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Developing Mathematics Creativity with Spreadsheets[1]

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The spreadsheet Microsoft Excel is the most widely used mathematical tool in today's workplace. Moreover, it is also an outstanding means for developing a surprisingly wide range of creative and innovative educational uses within such areas as mathematical modeling, visualization, and instruction. The spreadsheet's format provides us with a tool that closely parallels the way in which we naturally carry out problem solving, while the spreadsheet creation process itself illuminates the underlying mathematical concepts. In addition, the spreadsheet's visual layout allows us to introduce a broad variety of challenging and interesting topics, and to design creative demonstrations through eye-catching animated graphics. The material presented comes from actual classroom mathematics teaching experience in both industrially advanced and developing nations. A series of highly visual interactive illustrations from mathematics, the natural and social sciences, computing, engineering, and the arts provide a number of usable examples. The material discussed is applicable at diverse levels, ranging from schools and universities through adult education and in-service teacher development programs.

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ZDM Classification: U70
MSC2000 Classification: 97U70

INTRODUCTION

In many nations, mathematical educators are searching for new and effective ways to encourage the development of the mathematical creativity of students. Many of the new approaches incorporate the modeling of real-life examples and the use of computers. Part of the difficulty that arises in these endeavors is that traditionally the path to becoming effective in mathematics has involved a lengthy process of developing skills in algebra,

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calculus, and differential equations. Unfortunately, many students find these requirements to be both daunting and unattractive. This paper illustrates how we can use spreadsheets in developing mathematics creativity. While, we use the most popular spreadsheet, *Microsoft Excel*, other spreadsheets also work well.

In recent years, the spreadsheet has grown in popularity as a tool for doing and teaching mathematics. It is a natural tool for implementing algorithms, models, and applications. In addition, its powerful graphics enable us to create eye-catching animated visualizations of mathematical concepts. The naturalness of *Excel*'s operation, coupled with its availability on virtually every computer and that fact that it is the principal mathematical tool of the workplace, makes it an attractive teaching tool. We increasingly see mathematical uses of spreadsheets in education highlighted in mathematics texts, educational journals, and Web sites.

The author has used the approach discussed in this paper extensively in many settings. In the United States and Australia, courses for in-service mathematics teachers, numerical analysis classes for computer science, and as a demonstration tool in diverse mathematics courses used *Excel*. At the University of Vienna, a specially designed course in mathematical modeling for first year students of statistics and operations research uses this approach, while in Korea, the author uses *Excel* to create visualizations in calculus and linear algebra, and for creative student-designed projects in statistics.

Spreadsheets are also excellent tools for teaching in developing nations, where frequently other mathematics software is not readily available. *Excel* was used extensively in Papua New Guinea to introduce the fundamental elements of mathematics modeling and visualization to those majoring in all disciplines. Classes for the public also adopted this approach. These classes included both university graduates refreshing their skills in mathematics, and others for whom this would be their only university experience.

One key aspect in the use of spreadsheets is how a spreadsheet can enable mathematics teachers to establish a new approach to their profession. Rather than first needing to spend many years in learning algebra and calculus, often it is possible for students to design understandable models of mathematical concepts in a visual table format, and analyze those using ideas that are already familiar to them. In the process, we also can introduce students to algebraic ideas and notation as seems appropriate.

In our approach, rather than entering *Excel* expressions as formal equations, we use a "gesturing" system. Thus, in everyday explanations, if we want to explain how to compute the annual interest on a savings account, we might write the principal and the rate into two locations, and point to those values as we say, "we multiply the principal by the rate and list the answer as interest." Using *Excel* to "gesture" in the same way, we first enter the rate and the principal into two cells. We then use a mouse to click in the cell for the interest, and type = to indicate the start of a formula. We then click on the rate (Figure 1), type the multiplication sign, click on principal (Figure 2), and press the enter key to compute the interest (Figure 3). We never need to use algebraic notation or physical cell locations in this process.

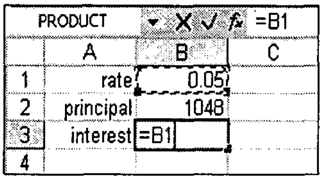

Figure 1. Gesturing I

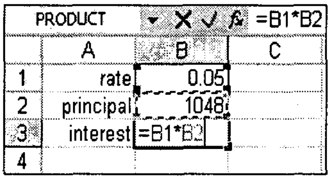

Figure 2. Gesturing II

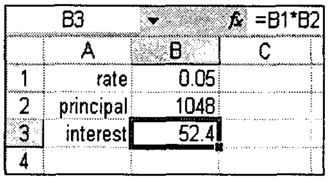

Figure 3. Output

In Figure 4 we use the arrow notation developed in Neuwirth & Arganbright (2004) to employ the spirit of our gesturing approach to describe the formulas. Dots in the display indicate a referenced cell, with an arrow showing where we use it in the formula. In the actual *Excel* construction process, the program will create the spreadsheet formulas (Figure 5) for us. After students have built a model in this straightforward manner, we then can use their experience in the creation process to discuss such algebraic expressions as $interest = rate \times principal$ or equations as $i = pr$.

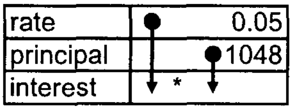

Figure 4. Arrow formulas

	A	B
1	rate	0.05
2	principal	1048
3	interest	=B1*B2

Figure 5. Excel formulas

Many people find that using the spreadsheet layout for organizing a model makes the related mathematics easier to comprehend. In fact, we can use our technique to create models for many advanced ideas, such as predator-prey models, that ordinarily require calculus and differential equations. Again, we need not ignore algebra and calculus ideas, as *Excel* records the formulas, and we can use the spreadsheet construction process to lead into discussions stemming from the equations. Example 1 below provides such an example.

In this paper, we present three illustrative examples. However, many more illustrations are readily available in the sources listed in the references, especially Neuwirth & Arganbright (2004)[2]. That book also contains a CD of fully functional *Excel* models designed in the spirit of this paper.

2 See also Neuwirth, Erich (2005): *Spreadsheets, Mathematics, Science, and Statistics Education*, http://sunsite.univie.ac.at/spreadsite/

EXAMPLES

Example 1. Newton's Law of Cooling.

In many mathematics courses, the topics of compound interest and geometric growth make excellent initial spreadsheet models, providing students with familiar and interesting settings. Since examples of this type are available elsewhere (Neuwirth & Arganbright 2004), here we will present a related example. Suppose that we remove a cake from an oven at certain temperature and place it in a room whose temperature is cooler. What happens to the temperature of the cake? Our experience says that it will gradually cool to room temperature.

We describe a model based on this observation to describe Newton's Law of Cooling. Our basic layout appears in Figure 6. A spreadsheet such as *Excel* employs an array of rows and columns. The usual way of describing a model is by listing formulas that use the cell locations. However, here we will use our gesturing approach, describing formulas through the innovative arrow notation of Neuwirth & Arganbright (2004). To begin, we enter descriptive headings as shown, together with the surrounding (or room) temperature and the initial temperature of the cake. We will count time in minutes down the first column, starting with time 0.

	A	B	C	D
1	temp sur	30	k	
2	temp init	200		
3		algorithm		
4	time	temp	temp diff	change
5	0			
6				
7				
8				

Figure 6. Newton's law of cooling layout

We compute the next minute by adding 1 to the cell above (Figure 7). This is a relative reference that will be adjusted accordingly as it is copied down the column to generate additional times. In this case, we show the referenced cell as a solid circle. We then copy this formula down the column. We indicate copying through shading. We copy the formula in a darker cell into the lighter cells. To continue developing the table, we next enter a simple formula to reproduce initial temperature as the temperature at time 0. Next, we compute difference between the temperature of the object and that of the room. However, when we copy this formula we will want the reference to the surrounding temperature to remain fixed as an absolute reference. We do this in *Excel* by pressing the F4 key when we enter a cell location. This puts $ signs on the reference indicating that it is an absolute reference (Figure 11). In our arrow diagrams, we show a pin drawn through a circle to indicate an absolute reference (Figure 7).

Next, we want to compute the change in temperature within the first minute. If they have completed a growth model earlier, students will have experienced a setting in which the change in a variable is proportional to a certain value. Here we use a similar idea. We assume that the amount of cooling in a unit time is proportional to the difference in the two temperatures. However, unlike a growth model, we do not know the value of the proportionality constant, *k*. Later we will see later how to use our model to determine it. For now, we estimate that during any minute the object loses 5% of the difference in temperatures. We will adjust this later. Since the temperature of the cake is decreasing, the change in temperature is the negative of the product of the proportionality constant and the temperature difference. When copying this formula, we need to ensure that the reference to the constant is absolute, and will not change when we copy (Figure 8).

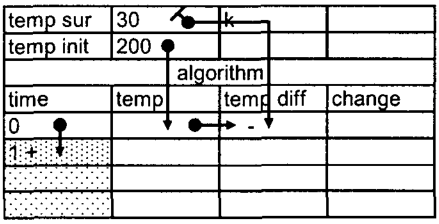

Figure 7. Newton cooling formulas I

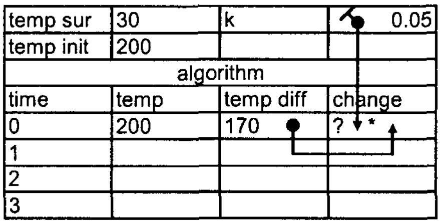

Figure 8. Newton cooling formulas II

We next compute the new temperature by adding the change in temperature (which of course is negative) to the previous temperature (Figure 9). Since we build the rest of our table in the same way, we can just copy the most recent entry in each column down that column.

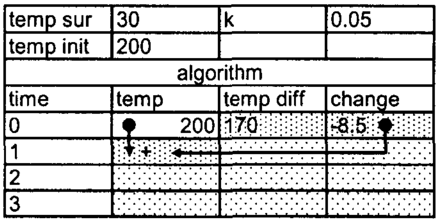

Figure 9. Newton cooling formulas III

We next show the resulting output display (Figure 10) and the underlying *Excel* formulas (Figure 11). Notice that relative references are adjusted in copying, while absolute references (shown by a pin in the arrow diagrams, and by $ in *Excel*) remain fixed.

	A	B	C	D
1	temp sur	30	k	0.05
2	temp init	200		
3	algorithm			
4	time	temp	temp diff	change
5	0	200	170	-8.5
6	1	191.5	161.5	-8.075
7	2	183.425	153.425	-7.67125
8	3	175.7538	145.7538	-7.287688

Figure 10. Newton's cooling model (output)

	A	B	C	D
1	temp sur	30	k	0.05
2	temp init	200		
3	algorithm			
4	time	temp	temp diff	change
5	0	200	=B5-B1	=-D1*C5
6	=1+A5	=B5+D5	=B6-B1	=-D1*C6
7	=1+A6	=B6+D6	=B7-B1	=-D1*C7
8	=1+A7	=B7+D7	=B8-B1	=-D1*C8

Figure 11. Newton's cooling model (formulas)

We have used no algebra or calculus in developing this model. This represents an important and different approach for doing mathematics. It allows us to study a surprisingly diverse range of models in a very natural manner. Moreover, the spreadsheet creation process provides us with a natural way to introduce a symbolic representation, allowing students to obtain a clearer picture of what is happening before the introduction of a symbolic representation.

Our model uses a difference equation approach. We will see that it is very close to the corresponding differential equation model. Figure 12 shows how we can use our model to derive a differential equation. We observe that the change cell shows us how to compute the amount of change in the temperature in one minute.

temp sur, y_s	30	k	0.05	
temp init, y_0	200			
algorithm				
time, t	temp, y	temp diff	change, dy	
0	y_0	y뻴$_s$	뻢(y뻴$_s$)	← dy/dt
1	y+dy			
2				
3				

Figure 12. Motivating a differential equation

This is essentially the rate of change in temperature in a unit time. Thus, the rate of temperature change is $-k(y-y_s)$, where k is the proportionality constant, y_s is the surrounding temperature, and y is the current temperature. Hence, the differential equation is $dy/dt = -k(y - y_s)$.

Now, we return to our model where we need to determine the value of k. One way is to do this is to measure the temperature of the cake at another time. Below we assume that we have found that after 10 minutes the temperature is 100 degrees. To create a graph (or chart) in *Excel*, we first use the mouse to select Columns A and B, and then click on the Chart Wizard button. From the options provided, we choose xy-graph type.

This type of graph plots points by their (x, y) coordinates (just as we do in mathematics), and connects successive points with straight-line segments to produce a smooth appearing curve. We can choose to show point markers, lines, or both. We will choose to plot both, and then later remove whatever we do not need. In addition, we want to plot the initial points, so in Columns E and F (Figure 13) we enter simple formulas to reproduce the values from cells above. We then use the mouse to highlight the resulting block, hold down on left mouse button, drag the block into the chart, and release the mouse button. In the resulting dialog box, we choose the new series option, with x values in the first column.

	A	B	C	D	E	F
1	temp sur	30	k	0.05	time 1	10
2	temp init	200			temp 1	100
3	algorithm				data points	
4	time	temp	temp diff	change	time	temp
5	0	200.00	170.00	-8.50	0	200
6	1	191.50	161.50	-8.08	10	100
7	2	183.43	153.43	-7.67		
8	3	175.75	145.75	-7.29		
9	4	168.47	138.47	-6.92		

Figure 13. Newton's cooling model (augmented)

Figure 14 shows the resulting graph, which incorporates some additional formatting. Here we notice that curve does not pass through the point for the second temperature. Thus, we need to adjust the value of k in order to produce curve in Figure 15.

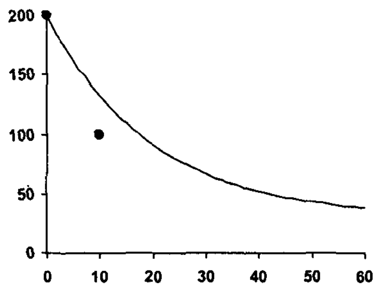

Figure 14. Graph for $k = 5.0\%$

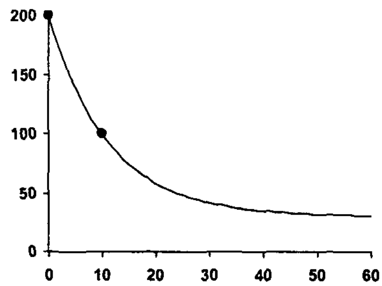

Figure 15. Graph for adjusted k

There are several ways to do this. Since a spreadsheet updates all computations immediately upon a change in any cell's value, one good technique to adjust the value of k in trial-and-error experimentation. Although we shall not do so here, we can also use Excel's built-in Goal Seek or Solver commands to adjust the value of k to force the cell that contains the temperature at time 10 to be 100 degrees (Neuwirth & Arganbright 2004).

However, another creative and visual way to accomplish our goal to link the value of k to a scroll bar. We can create a scroll bar by choosing the command View, Toolbars, Control Tool Box (with a Macintosh we must use the Forms toolbar). We want to link the scroll bar to the cell for k. However, in Excel a scroll bar can only take on non-negative integer values. Thus, we enter an auxiliary value; say 849, into Cell D2, and a formula in Cell D1 to divide this number by 10000, obtaining 0.0849.

On the resulting toolbar we click on the upper left button to enter the design mode, and then click on the scroll bar button. We then use the mouse to drag out a scroll bar. We right click in the scroll bar, and choose the option Properties. We then choose Cell D2 as the linked cell and set upper limits (here 0 and 2000).

We then toggle the design mode button to activate the scroll bar. Now as we move the slider of the scroll bar, the value of Cell D2 varies from 0 to 2000 in steps of size 1, causing the value of k in Cell D1 to vary from 0.0000 to 0.2000 in steps of size 0.0001. As we move the slider, the curve changes in a continuous fashion to adjust to the changing value of k. Our screen display appears in Figure 16.

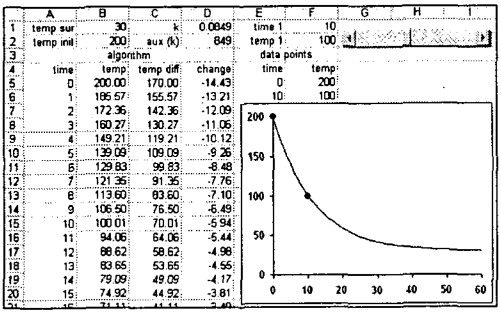

Figure 16. Screen display: Newton cooling model with scroll bar

Finally, if students know how to solve a variables separable differential equation, we can find that the solution of the differential equation for this model is $y = y_s + (y_0 - y_s)e^{-kt}$, where $k = -\ln((y_1 - y_s)/(y_0 - y_s))/t_1$. We use this to generate additional columns (Fig. 17) and find the difference between original difference equation and its estimated value of k, and the differential equation and its corresponding k. We can see that difference is quite small. Thus, our original difference equation model is quite good.

	G	H	I
1	temp 1a	difference	
2	100	2.55E-06	
3		k	0.08873
4	time	IVP temp	error
5	0	200	0
6	1	185.5657	5.67E-07
7	2	172.357	1.04E-06
8	3	160.2698	1.42E-06
9	4	149.2089	1.74E-06
10	5	139.0871	1.99E-06
11	6	129.8248	2.18E-06
12	7	121.3489	2.33E-06
13	8	113.5927	2.44E-06
14	9	106.495	2.51E-06

Figure 17. Differential equation solution and error of original model

Using the differential equation solution to create a graph allows us to examine an additional range of instructive aspects, and provides both students and teachers with a great opportunity to be creative. The modified model used to create the graphs in Figure 18 incorporates several sliders for the initial and surrounding temperatures as well as for the second time and the temperature at that point. We may pursue many other ideas as well. Figure 19 shows a graph that indicates the times for successive halving of the temperature difference. We also notice that the temperature difference decreases by half repeatedly over constant time intervals.

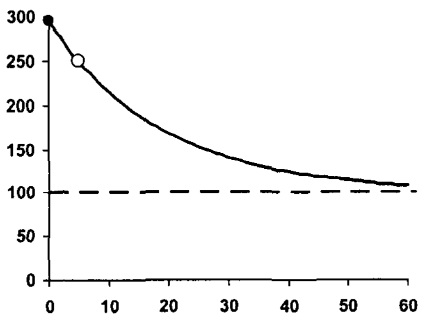

Figure 18. Additional graph features I

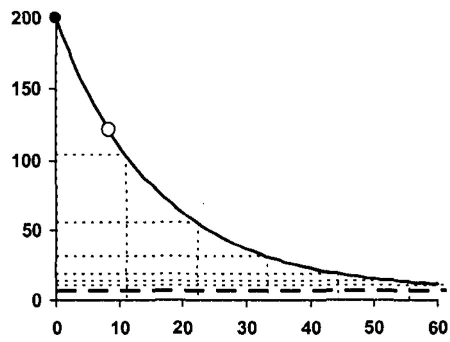

Figure 19. Additional graph features II

Students can use spreadsheets to design creative difference equation models for a diverse variety of topics that appear in differential equations books. Examples include models for mixtures and lake pollution, the spread of epidemics, and many others (see Neuwirth & Arganbright 2004).

Example 2. Polar Curves and Geometric Constructions.

In calculus, we study the creation of polar curves and the graphs of parametric equations. Although *Excel* does not provide these types directly, it is easy and natural to implement them as *xy*-graphs. First, we generate a degree count, $0 \le n \le 360$, down the first column, and use the built-in radians function in the second column (Figure 20). We then enter a formula for a polar curve in the third column. Here we use $r = a + b\cos ct$, where we enter the values of *a*, *b*, and *c* as parameters. Using $a = 1.0$, $b = 0.5$, and $c = 3$, we will generate the three-loop curve in the Figure 24. We next generate the (x, y) coordinates (Figure 21) as $x = r\cos t$, $y = r\sin t$ in the next two columns. We then use *Excel*'s chart wizard to create the curve as an *xy*-chart.

a,b,c	1	0.5	3	
			base curve	
n	t	r	x	y
0	RADIANS( )	+ *COS(*)		
1				
2				

Figure 20. Polar curve formulas I

a,b,c	1	0.5	3	
			base curve	
n	t	r	x	y
0	0	1.5	*COS( )	*SIN( )
1				
2				

Figure 21. Polar curve formulas II.

The construction of curves provides us with an excellent setting to encourage geometric creativity. In this example, we illustrate the creation of a special case of the inverse of a curve. For each point on a given curve, its inverse relative to the unit circle centered at the origin is the point that has same polar angle, but whose distance, *s*, is given by $s = 1/r$. To show the construction of the curve of the inverse, we use the next two columns to generate the unit circle as $x = \cos(t)$, $y = \sin(t)$, highlight those two columns, and drag them into the graph as a new series. We then repeat this process with the next two columns using the formulas $x_1 = \cos(t)/s$, $y_1 = \sin(t)/s$ to create points (x_1, y_1) on the inverse curve. We note that we also can use the equations $x_1 = x/s^2$ and $y_1 = y/s^2$ to generate the same curve, and that points inside the circle map to points outside the circle, and conversely.

a,b,c	1	0.5	3				N	3			
			base curve		circle		inverse				
n	t	r	x	y	xx	yy	xxx	yyy		XXX	YYY
0	0.000	1.500	1.500	0.000	1.000	0.000	0.667	0.000		IF(<= , ,)	
1	0.017	1.499	1.499	0.026	1.000	0.017	0.667	0.012			

Figure 22. Formula to trace out the inverse curve

We can be even more creative in showing how to trace out the constructed curve in a point-by-point manner. To do this we provide a counter cell that we label *N*. We then create two additional columns of *x* and *y* values using *Excel*'s If-Then-Else function: if a point's counter, n, satisfies $n \leq N$, then the formula reproduces the coordinates of the inverse curve, otherwise it just copies the last cell above (Figure 22). We then drag the x and y column block into the chart and delete the original inverse curve.

We then link a scroll bar to N. Then, as we increase N with the scroll bar, we see additional points of the curve traced out in a continuous fashion. We provide the numerical output when $N = 3$ in Figure 23.

a,b,c	1	0.5	3				N	3			
			base curve		circle		inverse				
n	t	r	x	y	xx	yy	xxx	yyy		XXX	YYY
0	0.000	1.500	1.500	0.000	1.000	0.000	0.667	0.000		0.667	0.000
1	0.017	1.499	1.499	0.026	1.000	0.017	0.667	0.012		0.667	0.012
2	0.035	1.497	1.496	0.052	0.999	0.035	0.667	0.023		0.667	0.023
3	0.052	1.494	1.492	0.078	0.999	0.052	0.668	0.035		0.668	0.035
4	0.070	1.489	1.485	0.104	0.998	0.070	0.670	0.047		0.668	0.035

Figure 23. Inverse curve model (output)

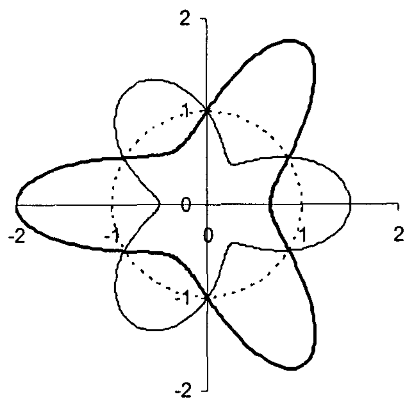

Figure 24. Curve, unit circle, and inverse curve

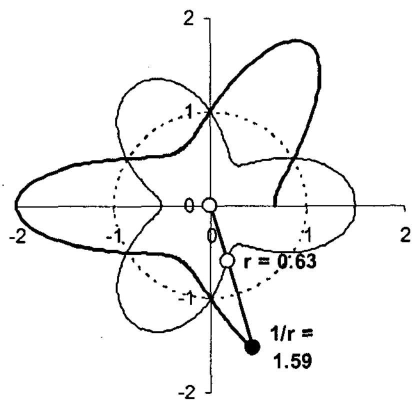

Figure 25. Tracing the inverse curve via a scroll bar

Figure 24 shows all three curves, while Figure 25 illustrates a step in tracing out the inverse curve. Figure 25 also incorporates the radius vector, to better illustrate the construction.

We can pursue many additional creative graphic projects based on our example. For example, we can investigate a variety of curves (try an asteroid, where $x = a\cos^3 t$, $y = a\sin^3 t$ with $a = 1.5$); we can generalize the construction by letting the center and radius of circle of inversion be parameters of our model; and we can illustrate other eye-catching constructions such as pedals of curves, conchoids, and evolutes (Arganbright 1993b). Additionally, we also can use the If-Then-Else construction to trace out the original parametric or polar curve Neuwirth & Arganbright (2004). Cycloids and related curves are especially interesting (Hill & Roberts 2001)[3]. Another animated tracing approach using matrices to trace out curves will appear in Arganbright (n. d.). The development of conic sections via classical definitions (*i.e.* a parabola as the set of all points that are equal-distant from a fixed point and a given line) appears in Hill & Roberts (2001). Each of these projects will expose students to new ideas at level that they can handle, enabling them to design clever animations while discovering new ideas. Geometry topics have proven especially useful in classes for in-service and pre-service mathematics teachers, giving them new ways to encounter geometric concepts and to provide them with examples that they can use in teaching.

Example 3. Drug Test Statistics.

The areas of probability and statistics provide us with a wealth of animations for spreadsheet models. These are especially attractive if they involve data from a student's life, country, or sports interest. For example, in the United States, it is common to perform drug tests on various groups of students, such as sports teams or even the entire student body. As our final example, we pursue the analysis of drug tests as they produce "false positives." In Figure 26 we show the layout of our model, but we do not list the underlying formulas. The formulas are quite elementary, and we encourage our readers to supply them.

Suppose that we test everyone in a school of 1000 students with a test for drug usage that we know to be 95% accurate. Further, suppose that only 6% of the students are actual drug users. Using the concept of expected value, we would expect the test to detect 95% of the 60 (or 57) of the users as positives. However, we would also expect the test to report 5% of the 940 (or 47) of the non-users as users as positives. We refer to the latter as "false positives." Thus, we would expect to have a surprisingly high percentage,

[3] The examples probably are not in their original paper, but are on the web site http://mathdemos.gcsu.edu/

45.2%, of those testing as positive for drug usage by the test to be false positives. Thus, it will be important to explain this to those who plan to use the results of the tests.

We want our model to incorporate a scroll bar to make it easy to examine the expected outcomes for various levels of accuracy. To do this, we employ another powerful *Excel* feature, the little-known, but very powerful, Data Table command.

	A	B	C	D	E	F	G	H	I	J
1					Number	Test Neg	Test Pos		Proportion	False
2	Population Size	1000		Users	60	3	57		of Users	Positive
3	Proportion Users	0.06		Non-Users	940	893	47			45.2%
4				Total	1000	896	104		0.000	100.0%
5	Accuracy Pct	0.95							0.001	98.1%
6	Auxilliary (Acc)	950		Proportion False Positive			45.2%		0.002	96.3%

Figure 26. Drug testing model (output)

In Figure 27 we use two columns (I, J) to create our data table. Leaving the top row blank initially, we use the first column to generate various values for the proportion of drug users, and employ the second column to hold the corresponding expected percentages of false positives. Because there is a great variation in the results when the proportion of users is very small, in the first column we start with 0 and use steps of size 0.001 for the next 10 values, and steps of size 0.01 thereafter. Next, we enter a simple formula (=G6) as the generic formula in the top of the second column. This provides the percentage of false positives for the current proportion of users that we have entered in Cell B3.

	G	H	I	J
1	Test Pos		Proportion	False
2	57		of Users	Positive
3	47			
4	104		0.000	
5			0.001	
6	45.2%		0.002	

Figure 27. Data table formula

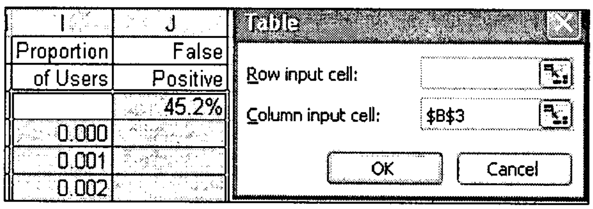

Figure 28. Data table dialog box

We next issue the command Data, Table. In the ensuing dialog box (Figure 28) we click the cell for the proportion of users (B3) as the Column Input Cell. After we click on the OK button, *Excel* successively substitutes the values in Column I into Cell B3, and stores the corresponding percentage of false positives from Cell G6 in the corresponding cell in Column J. From the resulting table, we create a *xy*-chart to see that for the cases in which there are a small number of drug users, the test yields many false positives. If we link a scroll bar to the cell containing the test accuracy, we can see how the results vary as the level of accuracy increases and decreases. Figures 29 and 30 provide two illustrations.

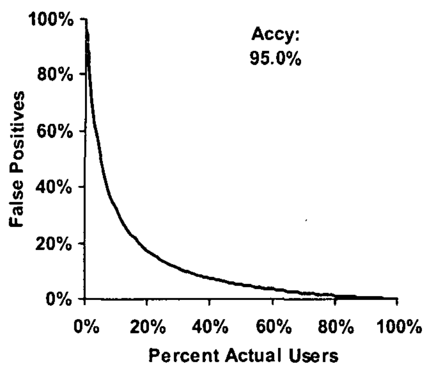

Figure 29. False positive graph I

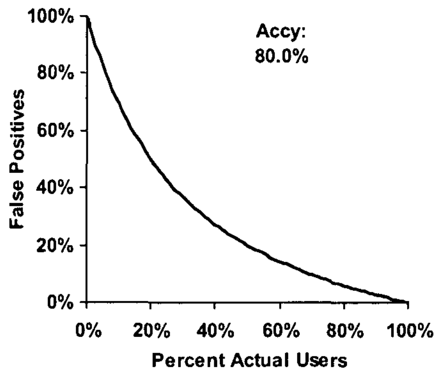

Figure 30. False positive graph II

CONCLUSIONS

As a good additional project, we can create a model that assumes that those testing positive are tested again using the same test, and determine the percentage of original population that we can expect to yield return two false positive tests. We can design other creative spreadsheet models from such statistical topics as probability distributions, Bayes' Theorem, confidence intervals, and hypothesis tests. The spreadsheet is a particularly effective for examining counter-intuitive situations, such as the classical birthday problem.

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