Progress in Superconductivity Vol.7 No.1 pp11-16 31 October 2005

INVITED PAPER

Josephson Junction Array for Voltage Metrology: Microwave Enhancement by Coupled Self-Generations in Series Array

K.-T. Kim*, M.-S. Kim, Y.-W. Chong

Korea Research Institute of Standards and Science, Daejeon, Korea

Received 18 August 2005

전압 측정표준용 조셉슨 접합 어레이: 직렬 어레이에서 상호 결합된 자체발진의 마이크로파 증진

김규태*, 김문석, 정연욱

Abstract

Coupling of non-linear oscillators have long been an interesting problem for physicists. The coupling phenomena have been frequently observed in Josephson junction series array, which have been used for Josephson voltage standard. Interestingly pronounced self-generation effect has been found during recent development of Josephson arrays for programmable Josephson voltage standard. But the coupling effect between the self-generations is not fully understood yet. We present harmonically approximated analytical solutions for coupled self-generations in the Josephson arrays, i.e., Superconductor-Insulator-Normal metal-Insulator-Superconductor (SINIS) array, externally shunted Superconductor-Insulator-Superconductor (es-SIS) array, Superconductor-Normal metal-Superconductor (SNS) array. We find that the coupling between the self-generated Josephson oscillations through microwave transmission line plays critical role in microwave property of the Josephson array.

Keywords : Coupled Josephson array, harmonic approximation, Self-generation, Programmable Josephson voltage standard

I. Introduction

For several decades, Josephson junction series array arranged on a microwave transmission line has been used for Josephson voltage standard (JVS) in many metrology laboratories. A few thousand junctions had to be integrated to obtain 1 V level, the usable level for practical metrology. The Josephson junction series array, now called conventional Josephson array, consists of Superconductor-Insulator-Superconductor (SIS) Josephson junctions [1~4]. The SIS junction has large hysteresis in current-voltage (IV) curve and very low damping in the dynamic behavior, because of large junction capacitance. The large hysteresis enabled the use of

*Corresponding author. Fax : +82 42 868 5018
e-mail : ktkim@kriss.re.kr

zero-crossing steps proposed by Levinsen [5]. Thanks to the employment of the Nb-Josephson junction technology developed by Gurvich [6], the Josephson arrays have become robust devices suitable for routine calibration. Now more than 40 SIS JVS are in use in metrology laboratories in the world. In spite of the success of the SIS JVS, it can be used only for DC, because it requires a process to select the desired step out of multiple steps equally available for a given dc bias current. Progress of Josephson technology in a few leading laboratories recently enabled to fabricate new Josephson arrays suitable for programmable dc voltage standards [7~12] or even ac voltage standards [13,14] based on calculable synthesis of arbitrary waveform. The new array consists of SINIS or SNS junctions showing reversible IV curve because of higher damping. The higher damping, caused by smaller junction capacitance enables one to control the junctions between the absolute zero voltage and the Shapiro step voltage in a fast and definite way. Since a few hundreds or thousands of junctions are employed in a single microwave branch, there are coupling phenomena between the junctions via the microstripline. When the junction impedance is closely matched with the transmission line's characteristic impedance, interesting but complicated microwave phenomena can happen in the Josephson arrays. For example, SINIS junctions array, in which the junction parameters are tuned near critical damping at about 70 GHz drive frequency, is expected to have rather large attenuation of about 0.05 dB per junction [15] because of the closer impedance matching. Thus, at the end of a long Josephson array with thousands of junctions, no microwave is transmitted and no Shapiro steps are expected. However, in experiment sizable Shapiro step with zero differential resistance have been observed even in 4,096 junction array in a single microwave line [16]. This astonishing phenomenon, possibly stimulating a hope for coherent microwave amplification by strong coupling, is attributed to self-generation effect from Josephson junctions, although no theoretically predictable explanation have been given yet. In recent research, the coupling effect began to be theoretically approached in search for a better design of es-SIS (externally shunted - SIS) array [17], but their harmonically approximated solutions were not complete without a stability check, which is important to determine whether the coupling behaves constructively or destructively.

In the present study, we focus on complete harmonic approximation of coupled self-generations in Josephson arrays for programmable Josepshon voltage standards (PJVS), i.e., Superconductor-Insulator -Normal metal-Insulator-Superconductor (SINIS) array, externally shunted Superconductor-Insulator-Supercondctor (es-SIS) array, Superconductor-Normal metal-Superconductor (SNS) array.

II. Mathematical Model

Fig. 1 shows an equivalent circuit for Josephson junction in an array. The coupling current I_{couple} from a Josephson junction embedded in a microwave transmission line is an ac component dumped into the transmission line out of the generated current, $I_g = I_c \sin\phi$ which is given by $I_{\text{couple}} = \tilde{V}_g/(2Z_0)$, where $\tilde{V}_g$ is ac component of self-generated voltage V_g given

$$i_{\text{couple}} = \tilde{v}_g / 2z_0 \tag{1}$$

$$\beta \dot{v}_g + v_g = \sin\phi \tag{2}$$

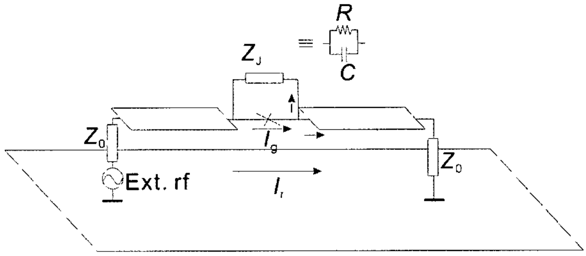

Fig. 1. Equivalent circuit for Josephson junction in an array as an rf generator and microwave transmission line, where self-generated rf current is divided into a coupling current and an internal junction current. For clarity only one junction is shown.

by equation $CdV_g/dt+V_g/R = I_c \sin\phi$ using Stewart-McCumber model [18,19]. These equations can be rewritten with dimensionless parameters $i = I/I_c$, $\tau = 4\pi e I_c R t/h = \omega_c t$, $vv = V/(I_c R)$, $z_0 = Z_0/R$ as followings

Here β is McCumber parameter $\omega_c RC$, z_0 is the transmission line impedance normalized to junction resistance R, I_c is junction critical current, and ϕ is phase difference between two superconductors of the Josephson junction. For simplicity, we assumed that both ends of the transmission line are ideally terminated, and only the ac component can propagate through the transmission line because typical Josephson array has no dc current path between two electrodes of the transmission line. When the junction impedance is larger and thus better matched to the transmission line impedance, the microwave coupling between junctions is enhanced as one can notice from the equations (1) and (2). But the enhanced coupling does not always mean constructive coupling, but whether it is constructive or not depends on phase relationships between rf currents given by dc and rf bias conditions and junction parameters. We will consider equally spaced identical Josephson junctions series array, the equation of k-th junction (k=0,1,2,..., N-1) out of N junctions is governed by the dc bias and external rf current as well as sum of all the coupling currents from the other N-1 junctions. The master equation including the effects of coupling and transmission line attenuation can be written as

$$\beta\ddot{\phi}_k + \dot{\phi}_k + \sin\phi_k = i_b + i_r e^{-\gamma k}\sin(\Omega\tau - \Gamma k) + (1/2z_0)\sum_{m\neq k} e^{-\gamma|m-k|}\tilde{i}_{couple}\Big|_{\tau\to\tau-\omega_c|m-k|l/c}$$

$$(\Gamma = \omega_c \Omega l/c) \tag{3}$$

Here γ is the transmission line attenuation parameter per junction, Ω is the reduced external microwave frequency given by $2\pi f/\omega_c$, Γ is the spatial phase increment per junction, l is the distance between junctions, c is the phase velocity of microwave in the transmission line, i_b and i_r are, respectively, the dc and rf bias current normalized to I_c. The tilde mark means ac component as before. Solution of equations (1)~(3) will give the effective microwave current distribution and at the same time junction phase motion with time. Although fully analytical calculation of these equations is not possible, harmonic approximation of the equations under assumption of zero attenuation (γ=0) is possible. The zero attenuation means zero or very small coupling effect, because both the coupling and attenuation are enhanced by the better impedance matching. The approximation will allow physical insight into the complicated coupling effects.

III. Calculations of Coupled Self-Generations

3.1 SINIS array

The very small coupling effect means large impedance mismatch (z_0>>1). In the approximation, using complex number notation $j=\sqrt{-1}$, we can assume the self-generated current of the m-th junction is generated in response to only external rf current of $i_r e^{j(\Omega\tau-\Gamma m)}$. It is divided into coupling current and junction current with ratio of junction impedance $R(1+j\beta\Omega)$ to the transmission line impedance Z_0 so that the coupling current can be approximately put as $\hat{i}_g e^{-j\Gamma m}/2z_0(1+j\beta\Omega)$, where i_g is equivalent to the self-generated current of the first junction (k=0), and the over-hat represents harmonic component. Therefore the effective rf current $i_{e,k}$, at the k-th junction can be approximately rewritten with $\theta = \arctan(\beta\Omega)$

$$i^*_{e,k} = i_r e^{-j\Gamma k} + \frac{e^{-j\theta}}{2z_0\sqrt{1+(\beta\Omega)^2}}\sum_{m\neq k}\hat{i}^*_g e^{-j\Gamma m}e^{-j\Gamma|m-k|} \tag{4}$$

Here the asterisk denotes complex variable. The summation can be divided into two parts: $m < k$ where the coupling current propagates to the left, and $m > k$ where it propagates to the right, to obtain the following expression.

$$
\begin{aligned}
i^{*}_{\mathrm{e},k} &= i_{\mathrm{r}} e^{-j\Gamma k} + \frac{e^{-j\theta}}{2z_0\sqrt{1+(\beta\Omega)^2}} \left\{ \sum_{m<k} \hat{i}_{\mathrm{g}}^{*} e^{-j\Gamma k} + \sum_{m>k} \hat{i}_{\mathrm{g}}^{*} e^{j\Gamma k} e^{-j2\Gamma m} \right\} \\
&= i_{\mathrm{r}} e^{-j\Gamma k} + \frac{e^{-j\theta}}{2z_0\sqrt{1+(\beta\Omega)^2}} \hat{i}_{\mathrm{g}}^{*} \left\{ k e^{-j\Gamma k} + e^{-j\Gamma N} \frac{\sin\Gamma(N-k-1)}{\sin\Gamma} \right\}
\end{aligned} \tag{5}
$$

where the exp($-j\Gamma k$) terms represent traveling waves propagating right, and the other term represents standing wave from the left-traveling coupling current combined with right-traveling external rf current. As external microwave current, at the first junction, is given as i_{r} sin($\Omega\tau$), the voltage can be put as $i_{\mathrm{b}} + \left\{ i_{\mathrm{r}} \big/ \sqrt{1+(\beta\Omega)^2} \right\} \sin(\Omega\tau - \theta)$. Then in the bias region around the 1st Shapiro step, the junction phase is given by

$$
\begin{aligned}
\phi &= i_{\mathrm{b}}\tau - \left\{ i_{\mathrm{r}} \big/ \Omega\sqrt{1+(\beta\Omega)^2} \right\} \cos(\Omega\tau-\theta) - \phi_{\mathrm{c}} \\
&= \Omega\tau - v_{\mathrm{r}} \cos(\Omega\tau-\theta) - \phi_{\mathrm{c}}
\end{aligned} \tag{6}
$$

Here, v_{r} is normalized rf voltage given by $i_{\mathrm{r}} \big/ \Omega\sqrt{1+(\beta\Omega)^2}$, and ϕ_{c} is a constant of integration. Using Fourier-Bessel series, the self-generated current $i_{\mathrm{g}} = \sin\phi$ can be written with harmonics of frequency Ω,

$$
i_{\mathrm{g}} = \sum_{n=-\infty}^{\infty} J_n(v_r) \sin\left((n+1)\Omega\tau - n(\pi/2+\theta) - \phi_{\mathrm{c}}\right) \tag{7}
$$

The constant ϕ_{c} can be determined from dc component of equation (6) for n = -1, which leads to $-J_1\cos(\theta-\phi_{\mathrm{c}}) = \Delta i_{\mathrm{b}}$, where Δi_{b} is the dc bias deviation from step center. We should note the solution for $\theta-\phi_{\mathrm{c}}$ is multi-valued with two kinds, i.e. $\theta-\phi_{\mathrm{c}} = \alpha$ or $\theta-\phi_{\mathrm{c}} = -\alpha$, if we define the $\alpha = \cos^{-1}(-\Delta i_{\mathrm{b}}/J_1)$, which lies between 0 and π. According to Kautz [14] motion of ϕ can be stable only when time average of cos ϕ is positive, and we can show that this condition is satisfied with $\theta-\phi_{\mathrm{c}} = \alpha$ when $J_1 > 0$, and with $\theta-\phi_{\mathrm{c}} = -\alpha$ when $J_1 < 0$. Therefore the harmonic component of i_{g} can be written as

$$
\begin{aligned}
\hat{i}_{\mathrm{g}} &= J_0\sin(\Omega\tau-\theta+\alpha) + J_2\sin(\Omega\tau-\theta-\alpha), \quad (J_1>0) \\
&= J_0\sin(\Omega\tau-\theta-\alpha) + J_2\sin(\Omega\tau-\theta+\alpha), \quad (J_1<0)
\end{aligned} \tag{8}
$$

Again with the complex number notation, substitution of Equation (8) into Equation (5) gives the effective current

$$
\begin{aligned}
i^{*}_{\mathrm{e},k} e^{j\Gamma k} &= i_{\mathrm{r}} + \frac{k}{2z_0\sqrt{1+(\beta\Omega)^2}} \left(J_0 e^{j(-2\theta+\alpha)} + J_2 e^{j(-2\theta-\alpha)} \right), \quad (J_1>0) \\
&= i_{\mathrm{r}} + \frac{k}{2z_0\sqrt{1+(\beta\Omega)^2}} \left(J_0 e^{j(-2\theta-\alpha)} + J_2 e^{j(-2\theta+\alpha)} \right), \quad (J_1<0)
\end{aligned} \tag{9}
$$

Here we dropped off the standing wave term because it is just oscillatory contribution that is of little interest. Whether the effective rf is constructive or destructive is mainly determined by the phase relationship between the external rf current and the coupling current. Fig. 2 is the calculated intensity graph showing the degree of coupling in the i_{b}-i_{r} plane where the degree of coupling was defined as the difference in effective rf currents between the first and the last junctions in the SINIS array. In the calculation of the graph, we included also the standing wave term, which was dropped off at the above equation. The degree of coupling shows a periodic behavior following the 1st order Bessel function as rf current is increased. We are mostly interested in the first period at the low rf current region, because practically available external rf sources have limited output power. In SINIS array, coupling is expected to be constructive in the region

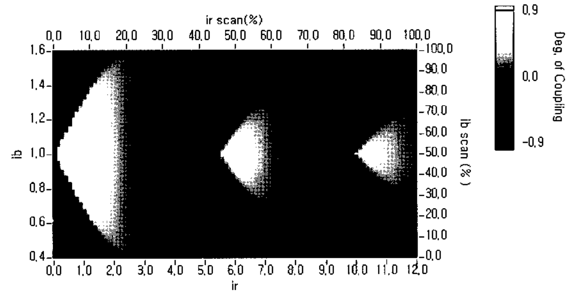

Fig. 2. Intensity graph of degree of coupling for 128 SINIS junctions array. β=1, Ω=1, z_0=50.

where rf voltage level is low, and destructive elsewhere.

3.2 SNS array

The harmonic approximation approach for SNS junction array is almost the same as the SINIS array except that the β should be replaced by zero in the above equations, because an SNS junction has no capacitance. This allows us to rewrite Equation (9) as following,

$$\begin{aligned} i^{*}_{\mathrm{c},k}e^{j\Gamma k} &= i_{\mathrm{r}} + \frac{k}{2z_0}\left(J_0 e^{j\alpha} + J_2 e^{-j\alpha}\right), \quad (J_1 > 0) \\ &= i_{\mathrm{r}} + \frac{k}{2z_0}\left(J_0 e^{-j\alpha} + J_2 e^{j\alpha}\right), \quad (J_1 < 0) \end{aligned} \tag{10}$$

Fig. 3 is the calculated intensity graph showing the degree of coupling for the SNS array. The coupling is expected to be constructive in the region where dc bias current is low, destructive in the region where dc bias is high at the rf current maximizing the 1st Shapiro step (i_{r}=1.85), and somewhat neutral elsewhere.

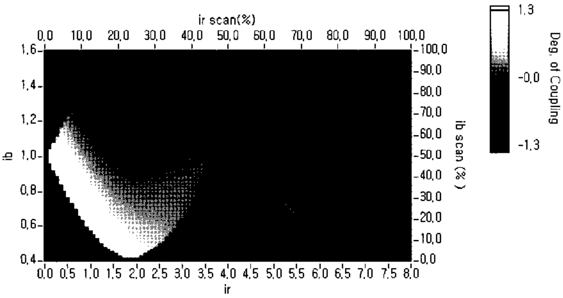

Fig. 3. Intensity graph showing the degree of coupling for 128 SNS junctions array. Ω=1, z_0=50.

3.3 Es-SIS array

In case of es-SIS junction, we can replace the junction resistance R of Z_J in Fig. 1 with $R_{\mathrm{s}}+j\omega L$, the effective impedance for a resistor–inductor series circuit. It is because the external shunt contains non-negligible inductance and the internal junction resistance R is large compared to the external shunt resistance R_{s}. For a typical es-SIS driven by about 70 GHz, most of the rf current flows through junction capacitance because of large impedance of the external shunt. In another words, the self-generated rf current see as if the junction has only capacitive internal impedance. It is divided into coupling current and junction current with ratio of junction impedance $R(j\beta\Omega)$ to the transmission line impedance Z_0 so that the coupling current can be approximately put as $\hat{i}_{\mathrm{g}} e^{-j\Gamma m}/2z_0(j\beta\Omega)$, which leads to the modified form of Equation (9);

$$\begin{aligned} i^{*}_{\mathrm{c},k}e^{j\Gamma k} &= i_{\mathrm{r}} + \frac{k}{2z_0(\beta\Omega)}\left(J_0 e^{j(-\pi+\alpha)} + J_2 e^{j(-\pi-\alpha)}\right), \quad (J_1 > 0) \\ &= i_{\mathrm{r}} + \frac{k}{2z_0(\beta\Omega)}\left(J_0 e^{j(-\pi-\alpha)} + J_2 e^{j(-\pi+\alpha)}\right), \quad (J_1 < 0) \end{aligned} \tag{11}$$

Fig. 4 is the calculated intensity graph showing the degree of coupling for the SNS array. The coupling is expected to be constructive in the region where dc bias current is high, destructive in the region where dc bias is low at the rf current maximizing the 1st Shapiro step (i_{r}=1.85) and somewhat neutral elsewhere. This is a mirror image to the graph for the SNS array with respect to the axis of i_{b}=1.

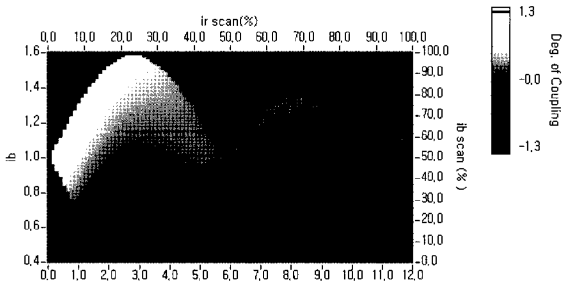

Fig. 4. Intensity graph showing the degree of coupling for 128 es-SIS junctions array. β=1, Ω=1, z_0=50.

IV. Conclusion

It should be noted that the harmonic approximation approach is limitedly applicable to a special case of very low coupling and very low attenuation. In the general cases of non-negligible attenuation, we need to consider the situation where all the external rf power is dissipated within the front part of the array and the self-generated waves interactively propagates to the rest of the array. To investigate the coupling effect, we also developed a virtual model of Josephson array (to be published) based and the results of simulation for low coupling using the virtual model which is in good agreement with the harmonic approximation of SINIS array. It proves that the harmonic approximation is a reasonable approach to estimate the coupling of the nonlinear oscillations of Josephson junctions in low coupling limit.

Although the harmonic approach has the limitations, it provided us an intuitive insight to the complicated microwave phenomena in the Josepshon arrays, helping us to design improved PJVS arrays where the microwave property is important.

References

[1] J. Niemeyer, L. Grimm, W. Meier, J. H. Hinken, E .Vollmer, Appl. Phys. Lett., 47, 1222 (1985).

[2] C. A. Hamilton, R. L. Kautz, et al., IEEE Trans. Instrum. Meas. 36, 258 (1987).

[3] Y. Sakamoto, H. Yoshida, T. Sakuraba, A. Odawara, Y. Murayama, T. Endo, IEEE Trans. Instrum. Meas. 40, 312 (1991).

[4] S. I. Park, K.-T. Kim, Y. H. Lee, R. D. Lee, IEEE Trans. Instrum. Meas. 42, 588 (1993).

[5] M. T. Levinson, R. Y. Chiao, M. J. Feldman, B. A. Tucker, Appl. Phys. Lett. 31, 776 (1977).

[6] M. Gurvitch, M. A. Washington, H. A. Huggins, Appl. Phys. Lett., 42, 472 (1983).

[7] C. A. Hamilton, C. J. Burroughs, and R. L. Kautz, IEEE Trans. Instrum. Meas. 44, 223 (1995).

[8] K. –T. Kim, H. K. Hong, J. Kim, K. W. Lee, Y. S. Song, IEEE Trans. Appl. Supercond., 11, 271 (2001).

[9] Y. Chong, C. J. Burroughs, P. D. Dresselhaus, N. Hadacek, H. Yamamori, and S. P. Benz, IEEE Trans. Instrum. Meas. 54, 616 (2005).

[10] H. Schulze, R. Behr, F. Mueller, and J. Niemeyer, Appl. Phys. Lett. 73, 996 (1998).

[11] K. -T. Kim, R. Behr, F. Mueller, Superconductor Science and Technology, 16, 1344 (2003).

[12] M. Ishizaki, H. Yamamori, A. Shoji, and P. D. Dresselhaus, and S. P. Benz, IEEE Trans. Instrum. Meas. 54, 620 (2005).

[13] S. P. Benz and C. A. Hamilton, Appl. Phys. Lett. 68, 3171 (1996).

[14] S. P. Benz, J. M. Martinis, P. D. Dresselhaus, and S.W. Nam, IEEE Trans. Instrum. Meas. 52, 545 (2003).

[15] R. Behr, H. Schulze, F. Mueller, J. Kohlman, J. Niemeyer, I. Y. Krasnopolin, International Superconductive Electronics Conference (ISEC'99), Berkeley, California, Extended Abstracts, 125 (1999).

[16] F. Mueller, H. Schulze, R. Behr, O. Kieler, B. Egeling, J. Kohlman, M. Khabipov, B. Balashov, and I. Y. Krasnopolin, International Superconductive Electronics Conference (ISEC'2001), Queenstown, New Zealand, Extended Abstracts, 225 (2001).

[17] J. Hassel, P. Helisto, L. Groenberg, H. Seppa, J. Nissila, and A. Kemppien, IEEE Trans. Instrum. Meas. 54, 632 (2005).

[18] R. L. Kautz, J. Appl. Phys. 52, 3528 (1981).

[19] K.-T. Kim and J. Niemeyer, Appl. Phys. Lett. 66, 2567 (1995).