# 제한된 피드백 정보를 사용하는 공간 다중화를 위한 부 공간 방식 기반 Precoding 기법

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# Subspace Method Based Precoding for Spatial Multiplexing with Limited Feedback

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요 약

본 논문에서는 제한된 피드백 정보를 사용하는 공간 다중화 기법에 사용될 수 있는 부 공간 방식 기반 precoding 기법을 제안한다. 제안된 precoding 기법은 수신기에서 다수의 공통 기저 집합(common basis sets)으로부터 용량을 최대로 하는 기저들을 선택하고, 선택된 기저들은 feedback 정보를 통해 송신기에 전달되고, 송신기에서는 선택된 기저들로 이루어진 precoding 행렬을 구성한다. 선택된 기저들은 해당 기저들이 속한 기저 집합의 index에 대한 feedback 정보와 선택된 기저 집합에서 각 기저들의 선택 여부에 대한 feedback 정보를 통해 송신기로 feedback된다. 선택된 기저 집합 index는 송수신기가 약속한 공통 기저 집합 중에서 해당 MIMO 채널의 부 공간의 좌표(coordinate)에 가장 적합한 좌표를 나타내고, 해당 좌표에서 상당한 양의 에너지를 포함하는 기저들을 선택함으로써 적은 feedback 정보량으로 MIMO 채널을 묘사한다. 시뮬레이션을 통해 제안된 부 공간 방식 기반 precoding 기법이 적은 feedback 정보량을 사용하면서 폐루프 MIMO 용량 (closed-loop MIMO capacity)에 근접하는 용량을 제공함을 보인다.

**Key Words** : MIMO, precoding, subspace method, limited feedback, spatial multiplexing

ABSTRACT

In this paper, for spatial multiplexing with limited feedback, we propose subspace method based precoding in which the active bases are selected at the receiver from a finite number of basis sets known at both receiving and transmitting ends, conveyed to the transmitter using limited feedback, and assembled into a precoding matrix at the transmitter. The selected bases are conveyed to the transmitter using feedback information on both the index of the selected basis set, which defines the most appropriate set of coordinates for describing a multiple-input multiple-output (MIMO) channel, and the principal bases maximizing the capacity in the selected basis set. We show that the proposed subspace method based precoding provides a capacity similar to that of the closed-loop MIMO even with limited feedback.

## Ⅰ. Introduction

Recently, there has been increasing interest in precoding as a method to improve the capacity of spatial multiplexing. Due to practical limitations on the uplink feedback load, current research has

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focused on precoding schemes that do not require transmit channel knowledge[1],[2],[3],[6]. The precoder is selected at the receiver and conveyed to the transmitter using a limited number of bits. The limited feedback represents an index within a finite set, or codebook, of precoding matrices. The receiver selects one of these codebook matrices using a precoder selection criterion. Precoder selection at the receiver can limit the amount of feedback, since there is typically a small number of precoding matrices.

In this paper, we propose a subspace method based precoding scheme with limited feedback, which provides a capacity level close to that of full transmit channel state information (CSI). The active bases used for transmission are selected from a finite number of basis sets known at both receiving and transmitting ends, each of which consists of $n_T$ orthonormal bases and represents a set of coordinates. The selected bases are conveyed to the transmitter using feedback information both on the index of a basis set, indicating the most appropriate set of coordinates for describing a multiple-input multiple-output (MIMO) channel, and on the principal bases having the significant amounts of energy in the selected basis set. The selected bases are then assembled into a precoding matrix at the transmitter. A precoder codebook design method [1]-[3] considerably reduces required feedback bits while providing a MIMO capacity close to that of full CSI. However, these methods will face further increases of required feedback bits to vary the number of substreams for multi-mode precoding[1] and will require the transmitter spatial correlation matrix to adjust the precoder codebook, designed for a spatially uncorrelated Rayleigh MIMO channel by using the Grassmannian subspace packing in [1], to a current spatial correlation condition[2]. On the other hand, the proposed subspace method based precoding scheme allows one to adjust a precoding matrix based on limited feedback information on the principal bases approximating the subspace of a MIMO channel. According to channel correlation conditions and the average received signal to noise ratio (SNR), the number of substreams and a precoding matrix can be adjusted without additional channel knowledge or feedback, which leads to a capacity close to that of the closed-loop MIMO, even with limited feedback.

## Ⅱ. System Model

We consider a spatial multiplexing system, with $n_T$ transmitting and $n_R$ receiving antennas. A precoding matrix consists of the active bases selected from a basis set among the $N$ basis sets. Let us define an active basis subset $A_{n*}$ in which $K_{A_{n*}}$ bases are selected from a basis set $\varepsilon_{n*}$ among $\{\varepsilon_n\}_{n=1,\cdots,N}$. The basis sets $\{\varepsilon_n\}_{n=1,\cdots,N}$ are designed offline and are known at both the transmitter and receiver.

The high-speed data stream is demultiplexed into several $K_{A_{n*}}$ independent substreams. The number of simultaneous substreams is adjusted up to $\min(n_T, n_R)$, according to the fading environments. The total transmit power $P_T$ is uniformly distributed over $K_{A_{n*}}$ independent substreams. The symbols are precoded by the corresponding active bases and are sent from transmitting antennas.

We further assume that the channel is flat fading and quasi-static. The signal at the receiver end is given by

$$y=\sqrt{\frac{P_T}{K_{A_{n*}}}}H\ E_{A_{n*}}x+n, \quad (1)$$

where $y$ is an $n_R\times1$ received signal vector and $n$ is an $n_R\times1$ additive white complex Gaussian noise vector with variance $\sigma^2$. The transmitted signal is a $K_{A_{n*}}\times1$ vector denoted by $x$ in which the $i$th element represents the symbol precoded by the $i$th basis in $A_{n*}$, $e_{A_{n*},i}$ and multiplied by $\sqrt{P_T/K_{A_{n*}}}$. A precoding matrix $E_{A_{n*}}$ is given by

$$E_{A_{n*}}=[\ e_{A_{n*},1}\quad e_{A_{n*},2}\quad \cdots\quad e_{A_{n*},K_{A_{n*}}}], \quad (2)$$

where the precoding matrix $E_{A_{n*}}$ consists of the selected bases in $A_{n*}$. The precoding matrix is chosen at the receiver and is sent back by using the feedback information on both the index $n*$ of the selected basis set and the active basis subset of the selected basis set, which can be implemented using $\lceil \log_2 N \rceil + n_T$ bits on a feedback channel. Here, $\lceil x \rceil$ is the smallest integer that is larger than or equal to $x$.

The basis sets $\{\varepsilon_n\}_{n=1,\cdots,N}$, which are designed offline and stored by both the transmitter and receiver, are used for all pairs of combinations between transmitter and receivers with various channel eigenstructures. Thus, we design $N$ basis sets, which each consists of $n_T$ orthonormal bases and define $N$ $n_T$-dimensional subspaces with large minimum distances. Finding subspaces with large minimum distances is known as the Grassmannian subspace packing problem[1]. In general, optimal packings are difficult to find. Therefore, in this study, we design $N$ basis sets using the systematic noncoherent constellation design in [4]. The $n_T$ orthonormal bases belonging to the initial basis set $\epsilon_1$, $\{e_{1,i}\}_{i=1,\cdots,n_T}$ are given by

$$e_{1,i}=\left[1 \quad e^{j\frac{2\pi}{n_T}(i-1)} \quad \cdots \quad e^{j(n_T-1)\frac{2\pi}{n_T}(i-1)}\right]^T. \quad (3)$$

$N$ different $n_T$-dimensional subspaces with large minimum distances are constructed by rotating the subspace defined by the initial basis set $_1$ by $\theta_n = 2\pi(n-1)/Nn_T$ through $n_T$-dimensional complex space [4]. The $n_T$ orthonormal bases belonging to $\varepsilon_n$, $\{e_{n,i}\}_{i=1,\cdots,n_T}$ are given by

$$e_{n,i} = \Phi_n e_{1,i}, \quad (4)$$

where

$$\Phi_n = \mathrm{diag}\{1,\ e^{j\theta_n},\ e^{j2\theta_n},\ \cdots,\ e^{j(n_T-1)\theta_n}\}. \quad (5)$$

The resultant $N \times n_T$ total bases are both maximally spaced and equiangular in $n_T$-dimensional complex space.

## Ⅲ. Subspace Based Precoder Optimization

The criterion used for joint optimization of a basis set $n*$ and an active basis subset $A_{n*}$ is expressed as

$$n*, A_{n*} = \arg\max_{A_n^j \text{ for } all\ n \in \{1,\cdots,N\}} C_{A_n^j}, \quad (6)$$

where $\{A_n^j\}_{j=1,\cdots,2^{n_T}-1}$ denotes a possible basis subset obtained by selecting $K_{A_n^j}$ bases among $n_T$ bases in $\varepsilon_n$ and $C_{A_n^j}$ denotes the capacity supported by the active bases in $A_n^j$. A closed-form solution for the criterion (6) cannot be found analytically, requiring a full search of all possible subsets of all basis sets $\{\varepsilon_n\}_{n=1,\cdots,N}$. The total number of all possible subsets is $N(2^{n_T}-1)$.

The capacity supported by the active transmit bases in $A_{n*}$ is given by

$$\begin{aligned} C_{A_{n*}} &= \log_2 \det\left( I_{n_R} + \frac{\rho}{K_{A_{n*}}} \tilde{H}_{A_{n*}} \tilde{H}_{A_{n*}}^H \right) \\ &= \sum_{i=1}^{K_{A_{n*}}} \log_2\left(1+\frac{\rho}{K_{A_{n*}}} \tilde{\lambda}_{A_{n*},i}\right), \end{aligned} \quad (7)$$

where $I_{n_R}$ is the $n_R \times n_R$ identity matrix and $\rho$ denotes the average signal-to-noise ratio(SNR) per receiving antenna. An unitary transformed channel matrix of $H$ by the precoding matrix $E_{A_{n*}}$ is denoted by $\tilde{H}_{A_{n*}}$, and the eigenvalues of $\tilde{H}_{A_{n*}} \tilde{H}^{H_{A_{n*}}}$ are denoted by $\{\tilde{\lambda}_{A_{n*},i}\}_{i=1,\cdots,K_{A_{n*}}}$.

At a low SNR, which requires basis selection for the efficient use of scare available power, the capacity can be expressed by Taylor series approximations, as follows:

$$\begin{aligned} C_{A_{n*}} &\approx \frac{\rho}{K_{A_{n*}}} \sum_{i=1}^{K_{A_{n*}}} \tilde{\lambda}_{A_{n*},i} \\ &= \frac{\rho}{K_{A_{n*}}} \mathrm{tr}\left(E_{A_{n*}}^H H^H H E_{A_{n*}}\right) \\ &= \frac{\rho}{K_{A_{n*}}} \sum_{i=1}^{K_{A_{n*}}} g_{A_{n*},i} \\ &= \frac{\rho}{K_{A_{n*}}} \sum_{i=1}^{K_{A_{n*}}} \sum_{l=1}^{\min(n_T,n_R)} \lambda_l \left| v_l^H e_{A_{n*},i} \right|^2, \end{aligned} \quad (8)$$

where $g_{A_{n*},i} = e^{H}_{A_{n*},i} H^H H e_{A_{n*},i}$ and $\lambda_l$ is the eigen-value of $H^H H$ associated with the eigenvector $v_l$. We assume that $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_{\min(n_T, n_R)}$ and $\left| v^{H_l} e_{A_{n*},l} \right|^2 \geq \left| v^{H_l} e_{A_{n*},i} \right|^2$ for $l \neq i$. Equation (8) shows that the approximated capacity is equal to the sum of the SNRs of the $K_{A_{n*}}$ substreams. Each SNR largely depends on the energy of the projection of $H$ in the direction of $e_{A_{n*},i}$, $g_{A_{n*},i}$, which physically represents the gain resulting from transmit beamforming by the basis $e_{A_{n*},i}$. Furthermore, the optimal active bases maximizing the capacity become $\{v_l\}_{l=1,\cdots,K_{A_v}}$ associated with the $K_{A_v}$ largest eigenvalues of $H^H H$, if CSI on $H$ is available at the transmitter, which is identical with the observations in [1]-[3]. Here, $K_{A_v}$ denotes the optimal number of active eigenvectors maximizing the capacity. If a set of coordinates defined by $\varepsilon_{n*}$ is sufficiently similar to that defined by the eigenvectors of a MIMO channel, then $K_{A_v} = K_{A_{n*}}$.

Obviously, each set of coordinates defined by a basis set $\{\varepsilon_n\}_{n=1,\cdots,N}$, provides a different active basis subset, maximizing the capacity on each set of coordinates, $\{A_n\}_{n=1,\cdots,N}$, which leads to a different capacity result. Therefore, the capacity achieved by subspace based precoding can be maximized by jointly (1) selecting the most similar set of coordinates defined by $\varepsilon_{n*}$ with a "natural" set of coordinates defined by the eigenvectors of a MIMO channel and (2) selecting the active basis subset $A_{n*}$, having the significant amounts of energy under the constraint that the total energy is given by $\sum_{i=1}^{n_T} g_{A_{n*},i} = \sum_{l=1}^{\min(n_T, n_R)} \lambda_l$. Particularly, in correlated fading environments, which can be modeled by a superposition of scattering clusters with limited angular spreads, those few bases that transmit beams focused in the direction of the scattering clusters convey the largest amounts of energy to the receiver. In such cases, the capacity can be maximized by concentrating most of the total power in some contributory bases, i.e., by selecting some contributory bases.

As a result, the channel matrix $H$ of dimension $n_T$ is projected onto a $K_{A_{n*}}$-dimensional subspace spanned by $\{e_{A_{n*},i}\}_{i=1,\cdots,K_{A_{n*}}}$, which results in reduction in the dimension of $H$ when $K_{A_{n*}} < n_T$. Such a reduction yields a capacity gain via efficient use of available power. For example, in the case of $g_{A_{n*},1} > g_{A_{n*},2} > \cdots > g_{A_{n*},n_T}$ at a low SNR, transmit beamforming by $e_{A_{n*},1}$ achieves the optimal capacity because $\rho g_{A_{n*},1} > \frac{\rho}{m} \sum_{i=1}^{m} g_{A_{n*},i}$ for all $m > 1$ in (8).

## Ⅳ. Simulation Results and Discussions

We consider spatial multiplexing with a uniform linear antenna array at the transmitter and receiver with spacings of $d_T = 4\lambda$ and $d_R = 0.5\lambda$, respectively. For each channel realization, a spatially correlated MIMO channel is generated by $H = R^{1/2_r} H_w R^{1/2_t}$, where $H_w$ is an $n_R \times n_T$ matrix with uncorrelated complex Gaussian entries. $R_r$ and $R_t$ are the correlation matrices at the receiver end and the transmitter end, respectively, and are given by [5]. We assume a uniform angular spectrum at both the transmitter and receiver sides, with angular spreads of $\Delta_T$ and $\Delta_R$, respectively. We assume that the receiving antennas are loosely correlated by letting $\Delta_R = 60°$, while the correlation between transmitting antennas varies with the angular spread $\Delta_T$. We assume two scenarios $\Delta_T = 5°$ and $\Delta_T = 30°$, which correspond to highly correlated and loosely correlated channels, respectively. For each channel realization, the angle of departure at the transmitter is uniformly generated over a range from $-60°$ to $60°$, while the angle of arrival at the receiver is fixed at

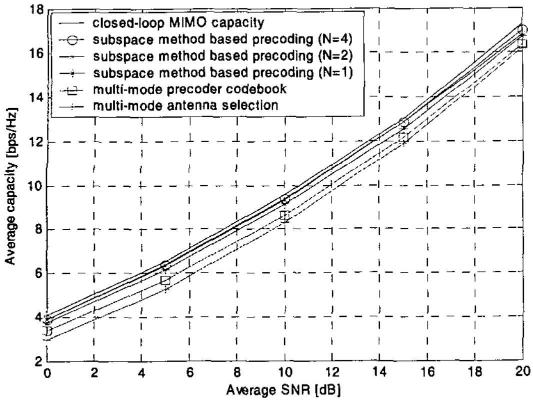


Fig. 1. Capacity comparison between precoding schemes in highly correlated channels when $n_T = n_R = 4$.

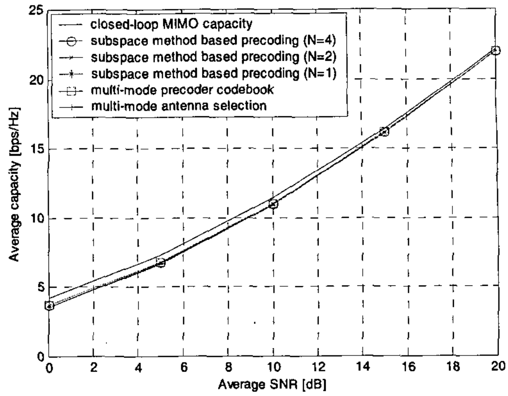


Fig. 2. Capacity comparison between precoding schemes in loosely correlated channels when $n_T = n_R = 4$.

$0°$.

Fig. 1 compares the average capacity between precoding schemes as a function of the average SNR per receiving antenna $\rho$ when $n_T = n_R = 4$ in highly correlated channels. For reference, the closed-loop MIMO capacity, which is achieved by transmitting via the eigenvectors and allocating power by water-filling solution, is presented. For comparison, the transmit antenna selection[6] and multi-mode precoding based on the precoder codebook design method[1] are also considered. The transmit antenna selection selects the optimal active antenna subset based on the capacity criterion and uses $n_T$ feedback bits each representing whether the corresponding antenna is used for transmission or not. The multi-mode precoding adjusts the number of substreams using precoder codebooks, finite sets of precoder matrices, for each of the supported substream configurations. We use the precoder codebooks in [7], in which 4 precoder codebooks $\{\Upsilon_i\}_{i=1,2,3,4}$, each consisting of 8 precoding matrices, i.e., $\Upsilon_i = \{F_{i,1}, F_{i,2}, \cdots, F_{i,8}\}$, are designed by the Grassmannian subspace packing criteria. The optimal precoding matrix maximizing the capacity is selected among 32 precoding matrices and fed back to the transmitter by using 5 feedback bits.

The results show that the proposed subspace method based precoding provides capacity close to that of the closed-loop MIMO when $N=2$ and $N=4$. A larger number of basis sets leads to a higher resolution in the space domain, which increases similarity of a set of coordinates defined by $\varepsilon_{n*}$ to that defined by the channel eigenvectors. The subspace method based precoding scheme using two basis sets is an efficient choice for achieving the capacity with least feedback bits because the subspace method based precoding shows minimal capacity improvement when $N \geq 2$. Even in the case of $N=1$, which requires no feedback for the basis set index, the subspace method based precoding shows a considerable capacity enhancement over transmit antenna selection even using the same 4 feedback bits. The capacity gain of the proposed subspace method based over transmit antenna selection is attributed to the coordinate transformation by basis set selection, which finds a set of coordinates for the most effective reduction in dimension of a MIMO channel.

When transmitting antennas are loosely correlated as shown in Fig. 2, the proposed subspace method based precoding schemes, each with $N$ basis sets, achieve the slightly higher capacity than transmit antenna selection and approach the capacity of closed-loop MIMO. With larger angular spreads at the transmitter side, the capacity enhancement of the subspace method based precoding over transmit antenna selection decreases. As the transmitter spatial correlation decreases, a preference of a MIMO channel in the directions of some eigenvectors disappears because the singular values of the MIMO channel become comparatively uniform. This decreases the additional gain of the subspace method based precoding over transmit antenna selection obtained by the coordinate transformation. Also, the results show capacity enhancement of the subspace method based precoding over the multi-mode precoder codebook scheme even using lower feedback bits in highly correlated channels. The capacity gap decreases as transmit correlation decreases. This indicates that the precoder codebook designed by the Grassmannian subspace packing for a spatially uncorrelated Rayleigh MIMO channel is no more optimal in correlated channels and requires a ''companding'' process using the transmitter spatial correlation matrix in [2]. On the other hand, the proposed subspace method based precoding scheme allows one to adjust a precoding matrix based on limited feedback information on the principal bases approximating the subspace of a MIMO channel.

## V. Conclusion

In this paper, we propose the subspace method based precoding for spatial multiplexing with limited feedback. Numerical results show that the proposed subspace method based precoding achieves the capacity close to that of the closed-loop MIMO even with limited feedback.

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