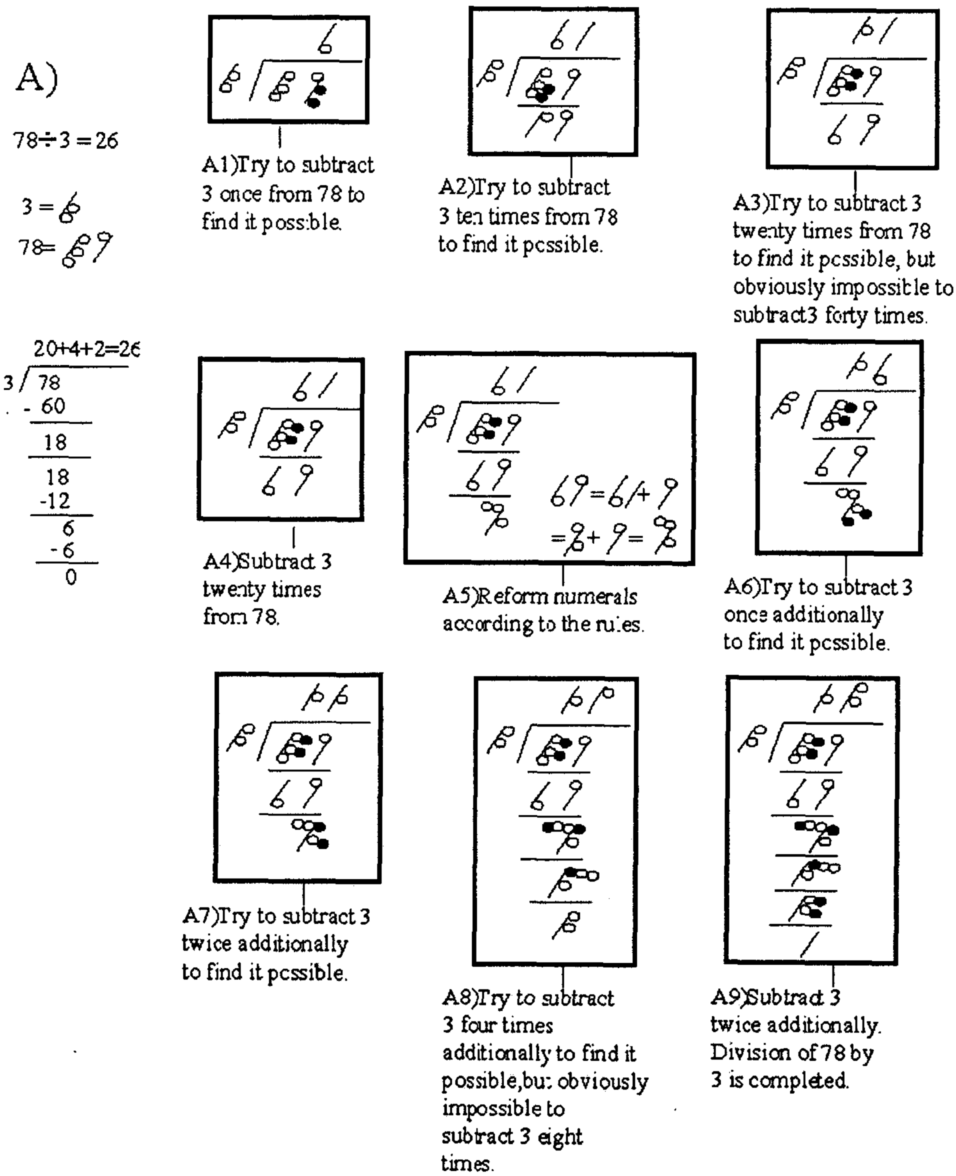


Figure 5. Examples of multiplication: A 7 × 2, 7 × 4 and 7 × 8; B 17 × 29, 17 circles in rank "ma" become 17 in Abacus Numerals and 17 circles in A rank and B order higher from "ma" become Abacus Numerals of 17 lifted by A ranks and B orders. The number 17 in triangle, diamond, and circle in the left hand side means 17 circles at the same rank and same order.

Examples of division are shown in Figure 6.


Arabic num. 0 1 2 3 4 5 6 7 8 9
Abacus num.
10
20
40
80


A)
78÷3 =26
3 =
78=
20+4+2=26
3 / 78
- 60
18
18
-12
6
-6
0
A1)Try to subtract 3 once from 78 to find it possible.
A2)Try to subtract 3 ten times from 78 to find it possible.
A3)Try to subtract 3 twenty times from 78 to find it possible, but obviously impossible to subtract3 forty times.
A4)Subtract 3 twenty times from 78.
A5)Reform numerals according to the rules.
A6)Try to subtract 3 once additionally to find it possible.
A7)Try to subtract 3 twice additionally to find it possible.
A8)Try to subtract 3 four times additionally to find it possible,but obviously impossible to subtract 3 eight times.
A9)Subtract 3 twice additionally. Division of 78 by 3 is completed.

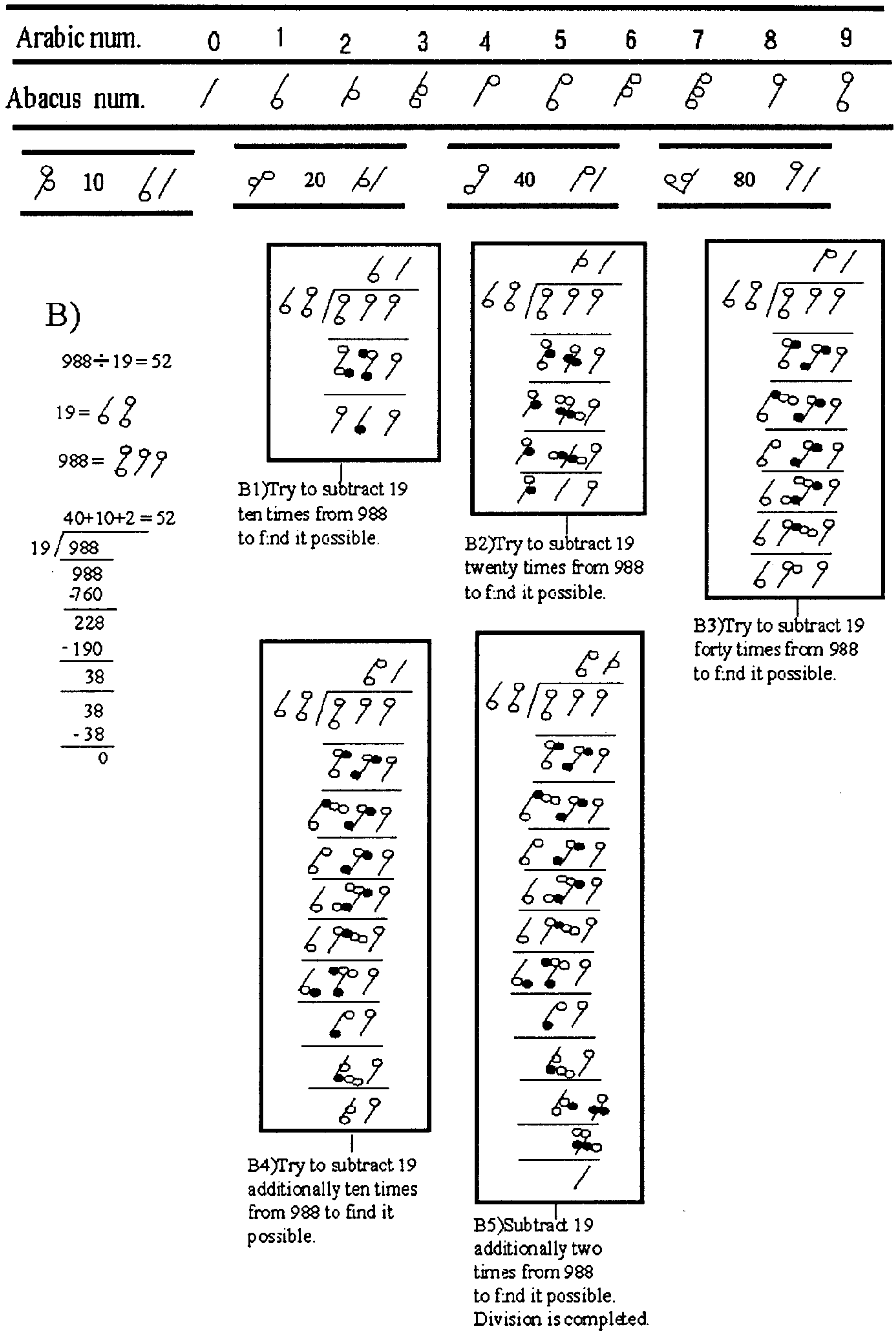


Figure 6. Examples of division: Actual calculation can be simplified by moving coins or stones instead of circles (Figure 7), as is the case of a traditional abacus.

In the same way, Abacus Numerals can be used for calculation of square roots without remembering multiplication table.

MATHEMATICS INSTRUCTION USING ABACUS NUMERALS

In September of 2004, approximately 90 pupils of Kuzuno Elementary School (4th grade) in city Yokohama and 170 pupils of Minami Ikuta Elementary School (3rd grade) in city Kawasaki. The learning time was 90 minutes. Approximately all pupils understood the principle of addition, and subtraction, and partly multiplication. Division was not taught to them because of the time was too short. Although they have already learned multiplication table, they are interested in addition, subtraction, and multiplication using Abacus Numerals.

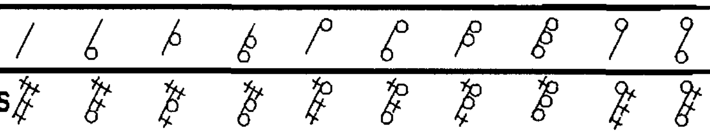
Prior to this, in 1996, the author used Abacus Numerals to instruct 5 mentally handicapped volunteers at the Niihari-En Institute with the cooperation of the president, Mr. Kyuugo Tsukada. The students had Down's syndrome and had an IQ of 40–32. They could understand Arabic numerals but could not perform the addition with carrying procedure or the subtraction with borrowing procedure. They could not perform multiplication. All students except one showed interest in the activity and after 4 days of instruction, 2 and half hours per day, 4 of the 5 volunteers were able to add, subtract and multiply, although with frequent errors if not assisted.

PROTECTED ABACUS NUMERALS

In order to prevent forgery, which can be performed easily by adding extra circles to A bacus Numerals, vacant positions are occupied by a "+" symbol to form protected Abacus Numerals (Table 3). The protected Abacus Numerals can be employed for important documents such as bonds.

Table 3. Protected Abacus Numerals against forgery.

Arabic numerals	0	1	2	3	4	5	6	7	8	9
Abacus numerals										
Protected numerals										



INTERNATIONAL NUMERATION SYSTEM

Using Abacus Numerals, an International Numeration System can easily be developed a priori. As are shown in Table 4 (*c.f.* Figure 1). The numeration to show the order can be developed from Figure 1 by changing the consonants, while vowels remain unchanged. In order to show the order of magnitude from 1 to 9 or from –1 to –9, the consonants are changed from "t" and "m" to "s" and "t". For example, 6×10^9 is changed to "miu satan" and 8×10^{-9} as "ta satakk". To show the order of magnitude from 10 to 90 or from –10 to -90, the consonants are changed to "f"and "p" instead of "t" and "m".

And to show the order of magnitude from 100 to 900 or from –100 to –900, the consonants are changed to "sh" and "k" instead of "t" and "m".

For example, 9×10^{977} is changed to "tama shakapiupatiutan" and 9.76×10^{-778} to "tama nikk miuma miu kiukapiupasakk". The number 1998 is numerated as "ma tuan tama tun tama tan ta (in)". Thus, the International Numeration System appears to provide a means of facilitating international communication (Berlitz 1995, Asian Culture Center for UNESCO 1989). Learning calculations including zero and negative numbers can be represented concretely by Abacus Numerals, and thus become understandable by intuition (*c.f.* Figure 4 C and D).

Table 4. International Numeration System.

Counting	0: ii 1: ma (in) 2: mu (in) 3: mua (in) 4: mi (in) 5: mia (in) 6: miu (in) 7: miuma (in) 8: ta (in) 9: tama (in) 10: ma tan 11: ma tan ma (in) 15: ma tan mia (in) 20: mu tan 300: mua tun 6,000: miu tuan 70,000: miuma tin 0.1: ma takk 0.02: mu tukk 0.0008: ta tuakk 0.0003: mua tikk 3.14: mua in ma takk mi tukk 1997: ma tuan tama tun tama tan miuma (in)
Order	1st: mu ina 10th: ma tana 19th: ma tan tama ina
Fraction	$\frac{7}{5}$: miuma miaono $10\frac{1}{7}$: ma tan (ii) in ma miumaono
Engineering	3.14×10^{999} : mua nikk ma mi shakafapasatan 6.02×10^{85} : miu nikk ii mu fatian 6.0×10^{-77}: miu nikk ii piupatiutakk

ABACUS FOR ABACUS NUMERALS

A newly proposed Abacus System is shown in Figure 6. This abacus can be constructed simply using an iron plate and magnets, cloth and coins, or lines on the ground and small stones, ceramic pieces, shells or grains (Pullan 1968).

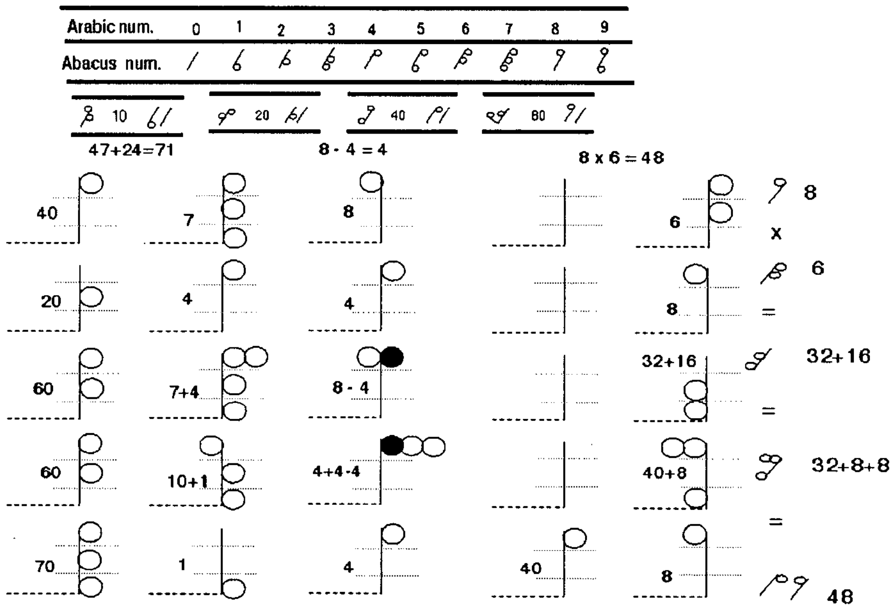


Figure 7. Proposed Abacus System: Two left columns: 47 + 24 = 71, middle column: 8 – 4 = 4, two right columns: 6 × 8 = 48.

DISCUSSIONS

The use of abacus numerals proposed in this work appears to be able to facilitate mathematics instruction. Since mathematics learning is basic to any science including higher mathematics and physics, the use of Abacus Numerals may facilitate gifted children in becoming better researchers early in their careers. Logical nature of calculation by Abacus Numerals seems to stimulate logical understanding of matters in early lives and hence enhance their creativity. Further, as the sciences and technologies develop, more time seems to be required only for learning basic knowledge before developing researches in sciences. Therefore, for a further advance of sciences in near future, innovation in instruction seems be required especially in mathematics, in reading and writing, and in international languages. It seems to be beneficial not only for gifted children but also for ordinary or even for mentally handicapped children. Introduction of Abacus Numerals is also desirable for society development to solve the problem of poverty and the prevention of contagious diseases since these are strongly related to ignorance due to lack of basic education (UNESCO 1990; Hayakawa 1995).

The use of base 8 systems instead of base 10 systems in common practice also appears to facilitate the application of the abacus numerals due to the simplified carrying and borrowing procedures.

ACKNOWLDGMENTS

The author is indebted to Dr. Nobuhiko Nohda and Dr. Jerry P. Becker for their valuable comments. The author is also thankful to members of Kuzuno Elementary School and Minami Ikuta Elementary School for their cooperation and support to teach pupils. He is indebted to Mr. Kyuugo Tsukada of Niihari-En Institute where Abacus Numerals was first learned.

REFERENCES

Asian Culture Center for UNESCO ED. (1989): *Talking with Asian Friends, 23 Asian Languages.* Tokyo: Kaigyusha Co. Ltd.

Berlitz Pub. (1995): *European Phrase Book.* Oxford: Berlitz Publishing Co. Ltd.

_____ (1995): *East European Phrase Book.* Oxford: Berlitz Publishing Co. Ltd.

Hayakawa, Y. (1995): *Abacus Numerals for Literacy against AIDS and Population Crisis.* Paper presented at the12th World Congress of Sexology, August 13, 1995, Yokohama, Japan

_____ (1995): *Universal Alphabet for Literacy against AIDS and Population Crisis,* 12th World Congress of Sexology, August 13th. Yokohama.

Ifrah, G. (1981): *Histoire Universelle des Chiffres, Seghers Ed.*, Chapter 28. Paris.

Papy, F. (1970): Teacher's Guide, *Papy minicomputer,* Mathematics Teaching, 50, 40. Macmillan Co.

Pullan, JM. (1968): *The history of the abacus.* London: Hutchinson & Co. Ltd.

UNESCO (1990): *World Declaration on Education for All and Frame Work for Action to Meet Basic Learning Needs.* Thailand: Jomtien.